Markov Switching Based Nonlinear Tests for Market Efficiency using the R$/US$ Exchange Rate

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Abstract

The present study aims at assessing the validity of the hypothesis of Weak Market Efficiency for the daily R$/US$ nominal exchange rate series, through a Markov Switching model. This article shows that the Markov Switching model is appropriate for capturing the dependence structure of this series, both in terms of the mean and the variance. The method is robust enough to capture the structural breaks observed in this period.

We conduct a series of procedures for the analysis of specification to show that changes in the variance structure can produce spurious patterns of persistence. We carried out two tests for Market Efficiency based on the estimated results of the Markov Switching model: Wald test and the Variance Ratio test.

1 Introduction

Traditionally, when Market Efficiency tests are carried out, we take for granted that the existing structure of the series is linear, even when the dependence structure is not only based on the mean and on variance, for instance, the existence of autoregressivity in the conditional volatility captured by ARCH models.

However, Market Efficiency tests based on linearity can be influenced by the existence of structural breaks caused by the change in parameters of the statistical mechanism that generates the series. The present article aims at showing that the validity of the Market Efficiency hypothesis is dependent on the accurate specification of the nonlinear series mechanism when applied to the daily log-return series of the R$/US$ exchange rate, a process that is remarkably liable to structural breaks.

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The article consists of three parts. The first part explains and applies the traditional Market efficiency tests. The second one estimates a Markov Switching model for all the parameters of an autoregressive process for the exchange rate log-return series. The third part proposes and applies two nonlinear Market efficiency tests based on the estimated Markov Switching Model.

2 Market Efficiency

The oldest and most important theory about asset pricing is known as *Martingale* Model, and its origin dates back to Cardano's manuscripts (1565), whose modern formulation was established by Bachelier [2] and Samuelson [21]. In brief, this theory postulates that the changes in the prices of assets (returns) cannot be systematically forecast.

This is the same as to say that, statistically, the returns of any assets are supposed to be a random i.i.d (independent and identically distributed) process. According to this model, any attempts to predict the future prices of an asset will not have a statistically significant explanatory power.

A model associated with the martingale process and which is widely represented in the tests for the forecastability of returns is the *Random Walk*. A Random Walk is represented by:

$$P_t = \mu + P_{t-1} + \varepsilon_t \tag{1}$$

This model shows that the asset price at time t+1 is given by the price at the immediately previous moment, a term of expected change known as *drift* plus an unpredictable error component. The random walk model can be obtained through the Martingale process by restrictions on the error term ε_t. The behavior of error term ε_t is extremely important,and restrictions on the behavior of this term produce three versions of the Random Walk model, as stated by Campbell, Lo and Mackinlay[4].

2.1 Random Walk I - IID Increments

The stronger version of the random walk model is the one in which increments at price P_t given by error term ε_t belong to the same distribution (identically distributed) and are independent. In addition, the original distribution can be used, which in the most common cases is the same as assuming that ε_t belongs to a normal distribution with zero mean and constant variance σ^2. Random Walk I, also known as RW1, is even more restrictive than the martingale model, since in this model the increments are nonlinearly uncorrelated and any nonlinear combination of the increments should also be uncorrelated.

2.2 Random Walk II - Independent Increments

The RW1 model is extremely restrictive; therefore, it should not be used in real financial series because it rules out the possibility of structural changes in the data-generating process, such as parameter changes, of which the most relevant are the changes in volatility. A more appropriate version is known as Random Walk II (RW2), which only determines that the increments should be independent, but not necessarily originate from the same distribution. This maintains the characteristic of linear unpredictability and allows for changes in unconditional volatility.

2.3 Random Walk III - Uncorrelated Increments

The general form of the random walk model is the one in which we only restrict process ε_t to being uncorrelated, which is known as RW3. For instance, financial series with ARCH effects can respect the behavior of RW3, once level returns may be uncorrelated, but squared returns have autocorrelation, which does not render the process independent. As this is the least restrictive form of random walk, it is more consistent with the behavior observed in real financial series. RW3 is usually the most widely tested form of random walk.

2.4 Market Efficiency - Definition

A concept that is linked to the properties obtained from the models mentioned above is that of *Market Efficiency*. The definition of market efficiency is related to the rational use of the publicly available information. A financial market is regarded as Efficient (informationally efficient) when the asset price reflects all the available information.

This definition shows that the current price is the best predictor for price at t+h periods ahead, since it incorporates all the information available up to the moment. Price changes cannot be systematically forecast if the market is informationally efficient. This takes us to the definition that price changes should be a martingale stochastic process.

As the definition of Market Efficiency is based on the joint use of the available information, the following taxonomy proposed by Roberts [21] includes three definitions of efficient markets, associating each of them with a relevant dataset. The definitions proposed by Roberts are:

1. Weak Efficiency - The prices reflect the information available from the series history. Market Efficiency Tests that use univariate time series models are an example of weak efficiency.

2. Semi-Strong Efficiency - The prices reflect all the publicly available information.

3. Strong Efficiency - The prices reflect all the public and private information available up to the moment. Here the possibility of insider trading is contemplated.

If a financial market is not efficient, then possibilities of excessive gains exist when the patterns found in the series by means of arbitrage are used. Thus, it is necessary to establish some statistical criteria for testing market efficiency. Market efficiency and rationality tests often try to find statistically significant correlations or patterns in the series.

The hypothesis of efficient markets can be tested in different ways, but all of them will attempt to find some data pattern. Some of the tests presented in the literature try to detect phenomena, such as Intraday effects, day-of-the-week effects, return seasonalities, autocorrelation tests, Runs test, Variance Ratio or profitability of filter rules, and technical analysis. In the present study, we aim at testing Weak Market Efficiency by checking the existence of nonlinear patterns in the observations of exchange rate log-returns.

2.5 Data Description

Our study sample consists of 1,882 daily observations, from July 1st 1994 to January 4th 2002. Figure 1 shows that the exchange rate has periods with different behaviors. At the beginning of the series, after the introduction of Real, we observe a period of fluctuation in which there is a tendency towards an appreciation of Real vis-a-vis the dollar. Afterwards, in the Exchange Rate Band Regime, we have a linear tendency towards the devaluation of Real. The exchange rate band regime could not resist the external crises that assailed the country during these periods, and was therefore replaced with a free exchange rate regime, which is represented by observation 1,337 in our sample. After this change, we note some periods of valuation and devaluation of the exchange rate (1).

Table 1 shows the descriptive statistics for the return and log-return series. The returns do not have a normal or log-normal distribution in the case of log-returns, since the skewness and kurtosis coefficients are far from the values associated with the normal distribution. The positive skewness indicates a tendency towards the appreciation of the exchange rate due to the prevalence of positive returns over the negative ones, whereas the values of the kurtosis coefficient indicate heavier tails than a normal distribution with the same mean and variance. The presence of excess kurtosis shows that extreme values (excessive gains and losses) are more frequent in this series than would be expected in an equivalent normal distribution.

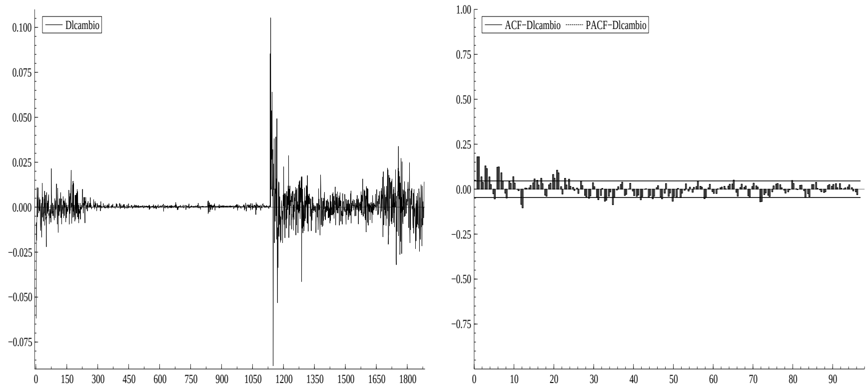

Figure 1: Exchange Rate Log-Returns and ACF

	returns	log-returns
mean	0.000753	0.000491
standard error	0.0141	0.007634
skewness	0.2451	1.8322043
kurtosis	30.44971	48.22841
Jarque-Bera	58759.63	160508.6
p-value- JB	0.000	0.000

Table 1: Descriptive Statistics

BDS Test					
Sample: 1 1882					
Dimension	BDS Stat.	Std. Error	Z Stat	Normal Prob	Prob.Btrap
2	0.059795	0.003039	19.67719	0.0000	0.0000
3	0.118890	0.004852	24.50301	0.0000	0.0000
4	0.169718	0.005809	29.21424	0.0000	0.0000
5	0.203105	0.006090	33.34803	0.0000	0.0000
6	0.226233	0.005909	38.28321	0.0000	0.0000

Table 2: BDS Test - Independence

The autocorrelation and partial autocorrelation functions (ACF and PACF) of the exchange rate log-return series are quite interesting. Note that there are several significant autocorrelations and that ACF and PACF do not allow us to clearly identify the order of an ARMA(p,q) model for this series. One should also observe that even though autocorrelations decrease as the interval between periods grows, they do not decrease exponentially as expected in an AR model, but they apparently decrease polynomially and, even after a significant number of periods there are still significant autocorrelations.

2.6 Market Efficiency Tests - Random Walk 1 and Random Walk 2

As previously commented, random walks I and II are too restrictive to be applied to real financial series. The hypothesis that the increments to the price series are identically distributed and independent cannot be sustained in situations of structural change and learning of agents.

To statistically confirm the violation of random walks I and II, we used the BDS statistics proposed by Brock, Dechert and Scheinkman [3]. Under the null hypothesis of IID for the increments, statistics has an asymptotic normal distribution. The power and size of this test for finite samples are established by Hsieh [12].

As expected, the BDS statistics refuses to accept at any significance level that the exchange rate log returns are an IID sequence.This way, there is some linear or nonlinear structure in the log returns. As a first step for the determination of the functional form of the existing structure, we tested the linearity of the process that generates the means for the data. We used the Teräsvirta [22] and White [16] tests for the nonlinearity of the mean.

Teräsvirta's procedure uses the linearity of the mean as null hypothesis. The test uses a Taylor-series expansion of the activation function of a neural network as a way to obtain the test statistics. The result obtained leads to the rejection of the null hypothesis that the mean of the process is generated by

	Neural Network Test - Teräsvirta	
χ^2 = 6.4885	df = 50	p-*value* = < 2.2e-16
	Neural Network Test- White	
χ^2= 36.2891	df=2	p-*value*=1.318e-08
	White Test - Regression - 12 lags	
χ^2= 29.2501	df = 2	p-*value* = 4.451e-07

Table 3: Linearity Tests for the Mean Process

a linear generating mechanism.White test also uses a neural network, which tests for the existence of nonlinearity in the series and can be used in the series itself or in the regression residuals.

Table 3 shows that the hypothesis for linearity of the mean is rejected by Teräsvirta and White tests.This table also shows that an autoregressive model with 12 lags cannot adjust all the existing structure to the series, and the test indicates that a nonlinear model is necessary.

To test whether the squared residuals of the series have some dependence structure, which would also lead to the rejection of the null hypothesis of IID on the BDS test, we carried out an ARCH test in the residuals of an AR(12) model for exchange rate log-returns. Under the null hypothesis of nonexistence of autoregression in the squared residuals, the sample size times the R^2 of this regression has a χ^2(n) distribution, where n is the order of the ARCH process in test. By using this regression we obtained an R^2 of 0.06401, which means a test statistics of 1876*0.06401=120.028, against a critical value of 21.02607 at 95%. We rejected the null hypothesis that there is no ARCH structure in the residuals.

By using the BDS tests, we rejected the hypothesis that the log-returns are independent and that they originate from the same distribution. The violation of this hypothesis, as pointed by the nonlinearity tests of the mean and by the ARCH test, shows that the violation of independence is possibly due to the existence of nonlinear structures in the mean and in the variance.

2.7 Random Walk III Tests

The most common way to carry out market efficiency tests is to use the linear random walk II model, which only presupposes uncorrelated increments. This model, as stated in section 2, is much less restrictive and therefore more easily respected for real series.

A quite simple way to test weak efficiency by means of the random walk model is to use the following regression:

$$R_t = \alpha + \sum_{i=1}^{n} \beta_n R_{t-n} + e_t \tag{2}$$

The null hypothesis of efficient market corresponds to a test that all β_n parameters are equal to zero against the alternative hypothesis that at least one of the β_n is statistically different from zero, which corresponds to the violation of market efficiency. This joint test can be performed by means of a traditional F test used to verify the existence of a regression.

We performed a first regression with 30 lags of exchange rate log-returns as explanatory variables. The result of F statistics of this regression was F(30,1820) = 7.438 with a *p-value* of 0.000, which rejects at any significance level that the 30 first lags are uncorrelated. To increase the test power, we limited the lags to 12, which corresponded to the significant parameters obtained from the t test. In the test with 12 lags, the statistics was F(12,1838)= 14.22, again with a *p-value* of 0.000, also rejecting that the 12 first lags are uncorrelated. An alternative form to carry out this procedure is to use a *Portmanteau* test for the n autocorrelations being tested. The *Portmanteau* test allowed us to obtain a statistics of 191.146, which corresponded to a *p-value* of 0.000, a result that is consistent with the one obtained through the regressions.

The most widely RW III test is the *Variance Ratio*. The Variance Ratio test (VR) was originally proposed by Cochrane [5] to verify the size of the Random Walk component in GNP. Under the null hypothesis that the process which generates the series is a random walk, the variance of returns of n-periods should be proportional to the number of periods.

The return of n periods is defined as

$$R_t^n = P_t - P_{t-k} \tag{3}$$

if the process that generates the series is a Random Walk, the returns of a period $P_t - P_{t-1} = \mu_t$ should have an IID distribution with means μ and constant variance σ^2. As the returns of n periods are an accumulation of successive μ_t the variance of R_t^n should be equal to $n \times \sigma^2$. The VR (Variance Ratio) statistics is defined as:

$$VR(n) = \frac{Var(R^n)}{Var(R^1)} \frac{1}{n} \tag{4}$$

which should be equal to one under the null hypothesis of a Random Walk. Lo and MacKinlay [18] derived the sample distribution of this test in finite samples and also an estimator for this statistics that is consistent as to the existence of heteroskedasticity. A complete derivation of the test and of the asymptotic distribution can be found in Campbell, Lo and MacKinlay [4], pages 48 to 55. We used the consistent estimator for heteroskedasticity

Variance Ratio			
q	VR	psi	*p-value*
5	1.4935	2.9559	0.0031
10	1.8392	3.2939	9.8817e-004
20	2.1866	3.3171	9.0973e-004
50	2.4678	2.8140	0.0049
75	2.1229	1.9574	0.0503
100	2.0267	1.5660	0.1173
300	1.8184	1.0049	0.3149

Table 4: Variance Ratio Tests

as well as the test distributions of Lo and MacKinlay [18] to construct the VR tests. We calculated the variance ratio for returns of 5, 10, 20, 50, 75, 100 and 300 periods, in order to analyze whether there could be a predictive power for both short and long time periods.

Considering the whole sample, the null hypothesis of random walk is rejected for returns up to 50 periods; however, for longer periods, this hypothesis is not rejected. This could be some evidence that we can have predictive power for short horizons, but for longer horizons, over 3 months, the hypothesis of weak market efficiency, represented by the random walk model, would be valid. This is an interesting phenomenon, since the evidence found in the literature points out that the random walk model could be valid for short time periods while, for long time periods, there would be predictive power, which goes against the results found herein.

However, the most important is to know that if we find some evidence of violation of market efficiency for the exchange rate, we have to assess whether such evidence is really significant. This means trying to find the most appropriate model to model the structure of the series and detect the necessity for a nonlinear process. We must verify whether the observed predictive power is so economically significant as to represent a real possibility of arbitrage in terms of the exchange rate, by using only the information available from the series past history. A statistical model recognizably able to capture this parameter change is the so-called *Markov Switching* model. Markov switching models belong to the class of *piece-wise* linear models, since the data generating process is linear within each regime.

One of the great advantages of Markov Switching model is that its economical interpretation is more trivial than the majority of nonlinear models, because as we are conditioned on the current regime, we will be using a linear autoregressive model whose behavior has already been exhaustively analyzed in the literature.

3 Markov Switching Model

The basic idea of Markov Switching model is to decompose a series in a finite sequence of distinct stochastic processes, or regimes, as more widely known in the literature. The current process in each regime is linear, but the combination of processes produces a nonlinear regime.

A simple example is the autoregressive model of first order, which is subject to changes in the autoregressive parameter and is represented by the following system:

$$\begin{array}{c} Y_t = \phi_1 Y_{t-1} + \epsilon_{1t} \,| se \, r = 1 \\ Y_t = \phi_2 Y_{t-1} + \epsilon_{2t} | se \, r = 2 \end{array} \tag{5}$$

where r represents the current regime. Thus, parameter ϕ_1 describes the behavior of the series when the current regime is 1, but if the current regime is 2, the parameter that will describe the behavior of the series will be ϕ_2.

As we have not observed the stochastic process that determines which the current regime is, we need some way to infer probabilities on which regime is current at period t. The basic idea of Markov Switching model is to describe the stochastic process that determines the switch from one regime to another by means of a Markov Chain. The Markov Chain is used to model the behavior of a state variable (or of a combination of variables) that determines which regime is current, as this variable cannot be directly observed.

A Markov Chain can be represented as follows: Suppose that the probability of a variable s_t assuming some particular j value, depending only on the previous value s_{t-1} is given by the following equation:

$$P\left\{s_t = j | s_{t-1} = k, ...\right\} = P\left\{s_t = j | s_{t-1} = i\right\} = P_{ij} \tag{6}$$

This process is described as a Markov Chain with n-states, whose probability P_{ij} indicates the probability of state i being followed by state j. If we observe that:

$$P_{i1} + P_{i2} + P_{in} = 1 \tag{7}$$

we can build the so-called transition matrix, where line i, column j, give the probability of state i being followed by state j:

$$P = \begin{bmatrix} P_{11} & P_{12} & ... & P_{1n} \\ P_{21} & P_{22} & ... & P_{2n} \\ ... & ... & ... & ... \\ ... & ... & ... & ... \\ P_{n1} & P_{n2} & ... & P_{nn} \end{bmatrix} \tag{8}$$

The main characteristic of this Markov transition matrix of first order is that the probability of transition to the next regime relies only on the current regime, which simplifies the modeling and, especially, the estimation methods.

3.1 Estimation and Specification Tests

To find the correct specification for the Markov Switching model that is more appropriate to our data, we used a combination of the general and specific procedures with an analysis of adequacy of the specifications. This procedure was carried out due to the extremely computationally intensive burden for the estimation of these models and also because the test distribution of the number of regimes does not have a standard distribution.

We departed from a general model with 10 regimes and 12 lags and regime switches in the intercept, in autoregressive parameters and in variance which, by using Krolzig [15] notation, is a MSIAH(10)-ARX(12) (*Markov Switching Intercept Autoregressive Heterocedasticity*) model. We also included a tendency variable in the specification of the model, which proved to be necessary by information criteria.

The models were estimated by means of the EM algorithm proposed by Dempsteir, Laird and Rubin [6] in the form of a BHLK (Baum-Lindgren-Hamilton-Kim) filter, using MSVAR software by Hans Martin Krolzig. In order to discuss this estimation method properly, we would need a presentation that drifts away from the context of application of the method proposed in the present article. The paper written by Krolzig [15] (chapters 5, 6 and 8) discusses this topic in further details.

3.2 Determination of the number of regimes

The major problem with the determination of the appropriate specification for a Markov Switching model is to determine the number of regimes. Tests used to determine the null hypothesis of n-1 regimes against the alternative hypothesis of n regimes do not have a standard distribution, since the null hypothesis is not identified due to the presence of nuisance parameters.

The usual procedure of testing this hypothesis by means of a Likelihood Ratio test is not valid, because the probabilities associated with the additional regime are not identified in the null hypothesis, thus violating the normal conditions of regularity of this test. Although some procedures used for the derivation of the asymptotic distribution have been proposed by Hansen [11] and Garcia and Perron [9], they are not valid for our general model and require the simulation of the data contained in a *grid* of values for the nuisance parameters, which would mean a time-consuming simulation for each specification tested. To determine the number of regimes, we will

Regimes	Log-Lik	Parameters	AIC	BIC	Nuisance	Restrictions
10	8944.3934	170	-9.3794	-8.8765	90	72
9	8924.5196	144	-9.3859	-8.9600	72	64
8	8908.4118	120	-9.3943	-9.0394	56	56
7	8883.6830	98	-9.3974*	-9.1016	42	48
6	8858.2377	78	-9.3856	-9.1549	30	40
5	8832.4047	60	-9.3772	-9.1998*	20	32
4	8769.3628	44	-9.3270	-9.1968	12	24
3	8611.4266	30	-9.1731	-9.0844	6	16
2	8309.3226	18	-8.8630	-8.8097	2	8
1	6516.8861	8	-6.9577	-6.9340	0	0

Table 5: Log-Likelihood and Information Criteria

use 3 methods. The first one consists in using information criteria, since Akaike and Schwartz criteria have shown to never underestimate the minimum number of regimes. The second method involves an approximation to the asymptotic distribution of the test, based on Ang, A. and Bekaert, G. [1]. The third method consists in carrying out specification tests to check the necessity for an additional regime (preferred procedure). Table 5 shows the log-likelihood, AIC and BIC information criteria, and the number of parameters, restrictions and nuisance parameters associated with each regime. We estimate MSIAH models with a number of regimes from 10 to 2, and a model with a regime that corresponds to the linear model. The number of autoregressive lags was selected through AIC and BIC information criteria; a number of 5 lags was considered to be appropriate.

According to table 5 the number of regimes selected by Akaike information criteria (AIC) corresponds to the model with 7 regimes, whereas Bayes (BIC) information criteria (BIC) included 5 regimes. Ang, A. and Bekaert, G. [1] show that the asymptotic distribution of the Likelihood Ratio test between n-1 and n regimes can be approximated by a chi-square distribution, where the number of degrees of freedom is given by the number of nuisance parameters of the model with n regimes plus the number of restrictions imposed by regime n on regime n-1. The test statistics is calculated in a usual fashion in likelihood ratio tests, LR=2(log-likelihood(n)-log-likelihood(n-1)), where n and n-1 are the models with n and n-1 regimes.

A test with a significance level of 1% indicates the necessity for a model with 5 regimes, whereas a significance level of 5% shows the necessity for a model with 6 regimes. This test is, however, based on an approximation to the correct critical values and therefore we need further support in order to decide on the optimal number of regimes.

The criterion used to decide on the necessary number of regimes arose

Test	Stat.	Dist	*p-value*
1 against 2	3854.873	Chi^2(10)	0.0000 **
2 against 3	604.208	Chi^2(14)	0.0000 **
3 against 4	315.8724	Chi^2(20)	0.0000 **
4 against 5	126.0838	Chi^2(28)	0.0000 **
5 against 6	51.666	Chi^2(38)	0.0487*
6 against 7	50.8956	Chi^2(50)	0.4381
7 against 8	49.4576	Chi^2(64)	0.9095
8 against 9	32.2156	Chi^2(80)	1.0000
9 against 10	39.7476	Chi^2(98)	1.0000

Table 6: LR Test - Ang e Bekaert- Number of Regimes

BDS Test	Residuals	6	Regimes		BDS Test		BDSSTDR7		
Dim	BDS Stat.	Std. Err.	z Stat	*p-value*	Dim	BDS Stat.	Std. Err.	z Stat	*p-value*
2	-0.002602	0.001363	-1.909946	0.0461	2	-0.001400	0.001307	-1.071312	0.2840
3	-0.004220	0.002159	-1.954688	0.0406	3	-0.000443	0.002068	-0.214278	0.8303
4	-0.003577	0.002562	-1.395865	0.1628	4	0.001066	0.002451	0.434681	0.6638
5	-0.003316	0.002661	-1.245971	0.2128	5	0.002315	0.002543	0.910344	0.3626
6	-0.002880	0.002557	-1.126457	0.2600	6	0.002775	0.002441	1.137098	0.2555

Table 7: BDS Test - Specification

from the idea of checking the necessity for an additional regime by means of specification tests. A test through which our viewpoint proves adequate within this context is BDS statistics, a robust test used to determine the presence of remaining structures both in the mean and in the variance of the process. Therefore, BDS statistics is efficient in checking whether the proposed specification can capture the whole structure of n-1 regimes.

Table 7 shows the results of the BDS statistics applied to the residuals of the models with 6 and 7 regimes. In the residuals of the model with 6 regimes, BDS statistics rejects at 5% that these regimes are IID in dimensions 2 and 3, showing some evidence that,with 6 regimes, there is still some uncaptured structure in the mean and/or variance. To check whether 7 regimes are enough to capture all the structure present in the mean and variance, we applied the BDS statistics to the residuals of the model with 7 regimes. The results of this test show that it was not possible to refute the residuals of the model with MSIAH(7)-ARX(5)as being IID, which indicates that we should work with 7 regimes, thus capturing all the dependence structure present in the exchange rate log-return series

3.3 Determination of the functional form

Contrariwise to the determination of the number of regimes, the specification test for the most appropriate functional form, in relation to the parame-

Standardized Residuals	Dist.	Stat.	*p-value*
Portmanteau(31)	Chi(26)	22.4019	[0.6665]
normality	Chi(2)	10.7503	[0.004] **
asymp. norm.	Chi(2)	9.7392	[0.0077] **
heterocedasticity	Chi(12)	12.6262	[0.3968]
hetero-χ test	Chi(27)	39.6994	[0.0546]
hetero Squared:	Chi(12)	12.6440	[0.3954]
hetero-χ Squared	Chi(27)	39.7478	[0.0541]
ARCH(5)	Chi(5)	5.44	[0.3536]

Table 8: Specification Tests - Residuals

Unrestricted Model	Restricted Model	Restrictions	Test Stat	Critical Value	*p-value*
MSIAH(7) 8883.683	MSIA(7) 7667.708	6	2341.94	14.0671	[0.0000] **
MSIAH(7) 8883.683	MSIH(7) 8743.989	35	279.38	49.8118	[0.0000] **
MSIAH(7) 8883.683	MSAH(7) 8866.492	6	34.38	14.0671	[0.0000] **
MSIAH(7) 8883.683	MSI(7) 7134.9259	42	3497.51	58.1240	[0.0000] **

Table 9: LR Tests - Functional Form

ters subject to Markov Switching, has a standard distribution. By using the principle of Likelihood Ratio, the test statistics is 2 (Log-likelihood (unrestricted model)- Log-likelihood (restricted model)), and the distribution is a chi-square distribution with the number of degrees of freedom corresponding to the number of imposed restrictions. We tested the general MSIAH (intercept changes, autoregressive parameters and variance) model against the other possible specifications. Table 9 shows that, according to the LR (Likelihood Ratio) test, the general MSIAH specification is the most appropriate specification, with changes in all parameters, since the other specifications, which determine that some parameters be constant throughout the regimes, are inappropriate.

The analysis of the residuals, presented in table 8 and in graph 2 shows that the prediction errors of the model have autocorrelation problems, ARCH and heteroskedasticity. However, the standardized residuals, that is, those divided by the variance that corresponds to the regime to which they have great probabilities of belonging, are uncorrelated, homoskedastic and do not have a conditional ARCH structure. The correction of the switching structure in the unconditional variance by the standardization of residuals shows that, for the exchange rate log-return series, most of the structure present in the series is generated by the changes in unconditional variance.

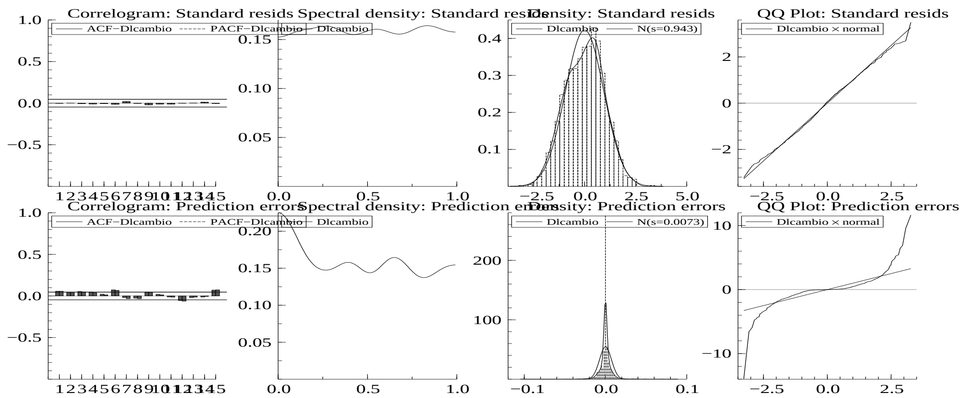

Figure 2: Residual Analysis

3.4 Estimated Model

The estimated MSIAH(7)-ARX(5) model corresponds to the following specification:

$$\begin{cases} y_{1t} = c_{1t} + \phi_{11} y_{1t-1} + \phi_{12} y_{1t-2} + \phi_{13} y_{1t-2} + \phi_{14} y_{1t-4} + \phi_{15} y_{1t-5} + \beta_1 t + \varepsilon_{1t} & \varepsilon_{1t} \sim N\left(0, \sigma_1^2\right) \\ y_{2t} = c_{2t} + \phi_{21} y_{2t-1} + \phi_{22} y_{2t-2} + \phi_{23} y_{2t-2} + \phi_{24} y_{2t-4} + \phi_{25} y_{2t-5} + \beta_2 t + \varepsilon_{2t} & \varepsilon_{2t} \sim N\left(0, \sigma_2^2\right) \\ y_{3t} = c_{3t} + \phi_{31} y_{3t-1} + \phi_{32} y_{3t-2} + \phi_{33} y_{3t-2} + \phi_{34} y_{3t-4} + \phi_{15} y_{3t-5} + \beta_3 t + \varepsilon_{3t} & \varepsilon_{3t} \sim N\left(0, \sigma_3^2\right) \\ y_{4t} = c_{4t} + \phi_{41} y_{4t-1} + \phi_{42} y_{4t-2} + \phi_{43} y_{4t-2} + \phi_{44} y_{4t-4} + \phi_{45} y_{t-5} + \beta_4 t + \varepsilon_{4t} & \varepsilon_{4t} \sim N\left(0, \sigma_4^2\right) \\ y_{5t} = c_{5t} + \phi_{51} y_{5t-1} + \phi_{52} y_{5t-2} + \phi_{53} y_{5t-2} + \phi_{54} y_{5t-4} + \phi_{55} y_{5t-5} + \beta_5 t + \varepsilon_{5t} & \varepsilon_{5t} \sim N\left(0, \sigma_5^2\right) \\ y_{6t} = c_{6t} + \phi_{61} y_{6t-1} + \phi_{62} y_{6t-2} + \phi_{63} y_{6t-2} + \phi_{64} y_{6t-4} + \phi_{65} y_{6t-5} + \beta_6 t + \varepsilon_{6t} & \varepsilon_{6t} \sim N\left(0, \sigma_6^2\right) \\ y_{7t} = c_{7t} + \phi_{71} y_{7t-1} + \phi_{72} y_{7t-2} + \phi_{73} y_{7t-2} + \phi_{74} y_{7t-4} + \phi_{75} y_{7t-5} + \beta_7 t + \varepsilon_{7t} & \varepsilon_{7t} \sim N\left(0, \sigma_7^2\right) \end{cases} \tag{9}$$

The estimated parameters are shown in table 10, along with the standard deviations and t statistics associated with each parameter. While there are regimes in which all the parameters are significant, as in regime 1, no parameter is statistically significant in regime 2. Another interesting fact is that the tendency is only different from zero in regime 7, but the presence of the tendency was confirmed by specification tests. The transition matrix is shown in table 12.

The figure 3 shows the estimated probabilities of each regime for each observation in the sample. The graph shows the forecast, filtered (using the information up to period t) and the smoothed probabilities (using the information of the whole sample to infer the probabilities at moment t). This graph shows us that the model associates 3 exclusive regimes (regimes 2, 3 and 4) with the exchange band regime, while the other regimes are present in the remaining regimes of the sample. Regimes 1, 5 and 6 are identified with the periods of free exchange rate variation, whereas regime 7 can be identified, according to table 10, with the periods of exchange rate crisis, thus combining high variance of the exchange rate with a tendency towards

Regime 1	Coef.	Std.Err.	*t-value*
Const	-0.0209	0.0019	-11.1947
Dlcambio_1	-0.1225	0.0940	-1.3037
Dlcambio_2	-0.3699	0.0892	-4.1483
Dlcambio_3	-0.5672	0.0996	-5.6919
Dlcambio_4	-0.2190	0.0906	-2.4176
Dlcambio_5	-0.7564	0.0791	-9.5628
Trend	0.0000	0.0000	0.4548
Std.Err.	0.0095764		

Regime 2	Coef.	Std.Err.	*t-value*
Const	-0.0004	0.0002	-1.9478
Dlcambio_1	-0.0007	0.0635	-0.0114
Dlcambio_2	-0.0083	0.0539	-0.1547
Dlcambio_3	-0.0427	0.0527	-0.8097
Dlcambio_4	0.0837	0.0484	1.7289
Dlcambio_5	-0.0021	0.0497	-0.0414
Trend	0.0000	0.0000	0.0422
Std.Err.	0.0042088		

Regime 3	Coef.	Std.Err.	*t-value*
Const	0.0002	0.0001	2.0211
Dlcambio_1	-0.0146	0.0663	-0.2201
Dlcambio_2	-0.1437	0.0622	-2.3103
Dlcambio_3	-0.0203	0.0607	-0.3343
Dlcambio_4	-0.1258	0.0526	-2.3922
Dlcambio_5	0.0795	0.0674	1.1790
Trend	0.0000	0.0000	0.0345
Std.Err.	0.0004381		

Regime 4	Coef.	Std.Err.	*t-value*
Const	-0.0000	0.0000	-1.9315
Dlcambio_1	-0.0150	0.0266	-0.5664
Dlcambio_2	-0.0873	0.0180	-4.8547
Dlcambio_3	0.0309	0.0154	2.0102
Dlcambio_4	-0.0646	0.0136	-4.7414
Dlcambio_5	-0.0241	0.0151	-1.5928
Trend	0.0000	0.0000	0.0333
Std.Err.	0.0001678		

Regime 5	Coef.	Std.Err.	*t-value*
Const	0.0021	0.0001	15.8368
Dlcambio_1	-0.1979	0.0713	-2.7755
Dlcambio_2	-0.1540	0.0745	-2.0686
Dlcambio_3	-0.1840	0.0735	-2.5049
Dlcambio_4	-0.1806	0.0752	-2.4018
Dlcambio_5	-0.2012	0.0758	-2.6533
Trend	-0.0000	0.0000	-0.2335
Std.Err.	0.0013201		

Regime 6	Coef.	Std.Err.	*t-value*
Const	-0.0000	0.0004	-0.1106
Dlcambio_1	0.1077	0.0486	2.2137
Dlcambio_2	-0.1328	0.0453	-2.9307
Dlcambio_3	0.1168	0.0458	2.5513
Dlcambio_4	-0.0053	0.0436	-0.1227
Dlcambio_5	0.0421	0.0420	1.0007
Trend	0.0000	0.0000	0.0860
Std.Err.	0.0084794		

Regime 7	Coef.	Std.Err.	*t-value*
Const	0.0924	0.0062	14.9538
Dlcambio_1	-0.2350	0.1496	-1.5707
Dlcambio_2	0.1601	0.1539	1.0405
Dlcambio_3	0.2282	0.1587	1.4377
Dlcambio_4	-0.1669	0.1628	-1.0256
Dlcambio_5	-0.2677	0.1648	-1.6237
Trend	-0.0001	0.0000	-3.6738
Std.Err.	0.0227350		

Table 10: Estimated Parameters

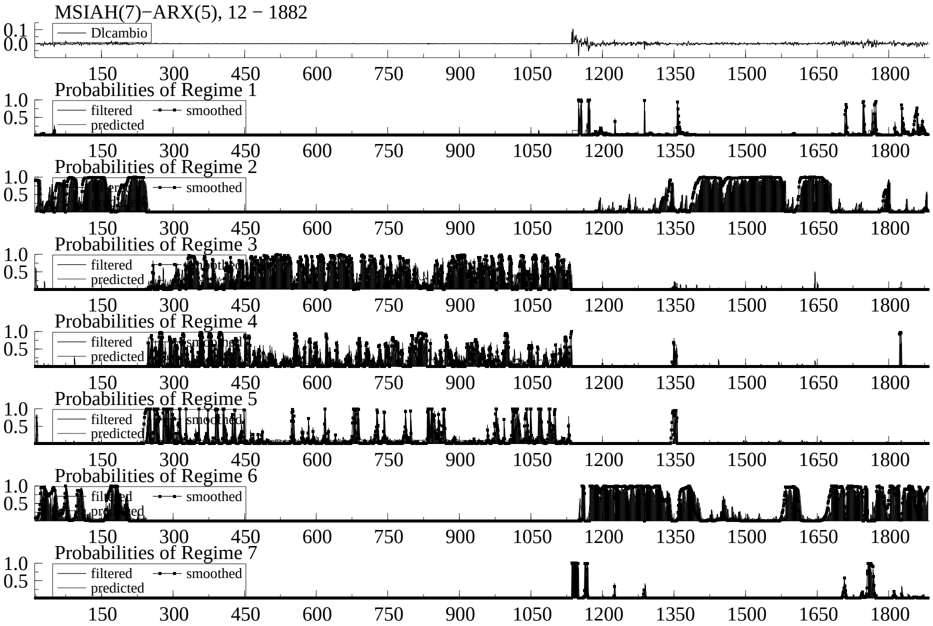

Figure 3: Estimated Probabilities

strong devaluation.

Table 13 shows the number of observations for each regime, and associates an unconditional probability with each one of the regimes. Regimes 2, 3 and 4, when added, have an associated probability of 0.5933, which corresponds approximately to the percent values of the sample of the band regime (1131/1882=0.6024). Regimes 4, 5 and 6 with a joint probability of 0,4985, correspond to free and controlled periods of fluctuation of the exchange rate. Regimes 1 and 7 correspond to regimes with low probabilities. Regime 1 is associated with periods of continuous exchange rate devaluation, whereas regime 7 is characterized by strong exchange rate devaluation and high volatility.

The association of regimes 3, 4 and 5 with the exchange band regime can be seen on the graph of probabilities associated with regimes and also on the transition matrix. The transition matrix shows that the transition probabilities of regimes 3, 4 and 5 for the other regimes are almost null, but very significant between them. We can interpret the existence of 3 regimes within the periods of exchange bands as the presence of 2 regimes acting as the upper and lower limits of the band, and a third regime as the exchange rate value within the limits of the band. By looking at the parameters estimated for these regimes, we can identify regime 3 as being the regime with normal values within the limits of the exchange band, while the values of regime 4 are close to the lower limit, and regime 5 contains the returns with

	regime1	regime2	regime3	regime4	regime5	regime6	regime7
regime1	0.7122	0.0009091	2.585e-008	6.118e-009	1.005e-007	0.2869	1.217e-011
regime2	8.171e-007	0.9744	8.937e-008	2.735e-008	0.004562	0.02103	3.011e-011
regime3	2.909e-011	9.485e-008	0.8714	0.06670	0.06194	9.380e-009	1.885e-015
regime4	0.003840	1.068e-007	0.1716	0.7283	0.09189	1.926e-008	0.004342
regime5	0.004077	6.174e-007	0.06717	0.1963	0.7324	7.645e-008	7.992e-015
regime6	0.01168	0.02171	1.034e-008	0.002224	4.566e-008	0.9558	0.008568
regime7	0.1250	8.225e-007	3.491e-011	8.977e-012	1.277e-010	1.128e-007	0.8750
Note: p[i][j]=Pr{s(t+1)=j|s(t)=i}							

Table 12: Transition Matrix

values close to the upper limit of the band.

This conclusion is consistent with the mean values of duration of these regimes, since the mean duration of regimes 4 and 5 is 3.5 days on average, while the mean duration of regime 3 is 7.8 days. The fact that the duration of these regimes in effect during the exchange band regime is short, shows that frequent interventions in the exchange rate market were necessary to keep the value within the intervals established by the Central Bank. We can observe that regime 2 is the most persistent, with an average duration of 38.08 days, followed by regime 6 with 22.63 days. These two regimes are characterized by the lowest correlations between the estimated regimes. The other regimes are much less persistent, with frequent switches.

An interesting characteristic is that regime 7, associated with moments of crisis in the exchange market, even with the lowest unconditional probability among all regimes, has the third highest mean persistence, lasting, on average, 8 days. This occurs because the probability of being in regime 7 compared with the probability of remaining in regime 7 is 87%, according to the transition matrix. It is also relevant to observe that regime 1, which is associated with periods of exchange rate devaluation, has a greater probability of preceding regime 7. This characteristic is consistent with the existence of extreme values and groups of high volatility in the series, which is frequently modeled through GARCH models.

The eigenvalues of the transition matrix are shown in table 14. Since the first eigenvalue is equal to one and the other eigenvalues are within the unit circle, the transition matrix is ergodic, as the transition matrix is also irreductible. Thus, the eigenvector associated with the unit eigenvalue present in 14 represents the ergodic probabilities of the process. This vector also indicates the unconditional probability of each regime, and therefore we build table 13. The fact that the transition matrix is ergodic confirms that our regime is stationary since, according to Hamilton [10] (pages 681 and 682) a Markov switching process with an ergodic transition matrix is always covariance-stationary.

	Number of Observations	Unconditional Probability	Duration
Regime 1	42.3	0.0233	3.47
Regime 2	420.7	0.2171	39.08
Regime 3	452.9	0.2388	7.77
Regime 4	256.4	0.1374	3.68
Regime 5	201.1	0.1062	3.74
Regime 6	457.3	0.2549	22.63
Regime 7	40.2	0.0222	8.00

Table 13: Regimes and Duration

real	1.0000	0.99353	0.95765	0.85702	0.73215	0.70606	0.60312

Table 14: Eigenvalues of Transition Matrix

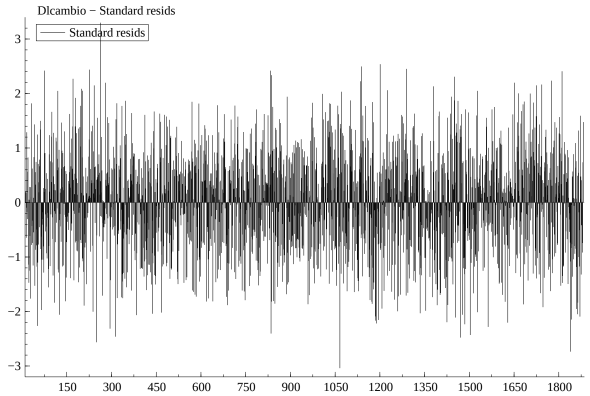

Figure 4: Standardized Residuals

Regime	Dist.	Test Stat.	*p-value*
Regime1	$\chi^2(7)$	270.004	[0.0000] **
Regime2	$\chi^2(7)$	6.98191	[0.4308]
Regime3	$\chi^2(7)$	11.3812	0.1228
Regime4	$\chi^2(7)$	162.51	[0.0000] **
Regime5	$\chi^2(7)$	288.391	[0.0000] **
Regime6	$\chi^2(7)$	17.2896	[0.0120]*
Regime7	$\chi^2(7)$	477.018	[0.0000] **
h0: $\mu_{1s} = \phi_{1s} = \phi_{2s} = \phi_{3s} = \phi_{4s} = \phi_{5s} = \beta_{trend} = 0 \mid s = Regime$			

Table 15: Wald Tests for Exclusion

4 Market Efficiency Tests in Markov Switching Model

The method proposed for testing market efficiency, which is similar to that presented in section 1.6, consists in testing whether the autoregressive coefficients of each regime are altogether statistically different from zero. To carry out this test, we used the principle of Wald test, in which we only have to estimate the unrestricted model, since the estimation of a restricted model would be computationally complex, and the asymptotic distribution of the Wald test is valid for the tests that were carried out.

The first exclusion test consisted in testing whether all parameters, except the estimated variance, were equal to zero, in contrast with the alternative hypothesis that at least one of them was different from zero. The objective of this test was to verify for which regimes the hypothesis of Market Efficiency was valid. We observed that, under the null hypothesis, the logarithm of prices is a random walk; therefore, the log-returns (first price difference in log) are uncorrelated.

As seen in section 1.6.2, this can be tested through a joint test, which verifies whether all the autoregressive parameters are equal to zero, against the null hypothesis that at least one of them is different from zero. Here we tested the strong hypothesis that the intercept and the tendency component are equal to zero as well, due to the fact that in the Markov switching model the need for inferences on these parameters, which are linked to the current regime, could also represent possibilities of arbitrage.

Table 15 shows us that we can reject the null hypothesis that all the parameters are equal to zero for regimes 1,4,5,6, and 7. The null hypothesis is not rejected for regimes 2 and 3. As the null hypothesis is not rejected in these regimes, we considered that the random walk III model would be valid, and in addition, the log-returns would have mean zero in these two regimes.

The value of the calculated Wald statistics shows us that the evidence of deviations in relation to Weak Market Efficiency is obtained at the periods of continuous exchange rate devaluation, which we identify with regimes 1 and 7, respectively. During regime 7, especially, the bet on exchange rate devaluation seems to lead to excessive gains, since the intercept component in this regime is very significant, showing a linear tendency towards devaluation while the regime is valid; however, we note that the risk of this stake is extremely high as the variance in this regime is much higher than that of other regimes.

The evidence collected by the Markov switching model shows that, while there are no significant tendencies towards a change in exchange rate values (poorly significant intercept components and tendency) in regimes 2,3,5,and 6, regime 1 can be identified as a period of high and relatively calm (in relation to regime 7) exchange rate devaluation. Regime 7 is, however, a period of strong exchange rate devaluation, accompanied by an extremely high volatility.

4.1 Variance Ratio in Markov Switching Model

The Variance Ratio tests used in section 2.7 whose critical test values derived from sample distributions simulated with Monte Carlo procedures have serially uncorrelated returns under the null hypothesis. To analyse the effect of the heteroskedasticity in the asset returns, Poterba and Summers [19] have studied the VR statistics behavior under homoskedastic and heteroskedastic conditions, and have found no significant differences in the results. However, in a later study, Kim, Nelson and Startz [13] have used a different strategy to define the VR test under heteroskedastic conditions. They have used a strategy that allows preserving the historical pattern of volatility present in the series.

The description of this procedure follows the presentation of Kim and Nelson [14], chapter 11, who adapt the stratification in the randomness of returns proposed by Kim, Nelson and Startz, to the volatility pattern presented by Markov Switching model in unconditional variance. Kim and Nelson's procedure [14] consists in using the returns standardized by the variance estimated for each period of the sample by the Markov Switching model in order to calculate the VR statistics. This way, the returns would preserve the information contained in heteroskedasticity and therefore the test would have more power by allowing the specific volatility patterns presented in the studied series to be considered for the test specification. As this procedure is based on the variance patterns presented by each series, it is necessary to derive the critical values for each sample via Monte Carlo methods.

The study conducted by Kim and Nelson was based on the estimation

q	VR	psi	*p-value*
5	1.0686	0.3733	0.373
10	1.1498	0.5393	0.5897
20	0.9579	-0.1071	0.9147
50	0.9504	-0.0821	0.9346
75	0.9537	-0.0640	0.9490
100	1.0179	0.0219	0.9825
150	1.0295	0.0309	0.9754
200	1.1194	0.1144	0.9089
300	1.1877	0.1635	0.8701

Table 16: Variance Ratios - Standardized Residuals

of the model for Markov Switching via Gibbs sampling methods, which directly allows us to obtain the random stratification used. Here, we will use a simpler procedure, whose aim is to test the value of Variance Ratios for risk-adjusted returns, that is, the returns standardized by the variances of Markov Switching model.

The Variance Ratios calculated for the standardized returns are shown in table 16. The estimated values show that we cannot reject the fact that the standardized log-returns originate from a random walk model, thus indicating that when we correct the returns by means of their volatility, the random walk model, rejected in regimes 1,4,5,6 and 7, cannot be rejected after risk adjustment.

This result is consistent with the results obtained from the literature on *Equivalent Martingale Measure*, which proposes that, after the adjustment for risks and dividends, the asset prices should be a martingale process. A brief presentation on the Equivalent Martingale Measure can be found in Ljungqvist and Sargent [17], pages 233-236.

An important conclusion of this result is to show that the violation of the random walk III model, that is, the existence of correlation between exchange rate log-returns is caused by changes in the structure of the series variance.

5 Conclusions

The discussion about Weak Market Efficiency produces two interesting results. The first one shows that there are regimes in the series in which the market efficiency is valid (regimes 2 and 3), while in the other regimes, we reject market efficiency by means of a Wald test; therefore, there is a possibility of arbitrage and excessive gains through exchange rate transactions, which only confirms some results observed in the exchange rate market in this period.

The fundamental issue is whether this nonlinear structure present in the Markov Switching Model is really a violation of the hypothesis of Weak Market Efficiency in terms of the exploration potential of these patterns with the aim of obtaining significant gains. The results obtained from the Markov Switching Model show that, even after the identification and estimation of the regimes associated with structural breaks in this series, some patterns of mean persistence can be found.

These remaining patterns correspond to the autoregressive processes that are statistically significant in regimes 1,4,5,6, and 7. To verify whether these patterns are actually a violation of the hypothesis of Weak Market Efficiency, it is necessary to analyze if these autoregressive structures are strong enough to resist the possibility of arbitrage. In this aspect, we note that regimes 4, 5 and 6 have an autoregressive structure, which in spite of being statistically significant, is very weak, and would not be economically feasible under the current conditions of transactions costs.

However, regime 1 and 7 represent real possibilities of speculative gain, since they are associated with periods of intense devaluation of the dollar vis-a-vis the Real, and correspond to the periods during which speculation actually occurred. Regime 1 is associated with the days that followed September 11, 2001, characterized by strong instability in the world markets. This regime is the only component of the series that is closer to being nonstationary, which can be seen by the eigenvalues associated with this process. Regime 7 is linked to the period right after the switch to the floating exchange caused by speculative attacks. It is also worth mentioning that regime 7 is the one with the highest mean duration among all the current regimes in periods of free exchange rate.

However Weak Market Efficiency continues valid, however, if we consider the exchange rate log-return series after the risk adjustment. This adjustment, which consists in standardizing the returns by using the variance associated with each return estimated by Markov Switching Model, allows us to reject the hypothesis that these adjusted returns stem from a random walk model, which is associated with the unpredictability of innovations that define the hypothesis of Market Efficiency. This evidence was obtained through the Variance Ratio statistics applied to the standardized returns.

The hypothesis of Weak Market Efficiency can also be justified within the context of a Markov Switching model by the computational complexity of making predictions when the set of parameters that are liable to regime switch includes autoregressive parameters. In this case, the out-of-sample forecast becomes a relatively complex nonlinear process and the number of possible predictions depends on the number of existing regimes, which grows exponentially with the number of periods ahead. In addition, the autoregressive structure is too weak, except in regimes 1 and 7. The level of uncertainty associated with these predictions is also a complex problem, as the construction of confidence intervals for the future predictions also becomes a nonlinear process.

Although the Markov switching model allows using the information available from the sample as a mechanism to investigate the inefficiency or misuse of such information, we have to make some considerations. The coefficients of determination of the model (0.43462 nonadjusted and 0.3011 adjusted) are much higher than those in the linear model, but even so they are just reasonable, considering the elaboration of predictions. The other problem is technical. Making out-of-sample forecasts in a MSIAH process is a complicated procedure, as the equation for the prediction in this model is a nonlinear process of the estimated parameters and data. The third restriction is that in this model, we have an additional element of uncertainty, which are the probabilities of the value observed in a specific moment belonging to each regime. It is also relevant to observe that the most significant data patterns were found in regime 7, which indicated a strong tendency towards devaluation but, at the same time, was the regime with higher variance and risk.

Therefore, the use of the Markov Switching Model as an arbitrage mechanism has to be carefully considered. Although Weak Market Efficiency is rejected by the data, we question whether it is really possible to explore the inefficiencies captured by the model in an effective way. A more comprehensive concept of Market efficiency that takes into consideration strategies that make the most of market inefficiency is therefore necessary.

The specific literature is the one in charge of analyzing whether the use of filter rules and a technical analysis will lead to relevant gains. An extended version of the analysis presented in this chapter could verify whether the gains obtained by the predictions made by the MSIAH model are significant, including transactions costs.

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