INFLATION, UNEMPLOYMENT AND MONETARY POLICY IN BRAZIL[1]

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Abstract

In this article we analyse the role of the Non-Accelerating Inflation Rate of Unemployment (NAIRU) in an inflation targeting model and also present some estimates of the NAIRU using Brazilian Data. We present a monetary policy model where the NAIRU gap plays a key role for inflation targeting, not because monetary authorities should close the gap every period, but because it helps predict future inflation. For the estimation of the NAIRU two different models are used. One is based on a transfer function estimation of a traditional Phillips curve approach. The second one is a signal extraction method where the NAIRU is the unobservable stochastic trend of the unemployment data. The NAIRU is estimated using both the IBGE and DIEESE data. The results show a linear Phillips curve for Brazil, and allow good estimates of the NAIRU. The estimations performed using quarterly data produced a time varying NAIRU. Our results are in line with the acceleration of inflation during the eighties and the desacceleration of inflation that follow the Real Plan.

Key word: NAIRU, Transfer Function Models, Kalman Filter, Inflation Targeting.

JEL Classification: C51/E52

1. Introduction

After almost three decades of high inflation rates and successive economic plans failures, the "Real Plan", implemented in 1994, brought down the inflation rate from almost 2,500% to just 2,48% per year. At the same time, the unemployment rate begun to increase to surprisingly high levels for the Brazilian standards. A relevant question in this framework is the estimation of an equilibrium unemployment rate.

The fact that the rising on unemployment can be originated from decreasing inflation is well known in economics since Phillips (1958) used English data to support an inverse relation between the rates of inflation and unemployment. In the present paper we will use the most recent concept of NAIRU (Non-Accelerating Inflation Rate of Unemployment). This concept was

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originated from Friedman's (1968) natural rate of unemployment and has been estimated for several countries [5].

In this paper we extend the results obtained in Portugal and Madalozzo (1998) by presenting some new estimates for the NAIRU for Brazil and developing a monetary policy model were the NAIRU gap is a useful construct for inflation targeting. The estimates are performed separately for the eighties and for the period of "Plano Real".

In the next section we will present a three equation model of inflation targeting and analyse the effects of uncertainty. In the next section the econometric models used to calculate the NAIRU for Brazil are presented. The results for quarterly data estimations are shown in the fourth section. Section five presents some conclusions and remarks.

2. On The Use of Short Run NAIRU for Inflation Targeting

The model we develop in this section is used to show how estimates of short run NAIRU can be used in an inflation targeting framework. It resembles the one employed by Svensson (1997b) with an output gap and a stochastic natural output level, and the results we derive are fully consistent with Estrella and Mishkin (1999). Monetary policy instrument, the interest rate, is assumed to affect inflation with a longer lag than the rate of unemployment. The model is described by the following equations

$$\pi_{t+1} = \pi_t - a_1(u_t - u_t^n) + \varepsilon_{t-1} \qquad (1)$$

$$u_{t+1} = \tilde{b}_1 u_t + b_2 x_t + b_3(i_t - \pi_{t+1|t}) + \tilde{\eta}_{t+1} \qquad (2)$$

$$x_{t+1} = c_1 x_t + \zeta_{t+1} \qquad (3)$$

$$u_{t+1}^n = d_1 u_t^n + \xi_{t+1} \qquad (4)$$

where π_t is the inflation rate, u_t is the unemployment rate, u^n_t is the short run NAIRU and x_t a persistent aggregate demand shock. $\pi_{t+1|t}$ is the expected rate of inflation in period $t+1$ conditioned on information available in t. All coefficients are non-negative and $0 \leq c_1, d_1 \leq 1$. The error terms are *i.i.d* disturbances, with zero means and given variances. The equilibrium value of all variables is zero, including the real interest rate $(i_t - \pi_{t+1|t})$. For the moment, assume that all parameters are known with certainty.

Define the NAIRU gap as $n_t = u_t - u_t^n$. Subtract u_{t+1}^n from (2) and rewrite the model as

$$\pi_{t+1} = \pi_t - a_1 n_t + \varepsilon_{t+1} \qquad (5)$$

$$n_{t+1} = b_1 n_t + b_2 x_t + b_3(i_t - \pi_t) + b_4 u_t^n + \eta_{t+1} \qquad (6)$$

[5] See Gordon (1997), Debelle and Laxton (1997), Nishizaki (1997) for NAIRU estimations for United States,

where

$$b_1 = \tilde{b}_1 + a_1 b_3 \qquad (7)$$

$$b_4 = \tilde{b}_1 - d_1 \qquad (8)$$

$$h_{t+1} = \tilde{h}_{t+1} - x_{t+1} \qquad (9)$$

The one period and the two period inflation forecasts are, respectively

$$E_t p_{t+1} = p_t - a_1 h_t \qquad (10)$$

$$E_t p_{t+2} / t\ (i_t) = p_t - a_1(1+b_1)n_t - a_1 b_2 x_t - a_1 b_3 (i_t - p_t) - a_1 b_4 u_t^n \qquad (11)$$

Where $E_t p_{t+2}$½$t\ (i_t)$ is the expectation of the inflation rate for period $t+2$, formed in t and conditioned to information available in t, including the monetary policy instrument i_t. The monetary authority conducts monetary policy with the objective of reaching a long run inflation target p^* and reducing in the short run NAIRU gap fluctuations around zero. In the long run, there is no unemployment target other than the natural rate (normalized to zero). The monetary policy instrument is the interest rate i_t, that affects unemployment one period ahead and inflation two periods ahead. The monetary authority objective can be summarized by the standard quadratic intertemporal loss function

$$E_t \sum_{t=t} d^{t-t} L(p_t, u_t) \qquad 0 < d < 1 \qquad (12)$$

with the period loss function

$$L(p_t, n_t) = \tfrac{1}{2}\left[(p_t - p^*)^2 + l n_t^2\right] \qquad (13)$$

where l ³$0$ is the weight on NAIRU gap stabilization. In the case of flexible inflation targeting (as opposed to strict inflation targeting, where $l=0$), the monetary authority places a positive weight on short run NAIRU gap stabilization, that is, $l>0$. To solve for the inflation target rule, consider first the solution for the simpler case of one year control lag for inflation[6]. The value function can be written as

$$V(p_t) = min\left\{\tfrac{1}{2}\left[(p_t - p^*)^2 + l n_t^2\right] + dE_t V(p_{t+1})\right\} \qquad (14)$$

subject to

$$p_{t+1} = p_t - a_1 n_t + e_{t+1} \qquad (15)$$

Where the NAIRU gap n_t can be considered the control. To solve for the first order condition, guess that the value function has the form

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Canada, Great Britain and Japan.

[6] See Svensson (1997b), appendix B.

$$V(p_t) = k_0 + \tfrac{1}{2} k(p_t - p*)^2 \tag{16}$$

with the k coefficients to be determined. Then the first order condition becomes

$$l n_t - d a_1 E_t V_p(p_{t+1}) = l n_t - d a_1 k(E_t p_{t+1} - p^*) = 0 \tag{17}$$

The solutions for the NAIRU gap and the inflation target rule are, respectively

$$n_t = \frac{d a_1 k}{l + d a_1^2 k}(p_t - p^*) \tag{18}$$

$$E_t p_{t+1} = p^* + \frac{l}{l + d a_1^2 k}(p_t - p^*) \tag{19}$$

Note that these solutions depend on k, that needs to be identified. Use the envelope theorem on (14) and (16) and with (19) equate the coefficients for $(p_t - p^*)$. The unique positive solution that fulfils k ³$1$ is given by

$$k = k(l) \equiv \tfrac{1}{2}\left[1 - \frac{l(1-d)}{d a_1^2} + \sqrt{(1 - \frac{l(1-d)}{d a_1^2})^2 + \frac{4l}{d a_1^2}}\right] \geq 1 \tag{20}$$

Now the two period control lag for inflation problem can be formulated as

$$V(p_{t+1}/t) = \min\left\{\tfrac{1}{2}\left[(p_{t+1}/t - p^*)^2 + l n_{t+1}^2/t\right] + d E_t V^*(p_{t+2}/t+1)\right\} \tag{21}$$

subject to

$$p_{t+2}/t+1 = p_{t+1} - a_1 n_{t+1} = p_{t+1}/t - a_1 n_{t+1}/t + (e_{t+1} - a_1 h_{t+1}) \tag{22}$$

where n_{t+1}½t is the control and the optimal interest rate is given by

$$(i_t - p_t) = \frac{1}{b_3} n_{t+1}/t - \frac{b_1}{b_3} n_t - \frac{b_2}{b_3} x_t - \frac{b_4}{b_3} u_t^n \tag{23}$$

Note that this problem is analog to (14) subject to (15), with p_{t+1}, n_{t+1} and $(e_{t+1} - a_1 h_{t+1})$ replacing p_t, n_t and e_{t+1}. This analogy leads to

$$p_{t+2}/t - p^* = \frac{l}{d a_1 k(l)} n_{t+1}/t \tag{24}$$

where $k(l)$ is given by (20). From (1), n_{t+1}½t = $-1/a_1$ (p_{t+2}½t - p_{t+1}½t). Use this to eliminate n_{t+1}½t from above and get

$$(p_{t+2}/t - p^*) = \frac{l}{d a_1^2 k(l) + l}(p_{t+1}/t - p^*) \tag{25}$$

or

$$p_{t+2}/t = p^* + c(l)(p_{t+1}/t - p^*) \tag{26}$$

note that 0£$c(l)<1$. Now solve for the instrument rule. From equations (23) and (24), we obtain

$$(i_t - p_t) = \frac{da_1 k(l)}{b_3(l + da_1^2 k(l))}(p_t - p^*) - \frac{da_1^2(1+b_1)k(l) + l b_1}{b_3(l + da_1^2 k(l))} n_t - \frac{b_2}{b_3} x_t - \frac{b_4}{b_3} u_t^n \quad (27)$$

or

$$(i_t - p_t) = f_p(l)(p_t - p^*) - f_n(l) n_t - f_x x_t - f_u u_t^n \quad (28)$$

Under flexible inflation targeting *(l>0)*, equation (25), the target rule equation, implies that the two period expected inflation should gradually approach the target. The reason for this result is that this strategy reduces short run NAIRU gap fluctuations. Additionally, equation (27), the instrument rule equation, says that if current inflation is above target, interest rate should be increased. The optimal interest rate also depends on predetermined current variables (the short run NAIRU gap, shocks and the discrepancy of the short run NAIRU level from its long run value), for they help explain future inflation. Note that equations (24) and (26) imply that the next period expected short run NAIRU gap will not be zero when future inflation deviates from target. The reason for this result is that the policy objective is not to drive unemployment towards the NAIRU, but to use the NAIRU gap as an indicator of the direction to move the instrument[7].

Under strict inflation targeting *(l=0)*, equation (25) implies that the target rule is reduced to $p_{t+2}½t = p^*$, that is, monetary authority should adjust its instrument for the two period inflation forecast to reach the target. Note that under strict inflation targeting *k(l)=1*, and therefore the instrument equation (27) will also depend on the predetermined variables. Although monetary authority does not aim to stabilise unemployment, taking short run NAIRU gap into account is still valuable, since it helps predict future inflation.

Consider now uncertainty about the coefficients in the model (5)-(6). Since the use of short run NAIRU estimates for inflation targeting concerns policy makers, it is interesting to ask how uncertainty of these estimates affects monetary authority reaction function. Simplify the model ignoring shocks, and rewrite it as

$$p_{t+1} = p_t - a_1 n_t + e_{t+1} \quad (29)$$

$$n_{t+1} = b_1 n_t + b_3(i_t - p_t) + b_4 u_t^n + h_{t+1} \quad (30)$$

Under strict inflation targeting, Svensson (1997a) showed that the solution to the period by period problem given by equations (31) to (34) where i_t is the control variable, is equivalent to the dynamic solution of the intertemporal loss function.

$$min\ d^2 E_t\left[\tfrac{1}{2}(p_{t+2} - p^*)^2\right] \quad (31)$$

subject to

[7] This result is also stressed by Estrella and Mishkin (1999).

$$p_{t+2} = p_{t+2}/t(i_t) + e_{t+1} - a_{1,t+1}h_{t+1} + e_{t+2} \tag{32}$$

$$p_{t+2}/t(i_t) = p_{t+1}/t - a_{1,t+1}b_1 n_t - a_{1,t+1}b_3(i_t - p_t) - a_{1,t+1}b_4 u_t^n \tag{33}$$

$$p_{t+1}/t = p_t - a_{1,t}n_t \tag{34}$$

Now, assume that there is model uncertainty in period t, resulting from uncertainty about coefficients a_1, b_1, b_3 and b_4. Let

$$a_{1,t+1} = a_1 + n_{a_1,t+1} \tag{35}$$

$$b_{1,t+1} = b_1 + n_{b_1,t} \tag{36}$$

$$b_{3,t+1} = b_3 + n_{b_3,t} \tag{37}$$

$$b_{4,t+1} = b_4 + n_{b_4,t} \tag{38}$$

Assume that $a_{1,t}$ is known with certainty in t. All n´s are *i.i.d.* stochastic disturbances with zero mean and given variances. For simplicity, assume that all cross equations covariances are zero. Realisation of disturbances in t are known in $t+1$[8]. Then write

$$a_{1,t+1}b_1 = a_1 + n_{1,t+1} \tag{39}$$

$$a_{1,t+1}b_3 = a_3 + n_{3,t+1} \tag{40}$$

$$a_{1,t+1}b_4 = a_4 + n_{4,t+1} \tag{41}$$

Again, the n´s are *i.i.d.*, have zero mean, given variances s^2_1, s^2_3, s^2_4 and zero cross equations covariances. Then write the constraint in t as

$$p_{t+2} = p_{t+1}/t - (a_1 + n_{1,t+1})n_t - (a_3 + n_{3,t+1})(i_t - p_{t+1}/t)$$

$$- (a_4 - n_{4,t+1})u_t^n + e_{t+1} - a_{1,t+1}h_{t+1} + e_{t+2} \tag{42}$$

with

$$p_{t+1}/t = p_t - a_{1,t}n_t \tag{43}$$

The two period inflation forecast now is

$$p_{t+2}/t(i_t) \equiv p_{t+1}/t - a_1 n_t - a_3(i_t - p_{t+1}/t) - a_4 u_t^n \tag{44}$$

Now we have all the ingredients to write down the solution to the problem given by the loss function equation (31). The relevant constraint is given by equation (44), and the first order condition leads to

$$p_{t+2}/t(i_t) - p^* = \frac{s_3^2}{a_3}(i_t - p_{t+1}/t) \tag{45}$$

Substituting equation (44) into (45), we find the instrument rule

$$i_t = p_{t+1}/t + \frac{a_3}{a_3^2 + s_3^2}(p_{t+1}/t - p^*) - \frac{a_1 a_3}{a_3^2 + s_3^2} n_t - \frac{a_4 a_3}{a_3^2 + s_3^2} u_t^n \qquad (46)$$

And from equations (45) and (46) we derive the target rule as

$$p_{t+2}/t(i_t) = p^* + \frac{s_3^2}{a_3^2 + s_3^2}(p_{t+1}/t - p^*) - \frac{a_1 s_3^2}{a_3^2 + s_3^2} n_t - \frac{a_4 s_3^2}{a_3^2 + s_3^2} u_t^n \qquad (47)$$

From the instrument rule equation (46), two implications are worth to note. First, uncertainty with respect to the policy multiplier coefficient *(a_3)* affects the solution. Brainard´s (1967) analysis applies, since the presence of policy multiplier uncertainty *($s^2_3 > 0$)* reduces the coefficients in the reaction function, making the policy maker more "conservative", that is, the policy maker will react to the discrepancies less than in the certainty equivalence case. Second, uncertainty with respect to the short run NAIRU does not affect the monetary authority reaction function. It does, however, affect the value of monetary authority loss function, since it increases the conditional variance of p_{t+2}[9].

The implications of the target rule equation (47) are also straightforward. Uncertainty about the short run NAIRU coefficient does not affect the two period inflation forecast, policy multiplier uncertainty motivates deviation of inflation forecast from the target and the two period inflation forecast approaches only gradually the target. This deviation is the sum of a fraction of one period inflation forecast discrepancy from the target, a proportion of short run NAIRU gap and a proportion of the NAIRU rate discrepancy from its long run value (assumed zero).

The conclusions derived above suggest that short run NAIRU estimates may be a useful construct for a monetary policy framework based on inflation targeting. Short run NAIRU estimates are potentially important in this policy approach because the current NAIRU gap level reveals information about future inflation and provides policy makers a signal for the adequate conduction of monetary policy.

3. NAIRU Models

In this section we present the econometric models to be used in the NAIRU estimation. The NAIRU estimations will be performed in two different ways. On one hand, as in Nishizaki (1997), we will use a traditional Phillips Curve approach based on a transfer function estimation. On the other hand, as suggested by Debelle and Laxton (1997), we will try to estimate the NAIRU as an unobservable stochastic trend in the unemployment data series.

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[8] These assumptions about the stochastic structure imply that there is no room for learning and experimentation by monetary authorities.

3.1 Econometric Models and Data Analysis

The concept of Phillips Curve states that

$$p_t = a + bU_t + p_t^e + e_t \qquad (48)$$

Where: p_t = inflation rate on period t;

U_t = unemployment rate on period t;

p_t^e = expectation of inflation rate to period t;

a = constant or level of equation;

e_t = residuals.

If the unemployment and inflation rates follow a standard behavior, like the Phillips Curve, then the NAIRU is the rate of unemployment when the inflation rate of the period is equal to the expected inflation rate. Therefore, we only need three series[10] to estimate the Brazilian NAIRU: infla tion, unemployment and expected inflation rates.

To capture the inflation rate at a national level, we use the National Consumer Prices Index (INPC) from IBGE. This price index measures the inflation on 11 main Brazilian cities. For unemployment rate we use both PME (Employment Monthly Research) from IBGE and PED (Research of Employment and Unemployment) from DIEESE/SEADE data[11]. Inflation quarterly data were obtained by the geometric mean of monthly data, while for quarterly unemployment rates we used the arithmetic mean.

We have to obtain also a data series for the expected inflation rate. Many models are used in economics to try to model expectations. One can use models with adaptive expectations, rational expectations or bounded rationality. In the period analysed, Brazilian economy presented high inflation rates, implying a high cost for forecasting errors. Thus, rational expectations seem to be the more appealing way to model the economic agents' expectations. The rational expectations hypothesis assumes that agents incorporate all available information to formulate expectations about a variable so that agents do not make systematic errors. Therefore, to obtain the expected inflation rate data, we have estimated several ARIMA models and choose the one that generated the best fit and prediction for the inflation rate.

[9] See Estrella and Mishkin (1999).

[10] For the three series we will use only quarterly data for the period 1982 to 1998.

[11] Since the DIEESE data series had different cities included in different periods, we are only using data for the state of São Paulo.

3.2 Transfer Function Model

When one variable appears to suffer influence from its past values and also from current and/or past values of other exogenous variables, the best modelling strategy is to use a transfer function[12]. Equation (49) specifies the transfer function model.

$$y_t = a + A(L)y_{t-1} + C(L)z_t + B(L)e_t \qquad (49)$$

where y_t = endogenous variable at time t;

z_t = exogenous variable at time t;

e_t = residuals

a = constant;

A(L), *B(L)* and *C(L)* = lags polynomials.

In order to be able to use this model, we have to assume that all the series are stationary and that z_t in exogenous for y_t. If we find non-stationary series, then the appropriate methodology would involve a cointegration analysis. On the other hand, if z_t is not exogenous one would have to use a VAR model to estimate the NAIRU.

In our case the exogenous variable can be the unemployment rate or some non-linear function of it, and supply shock such as woman's share in the work force. In our estimates we always use the unemployment rate in a linear fashion[13].

3.3 Unobservable Components Model (Structural Models)

The literature on unobservable components models (UCM) has grown quite significantly in the last few years, due mainly to the introduction of the Kalman filter to econometrics. The Kalman filter has helped make these models operational, giving an easy way to estimate the time varying unobservable components. Such models have been developed in both classical and bayesian statistics frameworks. The main difference between these two approaches is the estimation of the variances or hyperparameters. In the bayesian models the observational and evolution variances are obtained by variance learning and discount factors. In the classical models, on the other hand, the hyperparameters are estimated by maximum likelihood.[14] In this paper we are going to follow the classical approach.[15].

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[12] For details on transfer functions models see Enders (1995).

[13] Non-linear functions have presumably better results when the increase on inflation is more than proportional to the decrease on unemployment. We tested for non linearity but it led to worse estimates.

[14] See Mainhold and Singpurwalla (1983) and West and Harrison (1989) for a discussion on the bayesian approach.

[15] For a detailed presentation and discussion on this model see Harvey (1987 and 1989).

In this framework a time series is modelled by decomposition into its basic forming elements. That is, it is decomposed by a trend (m_t), cycle (y_t), seasonal (g_t) and irregular (e_t) components. [16]

$$u_t = m_t + g_t + y_t + e_t . \qquad (50)$$

The complete model also should specify the behaviour of each one of the individual components. A complete treatment of unobserved component models is beyond the scope of this article. Therefore, we will only briefly discuss the case of the "basic structural time series model". In the basic structural time series model only three components are used since the cycle component is omitted. [17] The trend, which will give us an estimate of the NAIRU, is modelled as a random walk with a time varying drift while the drift itself is a random walk. The seasonal component can be modelled by a combination of sine and cosine waves or by the introduction of seasonal dummies. The full model with seasonal dummies can be written as

$$\begin{aligned}
&u_t = m_t + g_t + e_t \\
&m_t = m_{t-1} + b_{t-1} + h_t \qquad (51) \\
&b_t = b_{t-1} + z_t \\
&g_t = -\sum_{j=1}^{S-1} g_{t-j} + w_t
\end{aligned}$$

where e_t, h_t, z_t and w_t are all normally distributed errors with variances s_e^2, s_h^2, s_z^2 and s_w^2. To simplify matters let us suppose that we are dealing with quarterly data, so that s = 4. Therefore, equations (51) can be cast in the state-space format as

$$y_t = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix} a_t + e_t$$

$$a_t = \begin{bmatrix} m_t \\ b_t \\ g_t \\ g_{t-1} \\ g_{t-2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} m_{t-1} \\ b_{t-1} \\ g_{t-1} \\ g_{t-2} \\ g_{t-3} \end{bmatrix} + \begin{bmatrix} h_t \\ z_t \\ w_t \\ 0 \\ 0 \end{bmatrix}$$

Once the models have been cast in the state space format, estimation of the unobservable components can be performed by the Kalman Filter and the estimations of the variances s_e^2, s_h^2, s_z^2 and s_w^2 by maximum likelihood.

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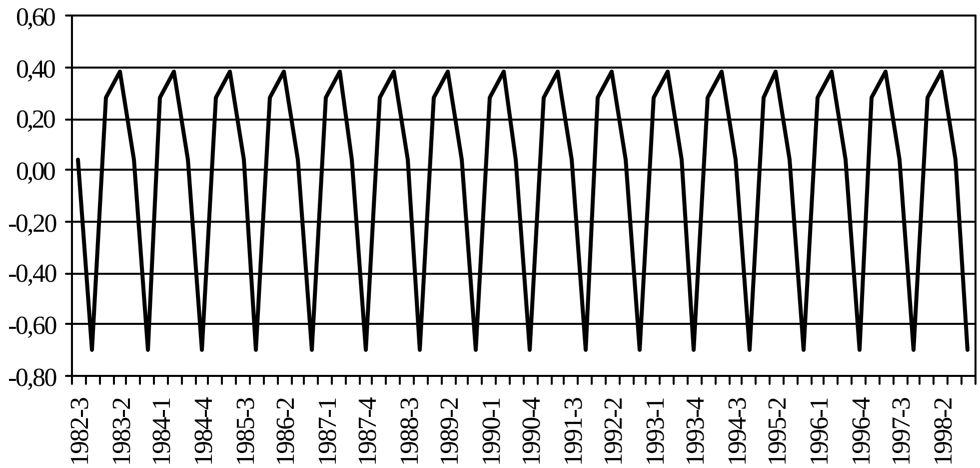
[16] The estimation of NAIRU using structural models was inspired on Debelle and Laxton (1997) but Portugal and Garcia (1996) also use the same process that used in this paper. In Corseuil, Gonzaga and Issler (1996), the structural model is again used to define the NAIRU on several metropolitan regi ons from PME of IBGE.

[17] The trend plus cycle model is discussed in Portugal (1993).

Graph 1
Seasonal Component of Quarterly Unemployment Rate from IBGE



In this specification we have a component to represent the level of the series (m_t) and another to represent the trend itself or slope coefficient (b_t). Therefore, series with no slope will have $b_t = 0$, while in the case of a constant slope we have $b_t = \beta \, (s_z^2 = 0)$. This allows us to extract the NAIRU from the trend of the unemployment series[18]. For all series analysed we found that the slope component is equal to zero, therefore, the trend of the series has only the level component. Sometimes one can be confused believing that the series has no trend because it has only the level component. But, as shown by Harvey (1989), the concept of trend still exists even when the slope is zero.

For all series analysed in this paper the seasonal component has fixed parameters, since the estimated variance of the residuals w_t is not significantly different from zero ($s_w^2 = 0$). Hence, there are no changes in the seasonal pattern of unemployment during the estimation period. This fact can be better seen on Graph 1, that shows the behaviour of the seasonal component of unemployment from the IBGE quarterly[19].

[18] There is nothing that imposes that the trend of the series must be its equilibrium value. In order to sustain that the behaviour of the unemployment rate is going to the NAIRU, we need other restrictions. In the Phillips Curve theory, this idea is based on the fact that the NAIRU will happen when the inflation expectations are satisfied. If the economic agents try to 'match' the next period inflation rate, it is not coherent to believe that they will always overestimate or underestimate the actual unemployment rate (the mistakes will follow a non-predictable pattern). Therefore, the NAIRU can be the trend of the unemployment rate series.

[19] The seasonal component of the DIEESE series show similar results.

4. NAIRU Estimates for Brazil

4.1 Estimations with Transfer Function Models

The initial step on estimating the transfer function model was specifying the proper function and testing the series for stationarity and exogeneity[20]. All series are stationary[21], which allows us to model them using the transfer function approach. Thus, the objective function to be estimated (52) follows the same specification of the equation (49).

$$\pi_t - \pi_t^e = \alpha + A(L)[\pi_{t-1} - \pi_{t-1}^e] + C(L)U_t + B(L)\varepsilon_t \tag{52}$$

Based on equation (52) the NAIRU can be calculated as[22]

$$\text{NAIRU} = \left[\left(\frac{-1}{C(L)}\right)\left(\alpha + B(L)\varepsilon_t\right)\right]$$

Even when using quarterly data, the $\pi_t - \pi_t^e$ series exhibits structural changes for some specific periods, mainly the beginning of price stabilisation plans. The first two quarters of 1990 and the second quarter of 1994 were the most affected. On the first quarter of 1990, the inflation rate was more than forty percentual points greater than the expected inflation. In the following quarter, we can observe exactly the inverse, with the expected inflation being greater than the actual inflation by the same proportion. To allow for these structural changes we inserted two dummy variables in equation (52) and tested for their significance. On the second quarter of 1994, period that the Real Plan was being implemented, we had again problems with the expectations. In this case, the expected inflation was thirty percentual points above the actual inflation. We, therefore, included the third dummy variable on the model and tested its significance. After the inclusion of the dummy variables we have equation (53) to be estimated.

$$\pi_t - \pi_t^e = \alpha + A(L)[\pi_{t-1} + \pi_{t-1}^e] + C(L)U_t + \gamma D_1 + \varphi D_2 + \delta D_3 + e_t \tag{53}$$

where D_1,D_2 and D_3 = dummy variables for the periods of expectations reversion

e_t = estimated residuals.

[20] The stationarity and exogeneity tests and the estimation of the transfer function model were performed using the software Stata 6.0. The tests results are on the appendix I.

[21] The stationerity of inflation rate have been questioned be some authors since the frequent structural breaks may generate biased this result (see Perron, Cati and Garcia (1995)). Even if the inflation rate series is I(1), it does not affect our results because we are modelling the difference between the actual and expected inflation. Therefore it is as if we were taking the first difference inflation rate. The Augmented Dickey-Fuller (ADF) tests show that there is no unit roots at 10%. Since unemployment data is bounded between 0% and 100% the unit root tests have to be made up from a transformation on the original data. Following Corseiul, Gonzaga and Issler (1996) we used a logit function [log U_t/(100-U_t)] before applying the ADF test. Following the procedure described on Enders (1995), we reject the null hypothesis of unit roots for both unemployment series.

[22] The inclusion of dummy variables in equation (5) may be important only when finding the estimated parameters, but not when calculating the NAIRU. To calculate NAIRU in the periods where the dummies were present, we made an arithmetic mean between the previous and next NAIRU.

The residuals in equation (53) do not necessarily have to be white noise. Therefore, the inclusion of relevant lagged residuals can improve our estimation by increasing the explanatory power[23]. Then, we construct a residual polynomial *B(L)* as in equation (54)

$$e_t = B(L)\varepsilon_t \qquad (54)$$

The degree of *B(L)* is given by the residuals autocorrelation functions (ACF) and partial autocorrelation functions (PACF). The polynomial *B(L)* may follow a pure autoregressive model (AR) or a combination of autoregressive and moving average process (ARMA). In our estimations, the most convenient model was an AR(1) for all cases. After defining the format and degree of *B(L)*, we include equation (54) on equation (53) and we get the final equation to be estimated. One should note that it is the inclusion of (54) that will generate a time varying NAIRU. If we find that e_t is white noise, the NAIRU will be fixed.

4.1.1 NAIRU for IBGE data (1982-1998)

The estimation of equation (53) using quarterly unemployment data from IBGE resulted in non white noise residuals. The final estimated equations was equation (55) [24]

$$p_t - p^e_t = \underset{(2.72)}{10.067} + \underset{(0.92)}{0.185}\,(p_{t-1} - p^e_{t-1}) + \underset{(0.71)}{0.123}\,(p_{t-2} - p^e_{t-2}) - \underset{(0.51)}{1.745}\,U_t$$

$$+ \underset{(5.88)}{25.789}\,D_1 - \underset{(6.83)}{55.533}\,D_2 - \underset{(5.55)}{38.772}\,D_3 + \underset{(0.16)}{0.385}\,e_{t-1} \qquad (55)$$

Number of obs. = 63 F(7, 55)= 27.71 R-Squared = 0.7791 Adj. R-Squared = 0.751

After some algebraic manipulation, setting inflation rate equal to expected inflation rate for each period and isolating the unemployment variable[25], we found the value of flexible NAIRU expressed by the equation (56)

$$\text{NAIRU} = U^*_t = (10.067 + 25.789\,D_1 - 55.533\,D_2 - 38.772\,D_3 + 0.385\,e_{t-1})/1.745 \qquad (56)$$

Graph 2 shows the values for the unemployment rate of IBGE for each quarter and the values found for NAIRU. By definition, when the unemployment rate equals the NAIRU, there is no pression for either acceleration or desacceleration of the inflation rate. When the unemployment is above (bellow) the NAIRU there are movements to desaccelerate (accelerate) the inflation rate. Notice that in the periods when the inflation rate was very high, naming the end of 80's and begging of 90's, the NAIRU was almost every period above the unemployment rate.

[23] The inclusion of lagged residual on equation (6) can cause a loss in consistency when using ordinary least squares estimation. Thus, we used Instrumental Variables estimation. To see a discussion on the convenience of using least squares or instrumental variables estimation, see Harvey (1990) pg. 243-244 and pg. 267-269. Enders, Sandler and Parise (1992), for example, estimate a transfer function model by least squares.

[24] The values in parenthesis are the standard deviations for each estimated parameter.

[25] For the unemployment series all time lags are taken to be the same.

This means that inflation rate could be increasing also because the unemployment was not in the equilibrium rate. On the second year of the Real Plan, the opposite begun to happen, that is the NAIRU was lower than the actual unemployment rate. This means that, in this period, the high unemployment could have helped to reduce the inflation rates.

Graph 2
Unemployment and NAIRU
Quarterly Data from IBGE

The results shown on graph 2 are quite intuitive. To confirm this intuition we formulate a simple test. We want to test that when the NAIRU was lower than the actual unemployment rate, the inflation rate was accelerating and when the NAIRU was greater than the unemployment rate, the inflation was desaccelerating. Therefore, we estimate by least squares a regression of the NAIRU gap (the difference between unemployment and NAIRU for each period) on the inflation rate. The results are shown on equation (57)

$$U_t - \text{NAIRU}_t = \underset{(0.23)}{0.18} - \underset{(0.01)}{0.07}\, p_t \qquad (57)$$

By direct observation of equation (57), the negative and significant coefficient in the inflation rate indicates the consistency of our estimation. The acceleration of inflation rate is related as expected with the increase on the NAIRU gap.

4.1.2 NAIRU for DIEESE data (1985-1998)

Following the same procedure used before, we also found non-white noise residuals for the estimation of equation (53) for the DIEESE quarterly unemployment data. Therefore, after including the residuals of the instrumental variable estimation of equation (53) and re-estimating by ordinary least squares we find the equation (55')

$$p_t - p^e_t = \underset{(3.75)}{14.43} - \underset{(0.44)}{1.558}\, U_t + \underset{(5.92)}{28.23}\, D_1 - \underset{(5.62)}{46.02}\, D_2 - \underset{(5.79)}{37.90}\, D_3 + \underset{(0.14)}{0.572}\, e_{t-1} \qquad (55')$$

Number of obs = 54 $F(5, 48) = 34.93$ R-Squared = 0.7844   Adj. R-Squared = 0.762

Given the coefficients in equation (55') one can find again a time varying NAIRU by equation (56').

$$\text{NAIRU} = U^*_t = (14.43 + 28.23\, D_1 - 46.02\, D_2 - 37.9\, D_3 + 0.572\, e_{t-1})/1.558 \qquad (56')$$

Graph 3 shows the behaviour of the estimated NAIRU and the DIEESE actual unemployment rate. A simple inspection of Graph 3 shows that the estimated NAIRU follows an expected pattern, that is for periods of high inflation the NAIRU is above the unemployment rate and vice-versa. For the period after the Real Plan, specifically the second half of 1995, the NAIRU becomes bigger than the unemployment and this gap appears to be persistent.

Graph 3
Unemployment and NAIRU
Quarterly Data from DIEESE

Using again a more general test we regressed the NAIRU gap on the inflation rate and the results are presented in equation (57')

$$U_t - \text{NAIRU}_t = \underset{(0.35)}{0.63} - \underset{(0.02)}{0.11}\, p_t \qquad (57')$$

Once more, the response of increasing the NAIRU gap is a decrease on the acceleration of the inflation rate. The equation (57') allows us to confirm that the estimated NAIRU is really consistent with the predictions one could make based on a Phillips Curve approach.[26]

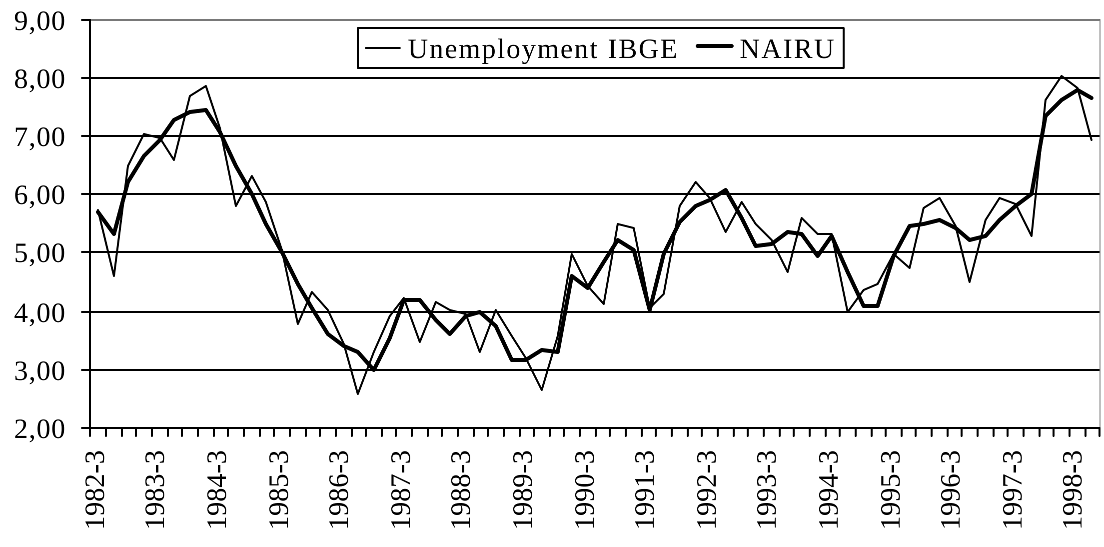
[26] It is not our objective here to discuss the different methodologies used by IBGE and DIEESE to measure the open unemployment rate in Brazil. But it is interesting to notice that the response of inflation to the NAIRU gap is different in each case, as shown by the different coefficients on equations (10) and (10'). This result may be due to the fact that for IBGE we are using the total unemployment rate, where as for DIEESE we are using data for São

4.2 Estimations with an Unobservable Components Model (Structural Model)

The structural model presented in equation (51) is used to estimate the trend in quarterly unemployment data from both IBGE and DIEESE. We have only a stochastic trend and a seasonal component in that model. In this case, the NAIRU is the trend component (m_t) for the series. The estimated variance of the seasonal residual w_t is not significantly different from zero, indicating that the seasonal pattern of unemployment rate had been constant for all series. Graphs 4 and 5 show the evolution of NAIRU and the unemployment rate for the quarterly IBGE and DIEESE data[27].

A simple inspection on graphs 4 and 5 shows that the estimated NAIRUs are completely different from the ones estimated with the transfer function approach. The main feature to be noticed here is that the estimated NAIRUs follow very closely the actual unemployment rate, as one would expect from a stochastic trend. We perform here the same test as before, regressing the NAIRU gap on the inflation rate, but in this case the estimated coefficient was not significantly different from zero. This means that the NAIRU gap did not have any influence on inflation which is not in accordance with the Phillips Curve predictions.

Graph 4
Unemployment and NAIRU – Structural Model
Quarterly Data from IBGE

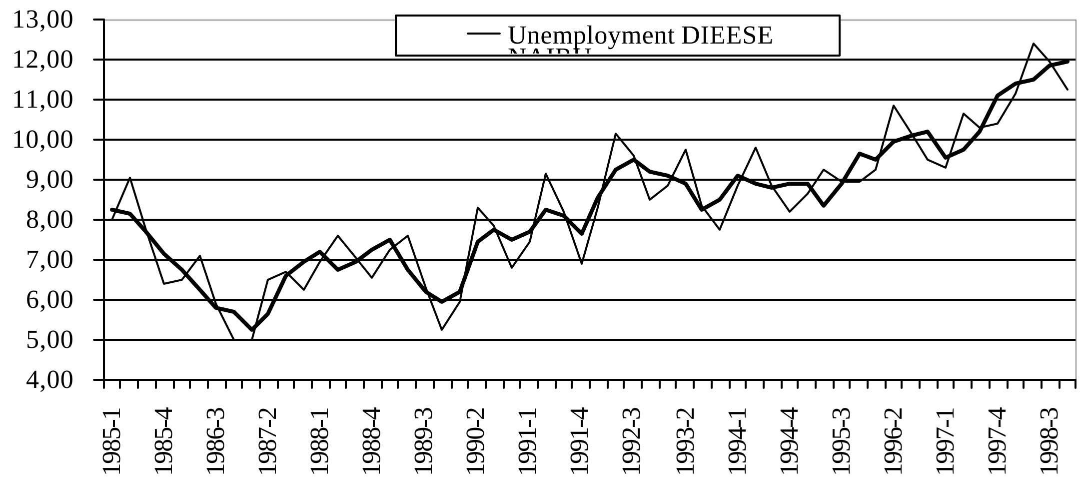


Paulo only. On the other hand, since São Paulo is responsible for more then one third of Brazilian GDP and, therefore, may be taken to be representative of what happens in the country as a whole, the difference in the inflation response could be caused by the different methodologies used to measure unemployment. In this case, it is crucial for any discussion relating inflation and unemployment to try to evaluate which is the best way to measure the open unemployment rate.

[27] All estimated NAIRUs are presented in the appendix II.

This result may lead to two different conclusions. On the one hand, one can say that this result allows us to conclude that the unemployment trend found by structural model does not represent a valid proxy for NAIRU. Regardless that the concept of NAIRU and structural unemployment are very close[28], we could not identify in our tests the trend component of the series as the representative value for the unemployment rate that does not accelerate the inflation for Brazil. Therefore, only the results for transfer function estimation can be considered valid to represent the relationship between unemployment and inflation in Brazil. On the other hand, if one believes in the business cycle theory, one can be very happy with the results of the structural model, since it allows us to conclude that output is always close to its potential level.

Graph 5
Unemployment and NAIRU – Structural Model
Quarterly Data from DIEESE



5. Final Remarks and Conclusion

Given the inflationary past of Brazilian economy and the various failures in price stabilisation plans that were implanted before the Real Plan, a convex Phillips Curve appears to be more intuitive than the linear one. The convexity would allow us to say that there exist a bigger price to be paid on unemployment for inflation reduction than the benefits that one can get by increasing employment when the inflation was accelerating. However, we could not derive this relation from our empirical results. On the other hand, the estimation of a linear trade-off between inflation and unemployment has statistic significance and appears to be adequate to the

[28] See Sachs and Larrain (1992), chap. XVI.

Brazilian data. In all of our estimations the unemployment rate is above the NAIRU for the period after the Real Plan. This result is consistent with the reduction of inflation that has happen since July 1994.

We need also to try to explain the difference, which appears to accelerate, between the employed population and the unemployed one. If the percentage of unemployed grows up proportionally to the decrease on the inflation rate as much as would decrease with the elevation on price levels, then the conclusion can be directly related to the credibility of the economic policies. We also could try to understand this factor by analysing the expectation of the economic agents on the other points of the stabilisation program. In a country like Brazil, which suffered failure of several economics plans in the past two decades, it is reasonable that the economic agents do not have a great amount of confidence on the implementation of another economic reform. The reversion of expectations does not come only with the stability of prices.

The increase in unemployment is well related to the changes on the equilibrium point of unemployment rate. This can be justified by the wrong conduction of the past economic policy. The longer the stabilisation on prices takes to occur, more penalisation will occur on the labour force by unemployment. But this does not mean that the unemployment must be forever this high. Policies of changing the equilibrium rate of unemployment can be applied bringing good results without implying on the return of inflationary process. The lower rates of NAIRU that we found during the Real Plan can be used to justify this kind of policy.

If we consider the latter results, then the main problem on the Brazilian labour market appears to be the structure of the employment/unemployment. The quality of jobs and the skills of the labour force can be questioned. The importance of more flexible labour regulations on reducing the equilibrium rate of unemployment is highlighted together with the incentive to invest on human capital. We think that these parameters should be deeply analysed to lower the unemployment without accelerating the inflation.

Finally, the monetary policy model we developed suggests that NAIRU estimates can play an important role in a monetary policy framework for inflation targeting. Two results from this analysis stress the importance of NAIRU estimates. First, uncertainty about these estimates does not appear to have an impact on the monetary authority reaction function. Second, NAIRU estimates are important not because they tell monetary authorities a target for unemployment, but because the current NAIRU gap helps predict future inflation and therefore provides a useful signal for the appropriate conduction of monetary policy.

APPENDIX I

a) Stationarity Tests:

We used the Augmented Dic key-Fuller (ADF) Test. The null hypothesis of existence of unit roots was rejected for all series, as can be seen on the Box A.

BOX A

	ADF	Table Value (10%)
π_t - π^e_t (quarterly series)	-7.527	-3.177
Quarterly Unemployment Series of IBGE	-2.629	-2.596
Quarterly Unemployment Series of DIEESE	-4.132	-3.177

b) Exogeneity Tests:

To test for exogeneity of the variable π_t - π^e_t in relation to the unemployment variables, we used a Granger Causality Test. The null hypothesis of non-causality can be tested by addin g the variable π_t - π^e_t on the unemployment regression. The results are:

Granger-Causality test for adding π_t - π^e_t to quarterly Unemployment Series of IBGE:
t = -1.226 $P>|t| = 0.225$

Granger-Causality test for adding π_t - π^e_t to quarterly Unemployment Series of DIEESE:
t = 1.33 $P>|t| = 0.190$

APPENDIX II

Table II.1 - Quarterly Data and NAIRU

Period	Unemployment IBGE	NAIRU Transfer Function	NAIRU Structural Model	Unemployment DIEESE	NAIRU Transfer Function	NAIRU Structural Model
1982-3	5.72	-	5.68	-	-	-
1982-4	4.62	-	5.32	-	-	-
1983-1	6.49	-	6.21	-	-	-
1983-2	7.03	6.24	6.65	-	-	-
1983-3	6.98	5.77	6.94	-	-	-
1983-4	6.60	6.53	7.30	-	-	-
1984-1	7.69	5.29	7.41	-	-	-
1984-2	7.85	6.44	7.47	-	-	-
1984-3	7.13	6.21	7.09	-	-	-
1984-4	5.79	6.04	6.49	-	-	-
1985-1	6.30	5.86	6.02	8.00	-	8.25
1985-2	5.88	5.77	5.50	9.03	-	8.15
1985-3	5.05	5.00	5.01	7.77	7.78	7.68
1985-4	3.78	5.94	4.48	6.40	9.05	7.12
1986-1	4.32	5.49	4.04	6.47	8.26	6.72
1986-2	4.00	4.04	3.62	7.10	5.81	6.22
1986-3	3.44	3.38	3.41	5.90	5.10	5.81
1986-4	2.59	4.92	3.29	4.97	6.93	5.68
1987-1	3.28	4.79	3.00	4.97	6.63	5.22
1987-2	3.93	7.01	3.55	6.50	10.26	5.62
1987-3	4.24	6.20	4.20	6.67	9.68	6.58
1987-4	3.48	6.32	4.18	6.23	9.91	6.95
1988-1	4.14	6.44	3.86	6.93	10.15	7.18
1988-2	4.01	6.23	3.62	7.60	9.22	6.72
1988-3	3.95	5.45	3.91	7.03	9.14	6.95
1988-4	3.30	6.05	4.00	6.53	9.87	7.25
1989-1	4.01	6.08	3.73	7.23	10.02	7.48
1989-2	3.56	5.84	3.18	7.60	9.76	6.72
1989-3	3.20	5.59	3.17	6.30	9.50	6.21
1989-4	2.64	8.79	3.34	5.23	13.80	5.95
1990-1	3.59	8.03	3.31	5.93	12.67	6.18
1990-2	4.98	7.28	4.60	8.30	11.53	7.42
1990-3	4.43	5.77	4.39	7.83	9.26	7.75
1990-4	4.13	7.41	4.83	6.77	12.87	7.48
1991-1	5.51	7.19	5.23	7.43	9.09	7.68
1991-2	5.44	5.67	5.06	9.13	8.44	8.25
1991-3	4.07	6.27	4.03	8.20	5.76	8.11
1991-4	4.29	6.87	4.99	6.90	11.34	7.62
1992-1	5.81	7.29	5.53	8.30	11.08	8.55
1992-2	6.20	5.69	5.82	10.13	9.08	9.25
1992-3	5.95	5.65	5.91	9.60	9.83	9.51
1992-4	5.36	6.36	6.06	8.50	10.52	9.22
1993-1	5.88	6.41	5.60	8.87	10.30	9.12
1993-2	5.49	6.60	5.10	9.77	10.66	8.89

Period	Unemployment IBGE	NAIRU Transfer Function	NAIRU Structural Model	Unemployment DIEESE	NAIRU Transfer Function	NAIRU Structural Model
1993-3	5.20	6.37	5.17	8.33	11.05	8.25
1993-4	4.67	7.09	5.37	7.77	11.70	8.48
1994-1	5.60	6.52	5.32	8.83	10.84	9.08
1994-2	5.32	7.62	4.94	9.80	12.88	8.92
1994-3	5.33	6.69	5.30	8.87	11.07	8.78
1994-4	3.98	5.77	4.68	8.20	9.26	8.92
1995-1	4.36	6.11	4.08	8.67	10.10	8.92
1995-2	4.48	5.44	4.09	9.23	7.52	8.35
1995-3	4.97	5.07	4.94	8.97	8.50	8.88
1995-4	4.75	4.83	5.45	8.93	7.69	9.65
1996-1	5.78	4.95	5.50	9.23	8.04	9.48
1996-2	5.95	5.09	5.57	10.83	7.80	9.95
1996-3	5.46	5.42	5.42	10.17	9.10	10.08
1996-4	4.51	4.93	5.21	9.50	8.20	10.22
1997-1	5.55	4.84	5.27	9.30	8.24	9.55
1997-2	5.95	5.22	5.56	10.63	8.15	9.75
1997-3	5.85	5.19	5.81	10.30	8.62	10.21
1997-4	5.30	5.20	6.00	10.40	8.51	11.12
1998-1	7.62	5.11	7.34	11.13	8.69	11.38
1998-2	8.02	5.89	7.64	12.40	9.09	11.52
1998-3	7.83	5.90	7.79	11.93	9.57	11.84
1998-4	6.94	5.74	7.64	11.23	9.16	11.95

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