Axial Impregnation of a Fiber Bundle.
Part 2: Theoretical Analysis

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This paper considers the numerical simulation of the axial capillary infiltration of an aligned fiber bed and the assessment of the input permeability and capillary pressure data by validating the simulation results with experimental data from Part 1 (1) of this study. Theoretical predictions of capillary pressure are close to the capillary pressure value determined from a long-term experiment and are used as input data in the infiltration simulations. The Carman-Kozeny equation is recommended for the prediction of permeability and the validation of the numerical simulations with experimental data for two infiltrating liquids, silicone oil and epoxy, yields recommended values for the empirical Kozeny constant for different ranges of fiber volume fraction.

1. INTRODUCTION

Part 1 of this study (1) includes experimental studies of the axial capillary impregnation of E-glass fiber bundles. The overall aim of this study is to determine the permeability and capillary pressure of fiber bundles of different porosities. Then, the values of these properties could be used to describe the axial infiltration of a unidirectional fiber bed or the axial impregnation of a fiber bundle, the latter being a basic unit in models of fabrics (2).

The determination of both permeability and capillary pressure of aligned fiber beds is based on the modeling of such flow as a combination of Poiseuille flows through parallel capillaries (3) of a mean radius equal to the hydraulic radius of the bed, where the latter is given as the ratio of the cross-section pore area normal to the flow direction divided by the perimeter of the fibers presented to the infiltrating fluid. Carman-Kozeny equation (4, 5) is based on this principle as expressed by

\[ \kappa = \frac{D_f^3 (1 - V_f)^2}{16K_o V_f^2} \]  

(1)

for unidirectional fiber beds, where \( \kappa \) is the permeability, \( D_f \) is the average fiber diameter, \( V_f \) is the fiber volume fraction of fiber bed (where \( V_f = 1 - \varepsilon \), and \( \varepsilon \) is the porosity) and \( K_o \) is the Kozeny constant considering tortuosity of flow paths and pore non-uniformity.

It is almost impossible to predict accurately the value of Kozeny constant theoretically in composites manufacturing, since aligned fiber beds do not form exact, known arrangements. Theoretical studies have been carried out for square and hexagonal arrays of solid cylinders (6-9, among others) using analytical or numerical methods to solve the flow problem. Gebart (8) estimated that in axial infiltration \( K_o = 1.78 \) for a square fiber arrangement and \( K_o = 1.66 \) for a hexagonal fiber arrangement. Choi et al. (9) found that the theoretical value of \( K_o \) for square packing was comparable to the values of \( K_o \) for hexagonal packing (\( K_o = 10 \) for \( V_f = 0.1 \)). At high \( V_f \), the predicted \( K_o \) values were lower for both types of fiber packing and there were considerable differences between the corresponding \( K_o \) values for the different types of packing (At \( V_f = 0.7, K_o = 1.5 \) for square packing and \( K_o = 4 \) for hexagonal packing). Skartsis et al. (10) presented data of experimental and theoretical results for axial flow through arrays of aligned cylinders (square and triangular arrays) where \( K_o \) is shown to vary in the range of 0.8 (\( V_f = 0.8 \)) to 6 (\( V_f = 0.2 \)).

Batch et al. (11) found that longitudinal flow through arrays of fibers really resulted in lower \( K_o \) values than the theoretical predictions because of increased permeability resulting from non-uniform fiber spacing. They determined an experimental value of \( K_o = 1.06 \) for axial infiltration of aligned glass fiber beds. Williams et al. (3) proposed \( K_o = 0.1-0.8 \) (\( V_f = 0.2-0.65 \)) for different fiber beds where \( K_o \) decreased with the fiber volume fraction. Gutowski et al. (12) found experimentally \( K_o = 0.7 \) for axial flow through a bed of aligned graphite fibers for \( V_f = 0.4-0.8 \), whereas Lam
and Kardos (13) determined experimentally $K_0 = 0.35-0.68$ (depending on the infiltrating liquid) for axial flow through a bed of aligned graphite fibers for $V_f = 0.57-0.75$. The dependence of $K_0$ on the infiltrating liquid in experimental studies may be attributed to the fact that wetting effects were not generally taken into account in the calculations for the evaluation of $K_0$ from the experimental data.

Equation 1 is generally also used for flow transverse to the fiber direction (13, 14) with a different value for the Kozeny constant so that the permeability along the fiber direction is about 19 times greater than the permeability transverse to the fiber direction. Lindsay (15) suggested that $K_0$ usually depends on the porosity of porous media and he reported values of $K_0 = 5.55$ for fibrous media and $K_0 = 3-7$ for porosities less than 0.8. Values of $K_0$ in the range of 7 to 180 have been reported for random glass fiber mats (16, 17).

The purpose of this study is to carry out an analysis of the axial infiltration of an aligned fiber bed taking into account the wetting effects in the form of capillary pressure. Following an iterative procedure, the permeabilities and capillary pressures will be determined for the determination of the permeability and capillary pressure in the axial infiltration of the fiber bundle. The values of $K_0$ in the Kozeny constant.

### 2. NUMERICAL MODELING OF THE AXIAL CAPILLARY INFILTRATION OF A FIBER BUNDLE

In the model, the fiber bundle was considered as a tow of unidirectional fibers and a relation based on Darcy's law was used to describe the vertical capillary flow through the bundle micropores

$$\frac{dh}{dt} = \frac{\kappa}{\mu \epsilon} \left[ \rho \frac{P_c - \rho gh}{h} \right]$$  \hspace{1cm} (2)

where $h$ is the flow height, $\mu$ is the fluid viscosity, $\rho$ is the density of the fluid, $g$ is the acceleration of gravity and $P_c$ is the capillary pressure. The micro-pores were assumed to be uniform or with an averaged pore size distribution, and no specific modeling was carried out on fiber twist in the bundle and any defects at the fiber surface. Equation 2 applies to Newtonian fluids. The value of permeability was estimated according to Eq 1 as a function of $V_f$ with $D_f$ being the diameter of the fiber filament.

As the infiltration starts, the weight of the column of liquid in the bundle acts against the capillary pressure, being responsible for slowing down the rate of impregnation, $dh/dt$. The theoretical value for the capillary pressure was estimated from the Young-Laplace equation as suggested (3, 18) for axial capillary flow through a bed of aligned fibers:

$$P_c = \frac{F}{D_f (1 - V_f)} \alpha \cos \theta$$  \hspace{1cm} (3)

where $\sigma$ is the surface tension of the liquid and $\theta$ is the contact angle at the liquid/solid interface. A form factor $F = 4$ was considered for axial impregnation of the fiber bundle, as derived from modeling the capillary flow parallel to fibers (3).

The finite difference method was used for the numerical solution of Eq 2 where the fluid advanced in length steps during the corresponding time steps. A numerical method was preferred to the analytical solution since the former could input (at the right time step) measured changes in the viscosity during a very long capillary experiment, if the viscosity changed as a result of temperature changes, for example. Figure 1 displays a flowchart of the numerical procedure as it was implemented in a FORTRAN 77 computer program. At every new step, the time was increased by a constant value and the length step was evaluated on the basis of an initial estimate of velocity. Then the velocity was re-evaluated according to the latest evaluated infiltration length and Eq 2, and this was the basis of an iterative numerical procedure to calculate the correct infiltration length. The end of the iteration was controlled by error monitoring, i.e. error smaller than a given tolerance for the iteration to end; otherwise the infiltration length was re-evaluated until numerical convergence.

This program allows for different Newtonian fluids by inputting the density and viscosity values. Other input data include the porosity of the fiber bundle or fiber bed and the diameter of the fiber filament. The numerical time step used in the computer simulations was usually of 1 second. However, if the final equilibrium position of the flow is required, this time step may be increased after long infiltration times so that less computational effort is used.

### 3. NUMERICAL RESULTS AND DISCUSSION

The aim of the computer simulations of capillary flow through a single bundle was (a) to use the output results of flow height obtained from the program to build graphs that would simulate the infiltration process and (b) to determine the permeability and capillary pressure of the fiber bundle. The values of $P_c$ and $\kappa$ were determined in a trial-and-error procedure as follows: A pair of $P_c$ and $\kappa$ values were guessed and given as input values to run the computer program. The success of the guess was assessed mainly by comparing the predicted curve of flow height as a function of time to the corresponding experimental data (1).

Figures 2 and 3 present a sensitivity analysis on the $P_c$ and $\kappa$ inputted values for the vertical capillary flow of silicone oil through an E-glass fiber bundle with a porosity $\epsilon = 0.50$. Figure 2 illustrates the influence of different inputted permeability values, namely 0.5, 1.0 and $2.0 \times 10^{-12}$ m², on the capillary flow curve. As expected, an increase in permeability is responsible for an increase in the rate of the flow height rise and the
Fig. 1. Flowchart of the computer program for the simulation of the vertical capillary infiltration of a fiber bundle or a bed of aligned fibers.

Fig. 2. Vertical axial capillary impregnation of a single fiber bundle by silicone oil. Predicted curves for different permeability values: $P_r = 5000$ Pa for all curves.
capillary flow takes less time to approach equilibrium. It is important to mention that the three curves presented in Fig. 2 eventually reach the same equilibrium height ($h_e = 0.596$ m), which is dependent only on the $P_c$ value. Likewise, the influence of changing the capillary pressure on the final curve can also be simulated, as is shown in Fig. 3. As expected, if the permeability is kept constant, an increase in $P_c$ also shifts the flow height curve upwards, reaching different equilibrium positions, according to the $P_c$ value.

Therefore, the flow curves for the bundle infiltration predicted from the computer simulations have to be regarded as a result of the combination of these two input parameters, $P_c$ and $\kappa$. Subsequent plots of predicted flow curves to be shown in this section will be compared with experimental data that cover only the duration of the experiment, as presented in Part 1 (1), but may be still far from the equilibrium position.

### 3.1 Silicone Oil

For all simulated flow runs for silicone oil and epoxy, whenever the pair of input values of $\kappa$ and $P_c$ had been obtained from the integral fitting procedure of the experimental data of the capillary experiments (1), the modeled curve satisfactorily represented the data for the duration of the experiment (19). For example, Fig. 4 shows the predicted flow curves for different pairs of inputted values for capillary pressure and permeability for experiment 8 along with the experimental data. As expected, the predicted curve from the simulation run which used input values $P_c = 259$ Pa and $\kappa = 194 \times 10^{-12}$ m$^2$ from Table 3 in Part 1 of this study (1) shows a fair representation of the experimental points.

If purely theoretical predictions are made the Carman-Kozeny equation yields $\kappa = 366 \times 10^{-12}$ m$^2$ (assuming $K_o = 0.3$) and Eq 3 yields $P_c = 1727$ Pa. If this pair of input values for $\kappa$ and $P_c$ are used in the infiltration simulations, the output flow height curve in Fig. 4 highly overestimates the experimental results. This is attributed to the fact that insufficient time has been allowed for the experiment, misleading the experimental estimations of $P_c$ and $\kappa$ (11, 20).

Equation 3 requires the determination of $\sigma$, $\theta$, $R_o$, $\varepsilon$ and $F$. Considering that experimental data exist for the four first variables (1) and $F$ has been derived from a theoretical analysis, the theoretical $P_c$ value according to Eq 3 might not be far from right (within the experimental error in the measurement of the properties and parameters required in Eq 3). Bearing that in mind, new curves can be generated in Fig. 4 for $P_c = 1727$ Pa until the experimental data is best matched, which happens for a permeability value around $\kappa = 18 \times 10^{-12}$ m$^2$.

Alternatively, the theoretical determination of $\kappa$ from the Carman-Kozeny equation requires the knowledge of $V_p$, $R_o$ and $K_o$ and, due to the large uncertainties regarding $K_o$, the determination of $\kappa$ is subject to questioning. Nevertheless, if $\kappa$ is considered to have been correctly estimated from Eq 1 ($\kappa = 366 \times 10^{-12}$ m$^2$ with a $K_o = 0.3$), a $P_c$ of about 190 Pa could be estimated to best fit the experimental data. This value of $P_c$ is rather low, lower even than the value estimated from the integral fitting procedure of the experimental data, which are known to be far from equilibrium, and, therefore, the experimental $P_c$ value has already been considered underestimated (1).

Subsequent case studies corresponding to infiltration experiments of Part 1 of this work (1) will use the
same procedure of providing input values for \( P_c \) and \( \kappa \) although only the flow curve predicted with input values from the integral fitting of experimental data and the predicted flow curve related to the theoretical \( P_c \) value (predicted from Eq 3) will be shown. Figures 5–7 display these curves and the experimental data from the capillary experiments 11, 13 and 14, respectively.

Similar conclusions can be drawn from the predictions related to these experiments as for experiment 8.

Figure 5 displays the experimental data from experiment 11 (1), the curve generated with input data \( P_c = 355 \text{ Pa} \) and \( \kappa = 79 \times 10^{-12} \text{ m}^2 \) from the integral fitting of the experimental data and the curve generated from the pair of input data \( P_c = 5117 \text{ Pa} \), derived from Eq 3.
Fig. 6. Vertical axial capillary impregnation of a single fiber bundle by silicone oil. Predicted curves for different pairs of input values of $P_c$ and $k$. Experimental data from capillary experiment 13 (1).

and $k = 2.8 \times 10^{-12}$ m$^2$, where this value of permeability resulted to the curve best approximating the experimental data.

Figure 6 displays the experimental data from experiment 13 (1), the curve generated with input data $P_c = 305$ Pa and $k = 88 \times 10^{-12}$ m$^2$ from the integral fitting of the experimental data and the curve generated from the pair of input data $P_c = 10,597$ Pa, derived from Eq. 3, and $k = 1.4 \times 10^{-12}$ m$^2$, where this value of permeability resulted to the curve best approximating the experimental data.

Figure 7 displays the experimental data from experiment 14 (1), the curve generated with input data $P_c = 434$ Pa and $k = 58 \times 10^{-12}$ m$^2$ from the integral fitting of the experimental data and the curve generated from the pair of input data $P_c = 7978$ Pa, derived from
Table 1. Vertical Axial Capillary Impregnation of a Single Fiber Bundle by Silicone Oil. Summary of Findings From the Theoretical Analysis and Computer Simulations.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>ϵ</th>
<th>$P_c$ (Pa)</th>
<th>$κ$ ($\mu$m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.81</td>
<td>1727</td>
<td>18</td>
</tr>
<tr>
<td>11</td>
<td>0.59</td>
<td>5117</td>
<td>2.8</td>
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<td>14</td>
<td>0.48</td>
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</tr>
<tr>
<td>13</td>
<td>0.41</td>
<td>10597</td>
<td>1.4</td>
</tr>
</tbody>
</table>

* Estimated from Eq 3.

from Eq 3, and $κ = 1.9 \times 10^{-12}$ m$^2$, where this value of permeability resulted to the curve best approximating the experimental data.

Table 1 summarizes the results obtained from the simulation runs for the axial bundle infiltration by silicone oil corresponding to experiments 8, 11, 13 and 14 of Part 1 of this study (1), where the $P_c$ values were derived from Eq 3 ($F = 4$) and the $κ$ values were selected such that the flow curves fit the experimental data. The cases are presented in order of falling values of bundle porosity resulting in a consistent fall of permeability values and corresponding increase of capillary pressure values. Examination of the corresponding Table 3 in Part 1 of this study leads to the conclusion that the integral fitting of experimental data resulted in misleadingly low $P_c$ and high $κ$ values, because of the insufficient duration of the experiments. Closer examination of the data for experiments 11, 13 and 14 in Table 3 of Part 1 also reveals that the expected trend of decreasing permeability and increasing capillary pressure with decreasing bundle porosity was not followed in the values determined from the integral fitting of the available experimental data.

### 3.2 Epoxy Resin

The same procedure was followed in the simulations of fiber bundle infiltration by resin as for the silicone oil. The simulated flow curves and the corresponding experimental data for experiment 19 are shown in Fig. 8. The curve obtained using input data $P_c = 226$ Pa and $κ = 177 \times 10^{-12}$ m$^2$ obtained from the integral fitting of experimental data shows a good representation of the experimental data. If the theoretical $P_c$ value derived from Eq 3 is used instead ($P_c = 3035$ Pa), $κ$ would be about $8 \times 10^{-12}$ m$^2$ for the flow curve to best fit the experimental data. Figure 9 is related to experiment 20 and shows the flow curve generated from $P_c = 260$ Pa and $κ = 141 \times 10^{-12}$ m$^2$ (obtained from the integral fitting of experimental data) and the flow curve generated from $P_c = 3199$ Pa (Eq 3) and $κ = 7.2 \times 10^{-12}$ m$^2$. Figure 10 is related to experiment 22 and shows the flow curve generated from $P_c = 203$ Pa and $κ = 193 \times 10^{-12}$ m$^2$ (obtained from the integral fitting of experimental data) and the flow curve generated from $P_c = 2875$ Pa (Eq 3) and $κ = 8.3 \times 10^{-12}$ m$^2$.

Figure 11 is related to the long-run experiment (1). The curve generated from input data $P_c = 9600$ Pa and $κ = 1.18 \times 10^{-12}$ m$^2$ from the integral fitting of the experimental data showed good agreement with the experimental data, as expected. Equation 1 yielded a theoretical value of $P_c = 10,267$ Pa and the corresponding flow curve close to the experimental data was generated with an input value of $κ = 1.1 \times 10^{-12}$ m$^2$. This pair of input values was close to the pair of values from the integral fitting of experimental data, which was attributed to the fact that the long duration of this experiment yielded a capillary pressure.
Fig. 9. Vertical axial capillary impregnation of a single fiber bundle by epoxy. Predicted curves for different pairs of inputted values of $P_c$ and $\kappa$. Experimental data from capillary experiment 20 (1).

value close to the theoretical estimate. Actually, the corresponding predicted flow curves are virtually coincident. If, on the other hand, the pair of input data are $P_c = 10,267$ Pa (from Eq 3) and $\kappa = 9.2 \times 10^{-12}$ m$^2$, estimated from the Carman-Kozeny equation (Eq 1) using $K_e = 0.3$, the resulting flow curve is clearly well above the experimental data. Besides, by using $\kappa = 9.2 \times 10^{-12}$, it was not possible to find an appropriate value of $P_c$ to fit the flow curve to the experimental data. (For example, see the curve for $P_c = 2100$ Pa and $\kappa = 9.2 \times 10^{-12}$ m$^2$ in Fig. 11.)

This long-run experiment provided trustworthy data that clearly show that the Kozeny constant should be different from 0.3. The theoretical $P_c$ value estimated

Fig. 10. Vertical axial capillary impregnation of a single fiber bundle by epoxy. Predicted curves for different pairs of inputted values of $P_c$ and $\kappa$. Experimental data from capillary experiment 22 (1).
by Eq 3, with F = 4, proved to be not only experimentally achievable for the long-run experiment, but also responsible for producing simulated curves with good agreement with the experimental data.

Table 2 summarizes the results obtained from the simulation runs of the fiber bundle infiltration by epoxy. Similar values of bundle porosity yielded similar values of capillary pressure and permeability, and lower values of porosity resulted in higher capillary pressure and lower permeability, as expected.

### 3.3 Determination of Kozeny Constant

Since the results for the flow of silicone oil and epoxy covered a range of fiber bundle porosities, the permeability values from the infiltration simulations presented in Tables 1 and 2 were plotted against \((1-\text{V}_f)^3/\text{V}_f^2\) to check the validity of the Carman-Kozeny equation (Eq 1). The results are presented in Fig. 12, including both infiltrating liquids, silicone oil and epoxy. Our conclusion is that the permeability values are independent of the infiltrating liquid, as the wetting effects have been taken into account separately via the capillary pressure in Eq 2. It is also taken that the data follow the Carman-Kozeny equation, which is recommended for the prediction of the permeability of the fiber bundle or a bed of aligned fibers as a function of fiber volume fraction and diameter of fiber filament.

However, for more accurate prediction of permeability as a function of fiber bundle porosity, or fiber volume fraction, two lines were fitted through the data via linear regression for different \(V_f\) ranges. These lines yielded \(k_0 = 5.8\) for \(V_f = 0.2-0.3\) and \(k_0 = 1.6\) for \(V_f = 0.5-0.6\). The higher value of \(k_0\) at lower \(V_f\) values is consistent with the findings of other investigators, especially those derived from theoretical studies (9, 10).

### 4. CONCLUSIONS

Computer simulations were carried out regarding the axial capillary infiltration of an E-glass fiber bundle using two infiltrating liquids, silicone oil and epoxy. The experiments (1) were generally of rather short duration to result in accurate determination of capillary pressure and permeability according to integral fitting of experimental data. Hence, the purpose of this study was to investigate whether it is possible to determine \(P_c\) and \(\kappa\) on the basis of experimental data of relatively short duration and existing theory.

The procedure recommended from this study includes the following steps: (a) apply Eq 3, using \(F = 4\), to evaluate theoretically the capillary pressure; (b) use this theoretical value of \(P_c\) as input value in the flow Eq 2 to fit the predictions to the experimental data in a trial-and-error procedure which involves

### Table 2. Vertical Axial Capillary Impregnation of a Single Fiber Bundle by Epoxy. Summary of Findings from the Theoretical Analysis and Computer Simulations.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>(\varepsilon)</th>
<th>(P_c) (Pa)*</th>
<th>(\kappa) ((\mu m^2))</th>
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</thead>
<tbody>
<tr>
<td>22</td>
<td>0.76</td>
<td>2875</td>
<td>8.3</td>
</tr>
<tr>
<td>19</td>
<td>0.75</td>
<td>3035</td>
<td>8.0</td>
</tr>
<tr>
<td>20</td>
<td>0.74</td>
<td>3199</td>
<td>7.2</td>
</tr>
<tr>
<td>Long-run</td>
<td>0.47</td>
<td>10,267</td>
<td>1.1</td>
</tr>
</tbody>
</table>

* Estimated from Eq 3.
Axial Impregnation of a Fiber Bundle. Part 2

![Graph](image)

**Fig. 12.** Plot to check the validity of Carman-Kozeny equation (1) and determine $K_p$. The points correspond to permeability values used as input data in infiltration simulations to fit the experimental flow data, where the input value for capillary pressure was predicted theoretically from Eq 3, using $F = 4$.

the input of different values of permeability; (c) the appropriate permeability value is the input value that results in the best fitting of experimental data. The practice proved very good, especially since the theoretical value of $P_c$ was close to the experimentally determined value for the long-term capillary experiment of 23 days with epoxy as the infiltrating liquid, which was closer to equilibrium.

It was not possible to apply the Carman-Kozeny equation (Eq 1) at the beginning of the procedure above because the correct value of $K_p$ was not known. The final task was then to determine the value of Kozeny constant, $K_p$, from the values of permeabilities determined according to the described procedure, given that the experimental data (1) covered a wide range of bundle fibre volume fractions. The experimental data from the flow experiments of both silicone oil and epoxy were combined on the same graph, which showed that it is possible that the data follow the Carman-Kozeny relation. Two different values for the Kozeny constant were determined in two $V_f$ ranges: $K_p = 5.8$ for $V_f = 0.2-0.3$ and $K_p = 1.6$ for $V_f = 0.5-0.6$.

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