SPREADSHEETS FOR THE CALCULATION OF THERMAL COMFORT INDICES PMV AND PPD

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Abstract. A set of spreadsheets developed in Microsoft Excel to calculate the thermal comfort indices, using the Fanger’s method proposed in ISO Standard 7730 is presented. The calculation method is based upon the determination, through an iterative process, of the clothing external temperature, being the PMV index calculated from a human body thermal balance equation where internal heat generation and heat exchanges with the surrounding environment are considered. The main objective of the work was to develop user-friendly simple software tools for the calculation of thermal comfort indices. Thus, different spreadsheets were prepared allowing the calculation with environmental data measured with different sets of sensors.

Keywords: PMV and PPD indices, Thermal Comfort, Software

1. INTRODUCTION

The improvement of the quality of life in modern societies and the awareness of public opinion regarding the non-inexhaustibility of fossil energy resources have led to a substantial increase of the attention devoted to the study of indoor environments, particularly regarding thermal concepts.

The application of international standards relating to environmental quality of indoor environments in which it is intended to obtain comfortable conditions, namely thermal, has often led to the need to calculate the PMV and PPD indices. This work, presents a set of worksheets, developed in Excel, with application to situations where the goal is to determine, from experimental measurements, the values of PMV and PPD.
2. CALCULATION METHOD

The method developed by Fanger (1972) and adapted in ISO Standard 7730 is based on the determination of the PMV index (Predicted Mean Vote) calculated from an equation of thermal balance for the human body (see Figure 1), involving the terms of internal generation and heat exchanges with the surrounding environment.

![Human Body Thermal Balance Diagram](image)

\[ S = M - W \pm R \pm C \pm K - E \pm \text{Res} \]

Figure 1 – Human Body Thermal Balance

The value of the thermal comfort index PMV, which is an estimate of the expected average vote of a panel of evaluators for a given thermal environment, is calculated by the method developed by Fanger (1972). He established a model of correlation between the subjective human perception, expressed through the vote of comfort on a scale ranging from -3 (very cold) to +3 (very hot), and the difference between the heat generated and the heat released by the human body, which corresponds to the following equation:
\[ PMV = (0.303e^{-2.100\cdot M} + 0.028) \cdot (M-W) - H - E_c - C_{res} - E_{res} \] 

(1)

where the different terms represent, respectively:

- \( M \) - the metabolic rate, in Watt per square meter (W/m²);
- \( W \) - the effective mechanical power, in Watt per square meter (W/m²);
- \( H \) - the sensitive heat losses;
- \( E_c \) - the heat exchange by evaporation on the skin;
- \( C_{res} \) - heat exchange by convection in breathing;
- \( E_{res} \) - the evaporative heat exchange in breathing.

In equation 1, the terms \( H, E_c, C_{res}, \text{ and } H_{res} \), correspond to the heat exchange between the body and the surrounding environment and are calculated from the following equations:

\[ H = 3.96 \times 10^{-8} \cdot I_{cl} \cdot [(t_{cl} + 273)^4 - (t_a + 273)^4] - f_{cl} \cdot h_c \cdot (t_{cl} - t_a) \] 

(2)

\[ E_c = 3.05 \times 10^{-3} \cdot [5733 - 6.99 \cdot (M-W) - p_a] - 0.42 \cdot [(M-W) - 58.15] \] 

(3)

\[ C_{res} = 0.0014 \cdot M \cdot (34 - t_a) \] 

(4)

\[ E_{res} = 1.7 \times 10^{-5} \cdot M \cdot (5867 - p_a) \] 

(5)

where:

- \( I_{cl} \) is the clothing insulation, in square meters Kelvin per watt (m² K/W);
- \( f_{cl} \) is the clothing surface area factor;
- \( t_a \) is the air temperature, in degrees Celsius (°C);
- \( t_r \) is the mean radiant temperature, in degrees Celsius (°C);
- \( v_{ar} \) is the relative air velocity, in meters per second (m/s);
- \( p_a \) is the water vapor partial pressure, in Pascal (Pa);
- \( t_{cl} \) is the clothing surface temperature, in degrees Celsius (°C).

The main problem in implementing the calculation method results from the fact that the term corresponding to the external temperature of clothing \( t_{cl} \) is unknown, a priori. This temperature must be determined by an iterative process, from an equation resulting
from a heat balance established for the clothing layer. It is considered that, in a steady-state regime, the heat flux transmitted by conduction through that same clothing from the inner layer, at skin temperature, until the outer layer, is equal to the sum of the heat exchange by convection and by radiation with the surrounding environment (see Figure 2), which is expressed by the following equation:

\[
\frac{(t_{sk} - t_{cl})}{l_{cl}} = 3.96 \times 10^{-8} \cdot \frac{f_{cl} \cdot [((t_{cl}+273)^4 - (t_{eq}+273)^4) + f_{cl} \cdot h_{cl} \cdot (t_{cl} - t_{eq})]}{(t_{cl} + 273)^4 - (t_{eq} + 273)^4},
\]

(6)

from which we can express \( t_{cl} \):

\[
t_{cl} = t_{sk} - l_{cl} \cdot 3.96 \times 10^{-8} \cdot f_{cl} \cdot [((t_{cl}+273)^4 - (t_{eq}+273)^4) - l_{cl} \cdot f_{cl} \cdot h_{cl} \cdot (t_{cl} - t_{eq})],
\]

(7)

where \( t_{sk} \) is the skin external temperature, calculated from:

\[
t_{sk} = 35.7 - 0.028 \cdot (M-W).
\]

(8)

Figure 2 – Thermal balance of clothing layer, in a steady-state regime.
In the iterative method used, a first guessed value of \( t_{cl} \) is introduced in the second member of the equation (7), in order to start the iteration procedure. In every subsequent iteration, the guessed value for \( t_{cl} \) will be the average between the initial and final values of the previous iteration. The calculation process is assumed converged, when the iterative change of \( t_{cl} \) is small enough (e.g., less than 0.001 °C).

In the method implemented in this work, the first guessed value for \( t_{cl} \) was taken by averaging the air temperature \( t_{a} \) with the temperature of the skin \( t_{sk} \). Once the value of \( t_{cl} \) is determined, the calculation of the PMV index is immediate, by applying the formulas 1 to 5; the values of the convection coefficient \( h_{c} \) and of the relation between the clothed and nude area \( f_{cl} \) were determined through the expressions presented for the purpose in ISO Standard 7730. The other index proposed in ISO Standard 7730 is PPD (Predicted Percentage of Dissatisfied) that quantifies the expected percentage of dissatisfied people in a given thermal environment.

Fanger concluded in his studies that the variation of PMV index can be approximated by an analytic expression that corresponds to a curve whose appearance is similar to an inverted Gaussian distribution (see figure 3), thus:

\[
PPD = 100 - 95 \cdot e^{-(0.03353 \cdot PMV^4 + 0.2179 \cdot PMV^2)} 
\]  

Figure 3 – Index PPD and PMV variation

Thermal comfort zones (A, B and C classes) are defined in by the ranges of PMV values from -0.2 to 0.2, -0.5 to 0.5 and -0.7 to 0.7, which correspond respectively to PPD values below 6, 10 and 15%. The analysis of Figure 3 allows to conclude that due to individual differences between people, even for the situation that is averagely considered by the population as thermal neutrality (PMV=0), the percentage of dissatisfied is 5%.
3. SPREADSHEETS

The main purpose of this work is to provide a set of simple tools for the calculation of thermal comfort indices PMV and PPD, in order to be able to apply it in an expedite way. Therefore, different worksheets were prepared, which apply accordingly to the set of sensors available and used to assess thermal environment.

3.1 – First version

The first spreadsheet directly applies the method recommended by ISO Standard 7730, determining the comfort indices based on the following input data related to environmental conditions - air temperature, mean radiant temperature, air velocity and partial pressure of vapour. In the graphical interface of the spreadsheet (see Figure 4) and in addition to the comfort indices, the values of the various heat fluxes in certain intermediate calculations are presented, which allows developing the physical sensitivity of the user to this topic.

The spreadsheet, of which a graphical interface for a given instance is presented in Figure 4, is divided into three sections: the first, located on the left side, is used for data input by the operator and the initiation and control of the calculation process; the second, in the middle, displays the results of intermediate calculations required to implement the method and, finally, the third section, located on the right side of the screen, where the output values of the two comfort indices, PMV and PPD, are presented.

For each situation, the user has to fill in the seven cells in the field of data entry at the top left corner, which are the only cells throughout the spreadsheet that are possible to alter, and then click on the button named "Run", which starts the calculation process that lasts less than one second.

On the left section, there is a field which checks the iteration convergence, where the value shown for the iterative variation of $t_{cl}$ must be equal to 0 when the calculation stops. The application range of Fanger’s method must be taken into account, which is designed to calculate the area of thermal comfort. To ensure the calculation convergence, the input data should be included within the limits of applicability defined in ISO Standard 7730:

- Metabolic level – $M$, from 46 W/m² to 232 W/m² (0,8 met to 4 met);
- Clothing insulation- $l_{cl}$, from 0 m² °C/W to 0,310 m² °C/W (0 clo to 2 clo);
- Air temperature– $t_{a}$, from 10 °C to 30 °C;
• Mean radiant temperature - $t_r$, from 10 °C to 40 °C;
• Relative air velocity – $v_{ar}$, from 0 m/s to 1 m/s;
• Water vapour partial pressure – $p_a$, from 0 Pa to 2700 Pa.

The spreadsheet also provides, as output, a graphical representation of the relative weights, in percentage, of the different parts of heat exchange fluxes between the individual and the surrounding environment. Figure 5 shows the graph obtained for the situation corresponding to the data in Figure 4. The case whose results are presented in Figures 4 and 5 corresponds to a situation considered comfortable. The results obtained may be used for a short summary of Fanger’s method bases.

The value of PMV is calculated from the heat balance of the human body, in this case -2.70 W/m², obtained by the difference between the heat produced [(MW) = 69.8 W/m²] and the sum of the exchanges with the surrounding environment (Q = 72.48 W/m²). Since the difference is negative, the body is in a net loss of heat, which means that the simulated person would feel cold, even though slightly.

![Figure 4 – Graphical Interface of basic spreadsheet](image-url)
Figure 5 – Breakdown of heat and mass fluxes for the situation presented in figure 4.

The calculated value of $-2.70 \text{ W/m}^2$ for the global heat balance represents the quantity within brackets in the second member of equation 1, used for PMV calculation. The transformation of the thermal balance value in the PMV index is made by multiplying the term inside the first brackets of the second member in equation 1, which corresponds to the human thermal perception modulation, when it is expressed in the so-called seven point Gagge scale (from -3, very cold, to +3, very hot).

In some situations it may happen that some of the terms of the heat flux exchanged by the human body with the environment, through the available transfer mechanisms, appear with negative sign. These situations may occur when the air temperature and/or the mean radiant temperature are higher than the external temperature of clothing, which means that the convection and radiation phenomena can represent heat gains rather than losses. In these cases, the graphical representation used in Figure 5 is not appropriate, thus, in the spreadsheet, there is an alternative representation. In the following figures, Figure 6 and Figure 7, the calculation for a discomfort situation, by excess of heat, and the respective breakdown of heat fluxes are presented. Analyzing the results presented in both figures, it appears that the overall balance value is positive and relatively high, $41.69 \text{ W/m}^2$, thus people would feel a warm sensation. Also, as the radiant temperature of the simulated situation is very high, 39 ºC, the radiation phenomenon, instead of representing a loss, represents a heat gain to the human body, hence the negative sign and backslash in the representation of Figure 7.
Figure 6 – Calculation carried out for a warm uncomfortable situation

Figure 7 – Terms used in human body thermal balance for the situation depicted in figure 6
3.2 – Additional versions

The widespread implementation of the evaluation parameters of thermal comfort in work environments has been hampered by the relatively high price of equipment dedicated exclusively to this type of testing. There are basically two possible ways to determine the values of PMV and PPD:

- With indoor climate stations that in addition to the sensors used for measuring the four environmental variables (t\text{ar}, t, V\text{ar} and p\text{a}), must also be capable of data processing in order to apply the iterative method described above, taking into account the metabolic rate and the clothing worn by the simulated person;

- From heated transducers which simulate the thermal behaviour of the human body. In this type of thermal comfort meters, developed by Madsen (1971), a winding of nickel wire ensures the heating of an ellipsoid body and provides an even surface temperature equal to the outer layer of clothing of the simulated person. This surface temperature value is calculated from the metabolism level data and the clothing insulation, being the comfort indices value derived from the electrical power which, at each moment, is supplied to the electrical resistance formed by the mentioned winding of nickel wire.

![Figure 8 – Graphical interface of first additional spreadsheet](image-url)
Despite the benefits known to be associated with the improvement of thermal conditions in the workplace in terms of workers’ health, or even in terms of their productivity, analysis of this kind have not been required by the authorities as often as it happens for other stimuli of discomfort. In order to enable the use of cheaper measurement equipment for the evaluation of indoor thermal environments the author developed four additional spreadsheets.

Apart from the basic spreadsheet already presented, some more versions were developed to ensure determination of PMV and PPD indices in situations where measurements were made with sets of sensors that, even though don’t measure directly all the four environmental parameters (\(t_a\), \(t_r\), \(p_a\) and \(v_w\)), allow their calculation.

The first of the additional spreadsheets allows the determination of PMV and PPD, if air temperature, mean radiant temperature, relative humidity and airspeed have been measured. The water vapour partial pressure \(p_a\) is calculated from the equations of the psychrometric chart, after calculating the partial vapour pressure for the saturated air situation, \(p_s\):

\[
p_s = e^{\left(\frac{-16.6536 \times 4030.183}{T_a + 235} \right)}
\]

\[
p_a = 1000 \times \frac{RH}{100} \times P_s
\]

ii) The second of the additional spreadsheets allows the determination of PMV and PPD from air temperature, globe temperature, relative humidity and airspeed values. As for the mean radiant temperature, it is calculated from an equation of thermal balance established for the blackbody globe, in which it is considered that, in steady-state regime, the heat flux by convection between the globe and the air and the heat flux by radiation between the globe and the surrounding areas cancel out:

\[
\varepsilon \cdot \sigma (T_r^4 - T_g^4) + h_{cg} (T_a - T_g) = 0
\]

where:

- \(\sigma\) – Stephan-Boltzman constant (W·m\(^{-2}\)·K\(^{-4}\))
- \(\varepsilon\) - emissivity (dimensionless)
- \(T_i\) – absolute temperatures (ºK)
Convection heat transfer coefficient of the globe (W·m\(^{-2}\)·°K)

Explaining the value of mean radiant temperature, \(t_r(°C)\), it comes:

\[
\bar{t}_r = 4\sqrt{(t_g + 273)^4 + \frac{h_{cg}}{e \cdot \varepsilon} (t_g - t_a)} - 273
\]

(iii) Another possibility to obtain the four environmental parameters is to use a combination of sensors where the thermo-hygrometer used in the previous situation is replaced by a psychrometer, which is a combination of two thermometers: one dry bulb and one wet bulb, mounted on a rotating support. The dry bulb thermometer allows the measurement of air temperature. The wet bulb thermometer provides a value where the air temperature is affected by the cooling effect caused by the evaporation of the wet sleeve which surrounds the sensing element. From the values of the two thermometers, using the psychrometric chart equations, it is possible to determine the water vapour partial pressure. For the measurement with a psychrometer, the user must ensure that the wet bulb thermometer sleeve is wet and that the psychrometer spins for a period of time with the sensor element rotating at a speed higher than 3 m/s, relatively to the still air.

Another possibility is to have a low speed anemometer and a WBGT meter, which incorporates the globe, the dry bulb and wet bulb thermometers. In this case, since the wet bulb temperature is measured without forced air circulation around the temperature sensor wrapped in the wet sleeve, that temperature is named of natural wet bulb temperature. The difference between the wet bulb temperature \(t_{wb}\), usually measured by a psycho meter and used on humid air diagrams and the natural wet bulb temperature, \(t_{nwb}\), is given by Malchaire (1976), based on studies carried out by Romero (1971) for situations where the air velocity is below 0.15 m/s:

\[
t_{nwb} - t_{wb} = \frac{0.16 \cdot (t_g - t_a) + 0.8}{200} \cdot (560 - 2 \cdot HR - 5 \cdot t_a) - 0.8
\]
iv) The fourth additional spreadsheet was developed for this situation, adding to the functionalities described for the previous spreadsheet, the possibility of correcting the natural wet bulb temperature, in order to be able to use the equations of the humid air diagram determining the water vapour partial pressure.

4. CONCLUSION

This set of simple software applications, while making easier the calculation processes by the technicians involved in the evaluation of thermal conditions in workplaces and allowing the conduction of tests with affordable sensors, they are also a valuable tool for training and teaching on thermal comfort. They allow distinguishing and evaluating the relative weights of the various concepts involved in the thermal balance of the human body in a given situation. The ability to quickly simulate different scenarios allows developing by the students the physical sense associated with the phenomena under concern.

REFERENCES


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