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Development of new equation to estimate the maximum soil depth by using the safety factor

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ABSTRACT: Soil depth is an important parameter which is hard to be measured and estimated in hydrogeomorphic and pedological studies. Furthermore, the soil depth is one of the most unknown variables in the hillslope system. Therefore, the objective of the present study was to develop a new equation to estimate the maximum soil depth by using the factor of safety. For this development, a uniform hydrological model was combined to an infinite slope model. The new equation is called MEMPS (*Modelo de Estimativa da Máxima Profundidade do Solo*). Differently from other equations to calculate maximum soil depth, MEMPS does not require the assumptions of complete saturation or total absence of water in the soil. Thus, MEMPS represents a condition closer to the reality when compared to other equations. Moreover, due to the incorporation of hydrogeomorphic parameters in its formulation, MEMPS acquires fitness to reflect the influence of hydrogeomorphic processes on the soil depth estimation.

1 INTRODUCTION

Soil depth, defined here as the vertical distance between the soil surface and the bedrock, is an important parameter which is hard to be measured and estimated in hydrogeomorphic and pedological studies. Furthermore, the soil depth is one of the most unknown variables in the hillslope system (Catani et al., 2010). That is why there are various attempts to estimate the soil depth. These attempts can be classified into two: field methods (Piegari et al., 2009, Bery, 2013) and mathematical methods (Saulnier et al., 1997, Segoni et al., 2012). Each attempt has its own good and bad points.

Borga et al. (2002) performed a sensitivity analysis of a slope stability model and demonstrated that depth variation can significantly change the slope stability results. It means that the successful mapping of landslide hazard areas depends upon the correct determination of the spatial distribution of soil depth in a basin.

As the soil depth measurement in a whole basin is quite difficult and time-consuming (Dietrich et al., 1995), it is usually required to use some mathematical formulations which estimate the soil depth distribution.

The process-based methods are those that use physically-based equations to describe various mechanisms affecting the soil formation and evolution. Though generating satisfactory results, they require that the physical processes acting in the study area are well known and that input parameters data are accurate. In this context, the objective of the present study was to develop a new equation to estimate the maximum soil depth by using the Factor of Safety (*FS*). This new equation resulted from the combination of an infinite slope stability model with a uniform hydrological model. The present study shows the equation development and comments its limitation use.

2 THEORETICAL DEVELOPMENT OF MEMPS

According to Selby (1993), the value of *FS* can be formulated:

$$FS = \frac{c + (\rho_s \cdot g \cdot z \cdot \cos^2\theta - g \cdot \rho_w \cdot h \cdot \cos^2\theta) \cdot \tan\phi}{\rho_s \cdot g \cdot z \cdot \sin\theta \cdot \cos\theta} \tag{1}$$

where FS = factor of safety; c = soil cohesion; ρ_w = water density; z = soil depth; h = height of the water column inside the soil layer; ρ_s = soil density; g = gravitational acceleration; θ = slope angle; and ϕ = soil internal friction angle (Figure 1).

Iida (1999) exercised a limit equilibrium analysis for an infinite slope model, and defined the critical

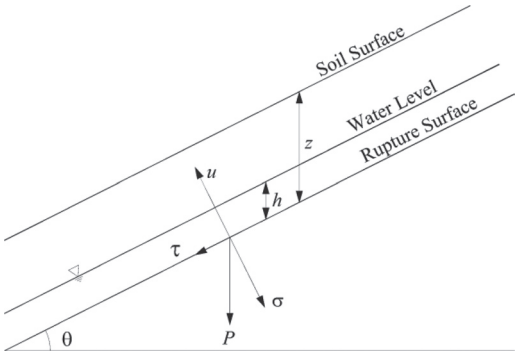

Figure 1. Representation of an infinite slope model. P = weight of the soil; τ = shear stress; σ = normal stress; u = pore-pressure. (From Michel et al., 2014).

soil depth as the maximum value of the depth that can settle on a hillslope, by considering that the *FS* value at a particular location does not reach lower values than 1.

In the case of water absence, where a soil layer is perfectly dry, the critical depth (z_{c0}) was described by:

$$z_{c0} = \frac{c}{\rho_s \cdot g \cdot \cos^2\theta \cdot (\tan\theta - \tan\phi)} \tag{2}$$

where z_{c0} = critical depth for dry soil. The z_{c0} value can be calculated, but it is never reached, since the soil depth slowly increases over time and periodic storms produce certain hillslope saturation (i.e. the assumption of absence of water is not satisfied), causing landslides. Therefore, Iida (1999) proposed, for the case that the soil is totally saturated, another equation to determine the critical depth (z_{c1}):

$$z_{c1} = \frac{c}{\cos^2\theta \cdot \left[\rho_s \cdot g \cdot (\tan\theta - \tan\phi) + \rho_w \cdot g \cdot \tan\phi\right]} \tag{3}$$

where z_{c1} = critical depth for the saturated soil. If the actual soil depth is smaller than z_{c1}, landslide will never occur even with very intensive rainfall. In this case, this location acquires immunity situation for a certain period. If the soil depth is greater than z_{c1}, the landslide can occur when the soil saturation reaches a threshold value. Iida (1999) demonstrated that his equations (Equations 2 and 3) had a good performance through a comparison with the values measured in the field.

As above-mentioned, Iida (1999) determined the maximum soil depth, by considering two extreme conditions of soil moisture: (i) the total absence of water in the soil column; and (ii) its complete saturation. In fact, both conditions hardly occur. The present study, therefore, extended his theory in order to treat the more real situation, by combining an infinite slope stability model to a uniform hydrological model.

Then, the present study modifies the Equation 1 and obtains:

$$FS = \frac{\left(\frac{c}{\rho_s \cdot g \cdot z \cdot \cos\theta}\right) + \cos\theta \cdot \left(1 - \frac{\rho_w}{\rho_s} \cdot \frac{h}{z}\right) \cdot \tan\phi}{\sin\theta} \tag{4}$$

The term h/z in Equation 4 represents the saturation degree of the hillslope soil. In a steady state hydrological model, which is commonly called TOPOG, O'Loughlin (1986) defined the wetness index (w) with the following equation:

$$w = \frac{h}{z} = \frac{q \cdot a}{b \cdot T \cdot \sin\theta} \tag{5}$$

where q = uniform steady state recharge; a = upstream basin area at one point; b = contour length; and T = soil transmissivity. Obviously, the value of w is limited to 1, and this condition means that the height of the water column inside the soil is equal to the soil depth. When the application of Equation 5 results in a value larger than one, is there overland flow. In Equation 5 saturated hydraulic conductivity (Ks) is considered constant throughout the soil profile, and then T can be calculated:

$$T = Ks \cdot z \cdot \cos\theta \tag{6}$$

By substituting the Equation 6 in the Equation 5, it is obtained:

$$\frac{h}{z} = \frac{q \cdot a}{b \cdot Ks \cdot z \cdot \cos\theta \cdot \sin\theta} \tag{7}$$

or

$$h = \frac{q \cdot a}{b \cdot Ks \cdot \cos\theta \cdot \sin\theta} \tag{8}$$

As the water balance of the soil layer is considered permanent, the Equation 7 is able to estimate an average behavior of the soil saturation level over time. The maximum value of h is equal to z. When the application of Equation 8 results in a h value larger than z, the soil is completely saturated and there is overland flow. Then, substituting the Equation 7 in the Equation 4, the following expression is obtained:

$$FS = \frac{\left(\frac{c}{\rho_s \cdot g \cdot z \cdot \cos\theta}\right) + \cos\theta \cdot \left(1 - \frac{\rho_w}{\rho_s} \cdot \frac{q \cdot a}{b \cdot Ks \cdot z \cdot \cos\theta \cdot \sin\theta}\right) \cdot \tan\phi}{\sin\theta} \tag{9}$$

The assumption, that the stability threshold in a hillslope occurs when *FS* is equal to 1, permits to estimate the maximum depth that the soil can achieve without causing instability, only based on soil strength characteristics, geomorphic characteristics of the basin and hydrological situations that take place. Hence, the Equation 9 can be modified as follows:

$$1 < \frac{\left(\frac{c}{\rho_s \cdot g \cdot z \cdot \cos\theta}\right) + \cos\theta \cdot \left(1 - \frac{\rho_w}{\rho_s} \cdot \frac{q \cdot a}{b \cdot Ks \cdot z \cdot \cos\theta \cdot \sin\theta}\right) \cdot \tan\phi}{\sin\theta} \tag{10}$$

Isolating the term z, the Equation 10 becomes:

$$z < \frac{\frac{c}{\rho_s \cdot g \cdot \cos\theta} - \frac{\rho_w}{\rho_s} \cdot \frac{q \cdot a \cdot \tan\phi}{b \cdot Ks \cdot \sin\theta}}{\sin\theta - \cos\theta \cdot \tan\phi} \tag{11}$$

The Equation 11 performs an estimate of the maximum depth reached by the soil under certain hydrological conditions. Then, by dividing the Equation 11 by $\cos\theta$ and treating the maximum soil depth, the value of the maximum soil depth (z_c) can be obtained:

$$z_c = \frac{\frac{c}{\rho_s \cdot g \cdot \cos^2\theta} - \tan\phi \cdot \frac{\rho_w}{\rho_s} \cdot \left(\frac{q \cdot a}{b \cdot K_s \cdot \sin\theta \cdot \cos\theta}\right)}{\tan\theta - \tan\phi} \tag{12}$$

The term in the brackets in the Equation 12 represents the water height of the soil layer (h) of the Equation 8. The maximum value of h must be equal or less than the value of z. Thus, it can be said that, under a threshold balance condition when h becomes equal to z, the critical depth becomes z_{c1} of the Equation 3. Then, the Equation 12 can be modified:

$$z_c = \frac{\frac{c}{\rho_s \cdot g \cdot \cos^2\theta} - \tan\phi \cdot \frac{\rho_w}{\rho_s} \cdot \min\left(z_{c1}, \frac{q \cdot a}{b \cdot K_s \cdot \sin\theta \cdot \cos\theta}\right)}{\tan\theta - \tan\phi} \tag{13}$$

The present study calls the Equation 13 *Modelo de Estimativa da Máxima Profundidade do Solo* (MEMPS) which means model to estimate the maximum soil depth, in Portuguese.

3 COMPARISON BETWEEN MEMPS AND IIDA (1999)

The MEMPS (Equation 13) has intermediate characteristics between two equations (Equations 2 and 3) of Iida (1999) which used an inversion of the *FS* equation to limit the maximum soil depth physically feasible on a slope. The difference between the equations of Iida (1999) and the MEMPS is graphically illustrated in Figure 2. For elaborating these graphs, the calculation was done under the following conditions: $c = 11.9$ kPa, $q = 0.005$ m/day, $a = 300$ m², $b = 5$ m, $\phi = 30.5°$, $Ks = 0.38$ m/day, $\rho_s = 1800$ kg/m³, and $\rho_w = 1000$ kg/m³. Considering that the aim of the present study is just to demonstrate the performance of Equation 13, and not

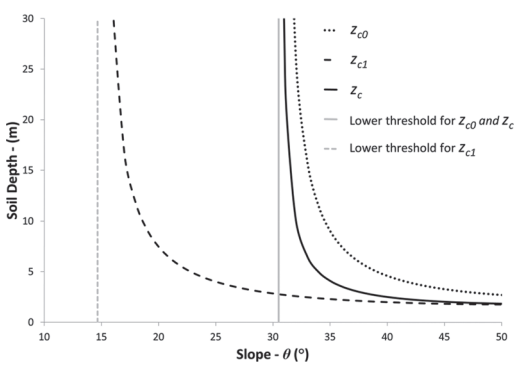

Figure 2. Variation of maximum soil depth values estimated by Iida's equations and MEMPS.

to modelling the actual maximum soil depth in a slope, some of these parameters were defined from Michel et al. (2014) and other ones were arbitrarily defined. The only care in the definition of these parameters was to select values of q, a, b and Ks that were not able to generate complete saturation in the slope.

It is clearly observed that the Equation 13 has its application limit. When the steady state recharge (q) used to define the soil depth distribution with MEMPS results in complete saturation of the soil, the results of MEMPS are equal to those with Equation 3. When the value of q is set to zero (i.e. there is no water in the basin) the results of MEMPS are equal to those with Equation 2. Thus, MEMPS can describe three different situations (absence of water, complete saturation and an intermediate condition) with only one equation. Because it, the performance of the model depends on the correct estimaton of q. Furthermore, this model is only applicable on slopes that are steeper than ϕ, i.e., $\theta > \phi$.

In Figure 2, it is observed that the wetness condition used by MEMPS is intermediate between complete saturation and total absence of water. For large values of soil depth, the wetness condition in MEMPS was more similar to absence of water. Instead, for small values of soil depth, the wetness condition in MEMPS was more similar to saturation.

The partially saturation condition in the zone above the water table was not took into account in this study. When analyzing the maximum stable soil depth, only the conditions developed in a possible rupture surface are relevant. In other words, only the conditions in the interface between soil and bedrock are taking into account, because this zone is the one more susceptible to failure.

In the application of MEMPS (Equation 13), it is necessary to set a uniform steady state recharge rate, in order to mimic a period of precipitation above the regular conditions, but without the capability to trigger landslides. The condition assumed generates a pattern of soil depth distribution in which the hillslope tends to its stability during normal or slightly more than normal rainfall seasons. When a heavy rainfall that is more than normal takes place, the soil gradually reduces its FS value and can reach unstable conditions.

Thus, for example, an initial condition of w related to the annual mean rainfall could generate a wet condition insufficient to cause the instability. When a heavy rainfall similar to that used to define the soil depth distribution occurs in the basin, the situation turns gradually from a stable condition to the threshold. If the rainfall persists and the wetness exceeds the condition used to define the soil depth, the situation evolves from limit equilibrium to unstable conditions.

The characteristic of MEMPS, in which initially there are stable conditions that gradually turn in to unstable conditions with the rainfall event, gives to this model the quality to be quite adequate to define initial conditions to slope stability modelling. Most of times, in the slope stability modelling, the initial conditions are of instability over a large part of the terrain (mainly in steep concave areas), even before the occurrence of the triggering event.

In regions where the landslides are triggered by hydrological factors, the initial conditions (before the rainfall or triggering event occurrence) of the terrain must be stable, what is achieved with application of MEMPS. Thus, with this more realistic scenario, the slope stability modelling would be able to describe the specific process involved in the landslide triggering.

Furthermore, in landslide warning systems, it is necessary to define the moment in which the terrain will turn from stable to unstable conditions. Thus, in these cases, an initial condition of stability is required. Therefore, MEMPS can be a useful tool for soil depth and landslide hazard mapping, and landslide warning systems.

4 FINAL CONSIDERATIONS

The present study developed a new equation, called MEMPS, to estimate the maximum soil depth through a combination of a steady state hydrological model with an infinite slope stability model. This equation does not need to have any of assumptions of Iida (1999), representing a condition closer to the reality. In future, it should be verified with field data.

Even if the MEMPS can be only used for areas where the slope angle is larger than the soil internal friction angle, i.e., mountainous basins, it is very useful. One of the goals of the development of this equation is to create more realistic initial scenarios for slope stability modelling. In these cases, MEMPS can be very useful both in to optimize landslide modelling and to improve landslide warning systems.

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