Op-Amp Design Overview

• Operational Amplifier
  – Stability
  – Compensation
  – Miller Effect
  – Phase Margin
  – Unity Gain Frequency
  – Slew Rate Limiting

• Reading:
Two-stage op-amp

VDD = +5V

All P: \( \frac{900}{10} \)

All N: \( \frac{350}{10} \)

Vi1

M1

M2

Vi2

V GS1

M6

M7

M8

V GS6

50\(\mu\)A

VSS = -5V

Vout
Analysis Strategy

- Recognize sub-blocks
- Represent as cascade of simple stages
Total op-amp model

Input differential pair    Common source stage
## DC operating point

<table>
<thead>
<tr>
<th>MOSFET</th>
<th>$I_D[\mu A]$</th>
<th>$V_{eff}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>25</td>
<td>0.235</td>
</tr>
<tr>
<td>M2</td>
<td>25</td>
<td>0.235</td>
</tr>
<tr>
<td>M3</td>
<td>25</td>
<td>0.247</td>
</tr>
<tr>
<td>M4</td>
<td>25</td>
<td>0.247</td>
</tr>
<tr>
<td>M5</td>
<td>50</td>
<td>0.350</td>
</tr>
<tr>
<td>M6</td>
<td>50</td>
<td>0.332</td>
</tr>
<tr>
<td>M7</td>
<td>50</td>
<td>0.332</td>
</tr>
<tr>
<td>M8</td>
<td>50</td>
<td>0.332</td>
</tr>
</tbody>
</table>
### Small signal parameters

<table>
<thead>
<tr>
<th>MOSFET</th>
<th>$I_D[\mu A]$</th>
<th>$V_{eff}$</th>
<th>$g_m[\mu A/V]$</th>
<th>$r_{ds}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>25</td>
<td>0.235</td>
<td>208</td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>25</td>
<td>0.235</td>
<td></td>
<td>800kΩ</td>
</tr>
<tr>
<td>M3</td>
<td>25</td>
<td>0.247</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>25</td>
<td>0.247</td>
<td></td>
<td>1.43MΩ</td>
</tr>
<tr>
<td>M5</td>
<td>50</td>
<td>0.350</td>
<td>285</td>
<td>715kΩ</td>
</tr>
<tr>
<td>M6</td>
<td>50</td>
<td>0.332</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M7</td>
<td>50</td>
<td>0.332</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M8</td>
<td>50</td>
<td>0.332</td>
<td></td>
<td>400kΩ</td>
</tr>
</tbody>
</table>

Note: Channel length modulation parameters

\[
\lambda_n = 0.050 \text{ V}^{-1} ; \quad \lambda_p = 0.028 \text{ V}^{-1}
\]
Total op-amp model: Low frequency gain

Input differential pair
\[ a_{v1} = g_m \left( r_{ds2} \parallel r_{ds4} \right) \]
\[ a_{v1} = (208 \mu A/V)(800k\Omega \parallel 1.43M\Omega) \]
\[ a_{v1} = 106 \]

Common source stage
\[ a_{v2} = g_m \left( r_{ds5} \parallel r_{ds8} \right) \]
\[ a_{v2} = (285 \mu A/V)(400k\Omega \parallel 715k\Omega) \]
\[ a_{v2} = 73 \]
Total op-amp model with capacitances

Gate of M5  Load: scope probe ≈ 10 pF

\[ C_g = (900 \mu m)(10 \mu m) \left( 4.17E - 4 \frac{F}{m^2} \right) \]

\[ C_g = 3.74 \text{ pF} \]
Total op-amp model with capacitances

First stage pole

\[ f_{p1} = \frac{1}{2\pi (r_{ds2} || r_{ds4}) C_{g5}} \]

\[ f_{p1} = \frac{1}{2\pi (800k\Omega || 1.43M\Omega)(3.74pF)} \]

\[ f_{p1} = 82kHz \]

Second stage pole

\[ f_{p1} = \frac{1}{2\pi (r_{ds5} || r_{ds8}) C_L} \]

\[ f_{p1} = \frac{1}{2\pi (400k\Omega || 715k\Omega)(10pF)} \]

\[ f_{p1} = 61kHz \]
Open loop transfer function

- **Product of individual stage transfer functions**

\[
A(j\omega) = \frac{g_{m1}(r_{ds2}\|r_{ds4})g_{m5}(r_{ds5}\|r_{ds8})}{1 + j\omega 2\pi(r_{ds2}\|r_{ds4})C_{g5}} \frac{1}{1 + j\omega 2\pi(r_{ds5}\|r_{ds8})C_{L}}
\]
Two-stage op-amp: Simulation Schematic
DC Operating Point Simulation

DC Response

OP POINT 2.885 mV

A: (2.11783 m -3.81929)  delta: (1.39772 m 7.02858)
B: (3.51555 m 3.20929)  slope: 5.02681 K
Bode plot (single-pole term)

- Magnitude, phase on log scales
- Pole: Root of denominator polynomial
Open loop Bode plot

- Product of terms: Sum on log-log plot
Open Loop Bode Plot Simulation

A: (15.592m 36.0451m)  delta: (255.918K -195.979)
B: (15.8489m -198.942)  slope: -766.698u
Stability example: Closed loop follower

- Negative feedback:
  Output connected to inverting input
Unity gain: Why bother?

• No buffer:  
Voltage divider
• Signal reduced due to voltage drop across $R_S$

\[ v_{out} = \left( \frac{R_L}{R_L + R_S} \right) v_{in} \]

• With buffer: 
No current required from source

\[ v_{out} = v_{in} \]
Problem: Instability

• Oscillation superimposed on desired output
• Output for zero input
• Why? Need...
Controls: ES3011 in 20 minutes

- General framework
  A: Forward Gain
  $\beta$: Feedback Factor
  fraction of output fed back to input
Example: Op-amp, Noninverting Gain

A: Forward Gain
Op-amp open loop gain
\[ V_{\text{out}} = A(V_+ - V_-) \]
Transfer function \( A(j\omega) \)

\[ \beta: \text{Feedback Factor} \]
\[ \beta = \frac{R_1}{R_1 + R_2} \]
Closed Loop Gain

- **Output**
  \[ v_{out} = A(v_{in} - \beta v_{out}) \]
  \[ v_{+} - v_{-} \]

- **Solve for \( v_{out}/v_{in} \)**
  \[ v_{out} = Av_{in} - A\beta v_{out} \]
  \[(1 + A\beta)v_{out} = Av_{in} \]

\[ \frac{v_{out}}{v_{in}} = \frac{A}{1 + A\beta} \]
Op-amp with negative feedback

• If $A\beta \gg 1$

\[
\frac{v_{out}}{v_{in}} = \frac{A}{1 + A\beta} \approx \frac{A}{A\beta} \quad \Rightarrow \quad \frac{v_{out}}{v_{in}} \approx \frac{1}{\beta}
\]

• Closed loop gain determined only by $\beta$

• Advantage of negative feedback:
  Open loop gain $A$ can be ugly (nonlinear, poorly controlled) as long as it's large!
Example: Op-amp, Noninverting Gain

\[ \beta: \text{Feedback Factor} \]

\[ \beta = \frac{R_1}{R_1 + R_2} \]

Closed loop gain

\[ \frac{v_{out}}{v_{in}} = \frac{R_1 + R_2}{R_1} = \frac{1}{\beta} \]
Reexamine closed loop transfer function

- Output with no input: infinite gain
- Infinite when $1 + A\beta = 0$
- Condition for oscillation:
  $$1 + A\beta = 0$$
- In general $A, \beta$ functions of $\omega$
- If there's a frequency $\omega$ at which $1 + A\beta = 0$: Oscillation at that frequency!
**Example: follower**

\[ \beta = 1 \quad \rightarrow \quad \frac{v_{out}}{v_{in}} = \frac{A}{1 + A} \]

- Use \( A(j\omega) \), solve for \( 1 + A = 0 \)
- No thanks!

\[
A(j\omega) = \frac{g_{m1}(r_{ds2}\parallel r_{ds4})g_{m5}(r_{ds5}\parallel r_{ds8})}{\left[1 + j\omega 2\pi (r_{ds2}\parallel r_{ds4})C_{g5}\right] \left[1 + j\omega 2\pi (r_{ds5}\parallel r_{ds8})C_{L}\right]}
\]
Reexamine condition for oscillation

\[1 + A\beta = 0 \rightarrow A\beta = -1\]

Magnitude and phase condition:

\[|A\beta| = 1 \quad \text{AND} \quad \angle A\beta = -180^\circ\]

- Easier to get from Bode plot
Look at original $A\beta$ for 2 stage op-amp

- Find $\omega$ at which $|A\beta| = 1$; Check $\angle A\beta = -180^\circ$?

Trouble!
Simulation $A\beta$ for 2 stage op-amp

AC Response

$A$: $(15.592M, 38.0461m)$
$B$: $(15.8489M, -196.942)$
Delta: $(256.918K, -196.979)$
Slope: $-766.698\mu$
Compensation: “Dominant Pole”

- Move one pole to lower frequency
- How?
Compensation: “Dominant Pole”

- Need to increase capacitance by $\approx 1000X$:
  BAD! Die area cost
Miller Effect

- Impedance across inverting gain stage $G$
- Reduced by factor equal to $(1+G)$
Math for Miller effect

\[ i_x = \frac{v_x - (-Gv_x)}{Z} \]

\[ i_x = \frac{v_x (1 + G)}{Z} \]

\[ \frac{v_x}{i_x} = Z_{in} = \frac{Z}{(1 + G)} \]

- Impedance across inverting gain stage G
- Reduced by factor equal to (1+G)
Example: Impedance is capacitive

- Capacitance multiplied by \((1+G)\)

\[
Z_{in} = \frac{Z}{1 + G}
\]

\[
Z = \frac{1}{sC} \quad \rightarrow \quad Z_{in} = \frac{1}{s(1 + G)C_{eq}}
\]

- Equivalent capacitance higher by factor \(1+G\)
- Problem for high bandwidth amplifiers
- Opportunity for compensation ...
Miller Compensation

• Need effect of large capacitance
• Use Miller effect to multiply small on-chip capacitance to higher effective value
• Effect of large capacitance without die area cost of large capacitance
New schematic

- Add $C_C$ across 2nd stage
New transfer function

AC Response

\[ \text{dB20}(\text{VF}(''/\text{vout}'')) \]

\[ \text{phase}(\text{VF}(''/\text{vout}'')) \]

freq (Hz)

A: (64.359K 3.34525)  
delta: (-875.82 -128.519)  
B: (83.4832K -125.174)  
slope: 148.741m
New step response

• No oscillation!
"Phase margin"

- How stable is new transfer function?
- Phase margin = Phase lag at $|A\beta| = 1$ minus (-180°)
Dominant pole op-amp model

- Simpler model with dominant pole from $C_C$
Approximate dominant pole transfer function

\[ A(j\omega) \approx \frac{g_{m1} \left( r_{ds2} \| r_{ds4} \right) A_2}{1 + j\omega \left( r_{ds2} \| r_{ds4} \right) A_2 C_C} \]

\[ A_2 = g_m5 \left( r_{ds5} \| r_{ds8} \right) \]
Unity gain frequency

- Depends only on
  - Input stage transconductance \( g_m \)
  - Compensation capacitor \( C_C \)

\[
|A(j\omega)| \approx \frac{g_{m1}(r_{ds2}||r_{ds4})A_2}{\omega(r_{ds2}||r_{ds4})A_2C_C}
\]

\[
|A(j\omega)| = 1 \text{ at } \omega_T
\]

\[
\omega_T \approx \frac{g_m1}{C_C}
\]
Slew rate

- \( I = C \frac{dV}{dt} \)
- Only limited current \( I_{\text{BIAS}} \) available to charge, discharge \( C_C \)
• $I = C \frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{I_{BIAS}}{C_C}$
Summary Op-amp:

- Stability
- Compensation
- Miller effect
- Phase Margin
- Unity gain frequency
- Slew Rate Limiting