Fractal structure in the Chinese yuan/US dollar rate

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Abstract

Price changes of the Chinese yuan/US dollar rate are found to display a Sierpinski triangle in an Iterative Function System clumpiness test. This fractal structure commonly emerges in "the chaos game", where randomness coexists with deterministic rules. We show that a threshold model with four states, two deterministic and two stochastic is able to replicate the properties of the yuan/dollar returns in general, and the Sierpinski triangle in particular.

Keywords: Autocorrelation Function; Foreign Exchange Rates; Iterative Function System; Sierpinski Triangle; Threshold Models

1. Introduction

Regime switching models include Markov switching and threshold models. In a Markov switching model, a regime that occurs at time t cannot be directly observed and is determined by a random variable R_t. For r regimes, R_t can be assumed to take values from 1 to r. The Markov regime switching model usually assumes that R_t is a first order Markov process where the current regime depends only on R_{t-1}. This means that a regime is defined by the probability of transition between regimes.

Threshold models with non additive noise (e.g. Tong 1990) also assume that a regime is determined by a unobservable random variable; however, the variables are independent from one another. What is more (and interestingly) realizations of these classes of models can be fractals.

This paper assesses the behavior of the Chinese yuan/US dollar exchange rate returns over time by considering a suggested threshold model with both additive and non additive noise. We employ classic times series tools (such as the autocorrelation function, partial autocorrelation function, and Ljung-Box test) only to have the null hypothesis that the series follows a simple uncorrelated random process rejected. However, when we employ a chaos-game algorithm known as Iterated Function System (IFS) (Barnsley 1988), the series turns out to be "pseudo-random" in that the data clump together in a fractal structure easily recognizable as a Sierpinski triangle. This is suggestive of some sort of global determinism in the data. Both the deterministic and stochastic structure are best visualized by a phase diagram of the time series which plots current values against lagged ones (Tong 1990). We then put forward our threshold model with additive and non additive noise that seems to fit well the yuan/dollar rate behavior.

The paper is organized as follows. Section 2 presents the data, Section 3 analyzes them, and Section 4 concludes.

2. Data

The data set of daily Chinese yuan/US dollar rates was collected by the Federal Reserve Bank of New York from a sample of market participants. The yuan/dollar rates are noon buying rates in New York from cable transfers payable in the foreign currency. The data set is available online from http://www.federalreserve.gov/releases/H10/hist/. The sample covers the time period ranging from 2 January 1981 to 1 November 2002 and constitutes a series of 5426 data points. As standard, "holes" from weekends and holidays are ignored and analysis concentrates on trading days.

Price changes $X(t) = Z(t) - Z(t-1)$ are initially taken, where $Z(t)$ is a yuan/dollar rate at time t. Figure 1 displays the time evolution of $Z(t)$ and $X(t)$ where the spikes correspond to episodes of foreign exchange intervention from the part of the Chinese monetary authorities.

3. Analysis

Figure 2 displays the logarithm of the probability density of the yuan/dollar price changes by dropping four major episodes of intervention. We define these four structural breaks to occur whenever $X(t) > 0.05$. Since a symmetric joint distribution emerges, Gaussian stationary ARMA models are not appropriate to describe the data. Indeed there is marked asymmetry as well as a peak higher than that expected for a Gaussian distribution.

Figure 3 shows the autocorrelation function and partial autocorrelation function of $X(t)$. The autocorrelations are not significantly different from zero after the first lag. Thus we cannot discard that $X(t)$ is a uncorrelated random process. We also carried out other statistical tests to further check for randomness. Results are in Table 1. The extra tests all fail to identify known autocorrelation structures. There is no evidence against the hypothesis of pure randomness.

By contrast, an IFS clumpiness test gives us a different result and casts doubt on the complexity of the series. In a picture displaying an IFS clumpiness test, white noise fills it uniformly whereas correlated noise generates localized clumps. For white noise, the figure generated tends toward the orbit. For correlated noise, the plot reveals suggestive particular features. The test was carried out using *Chaos Data Analyzer* (Sprott 1995). Clumps appear for the entire period (upper panel in Figure 4). Surprisingly, the data happen to idiosyncratically clump together and form a Sierpinski triangle. We checked for and found that this fractal emerges for some transformations of the original data. This is the case for log differences ($\log Z(t) - \log Z(t-1)$), natural log differences ($\ln Z(t) - \ln Z(t-1)$), relative differences ($(Z(t) - Z(t-1))/Z(t)$), and relative changes ($(Z(t) - Z(t-1))/\{(Z(t) + Z(t-1))/2\}$).

It is reasonably well established that the presence of a Sierpinski triangle is somewhat suggestive that a system is locally random and globally deterministic because the Sierpinski triangle is a self-similar fractal that results from a deterministic rule implemented in a random fashion. In literature, this feature is dubbed "the chaos game" (Barnsley 1988; also Peitgen, Jurgens and Saupe 1992, Chapter 6). Details on the implementation of the IFS together with its relation to the chaos game can be found elsewhere (e.g. Mata-Toledo and Willis 1997). The finding that the Sierpinski triangle appears in the yuan/dollar returns may be explained by the fact that the Chinese authorities might have been playing the chaos game when pegging their currency. The yuan is convertible for current account transactions in the balance of payments but not convertible for capital transactions. Thus it is not come as a surprise that the Chinese authorities might have been engaging in stable rules of intervention in the current account.

The intricate nested pattern of the Sierpinski triangle can also result from simple rules set at each step in a computer program known as "cellular automaton". Here a particular rule might be as follows. A cell should be black whenever one or the other, but not both, of its neighbors were black on the step before (Wolfram 2002, p. 25). One might think of "black" or "white" as "sell" or "buy" dollars and try to accurately infer the rule actually pursued by the Chinese monetary authorities. But we leave this suggestion for future research.

Figure 5 shows a plot of $X(t)$ against $X(t-1)$ where points are not tracked and the four jumps are left out. First, a great deal of data points cluster at $X(t) \approx 0$. Secondly, some observations seem to follow rule $X(t) = X(t-1)$. Also, a straight line obtains in a regression of $X(t)$ on $X(t-1)$; and this is suggestive of an autoregressive state. We put forward that these distinct features seem to suggest that the data can be captured by a threshold model with both additive and non additive noise. As observed, realizations of these models can be fractals.

The random mechanism we empirically assessed in the yuan/dollar price change is a sequence of independent trials with four states, namely

(1) $X(t) \approx 0$ with approximate probability of 0.6;

(2) $X(t) = X(t-1)$ with probability of 0.12;

(3) $X(t) = 0.5\ X(t-1) + 0.0003 + \varepsilon(t)$ (where $\varepsilon(t)$ is an independent white noise process with zero mean and standard deviation equaling 0.006) with probability of 0.279; and

(4) $X(t) = 1.5\ \eta(t)$ with probability of 0.001 ($\eta(t)$ is also an independent white noise process with zero mean and standard deviation of 1).

Table 2 summarizes the four states.

We assume that such a mechanism is independent of $X(s)$, $\varepsilon(s)$ and $\eta(s)$ at time t for $s < t$. States 1 and 2 are deterministic whereas states 3 and 4 are stochastic. State 3 is a first order autoregressive process with the first autocorrelation set to 0.5, and state 4 is supposed to capture the four episodes of intervention leading to the vertical shifts.

To assess the validity of our model, Figure 6 presents a simulated realization using 5000 data points. The lower panel in Figure 4 shows that the Sierpinski triangle is able to emerge with the simulated realization of our model. Also, Figure 7 shows a phase diagram with no jumps that almost entirely replicates the one at the bottom in Figure 5. Thus our threshold model with additive and non additive noise seems to model the yuan/dollar returns well.

4. Conclusion

Price changes of the Chinese yuan/US dollar rate display a Sierpinski triangle in an IFS clumpiness test. An explanation tentatively advanced for the presence of this type of determinism in data is that the Chinese authorities might have been playing "the chaos game" with their currency.

Since the Sierpinski triangle is a self-similar fractal that results from a deterministic rule implemented in a random fashion, we employ a model to capture both determinism and randomness. Our threshold model with additive and non additive noise seems to replicate the essential features of the yuan/dollar rate series in general, and the emergence of the Sierpinski triangle in particular.

Test	Null Hypothesis H_0	Alternative Hypothesis H_A	Statistic of the Test	P-Value	Decision
Ljung-Box Type[1]	White Noise	Non White Noise	4.67 With 120 Degrees of Freedom	~1.000	We Cannot Reject H_0
Durbin-Watson[2]	$\rho(h) = 0$ Where $\rho(h)$ is the Correlation Between $X(t)$ and $X(t+h)$	$\rho(h) \neq 0$	About 2.00 For $h = 1, 2, 3, \ldots 200$.	> 0.5000	
McLeod–Li[3]	Data Set is an IID Gaussian Sequence	Data Set is Not an IID Gaussian Sequence	0.07 With 50 Degrees of Freedom	~1.000	
Turnings Points[3]	Data Set is an IID Sequence	Data Set is Not an IID Sequence	501	~1.000	
Difference–Sign[3]			405	~1.000	
Rank Test[3]			489.248	~0.9965	

Table 1. Autocorrelation check for the presence of white noise.

Notes:
1 SAS/ETS/PROC ARIMA Version 8.2
2 SAS/ETS/PROC AUTOREG Version 8.2
3 ITSM96, Program PEST (Brockwell and Davis 1991)

$X(t)$	Probability
$X(t) \approx 0$	0.600
$X(t) = X(t-1)$	0.120
$X(t) = 0.5\, X(t-1) + 0.0003 + \varepsilon(t)$	0.279
$X(t) = 1.5\, \eta(t)$	0.001

Table 2. Model of four states for the data.

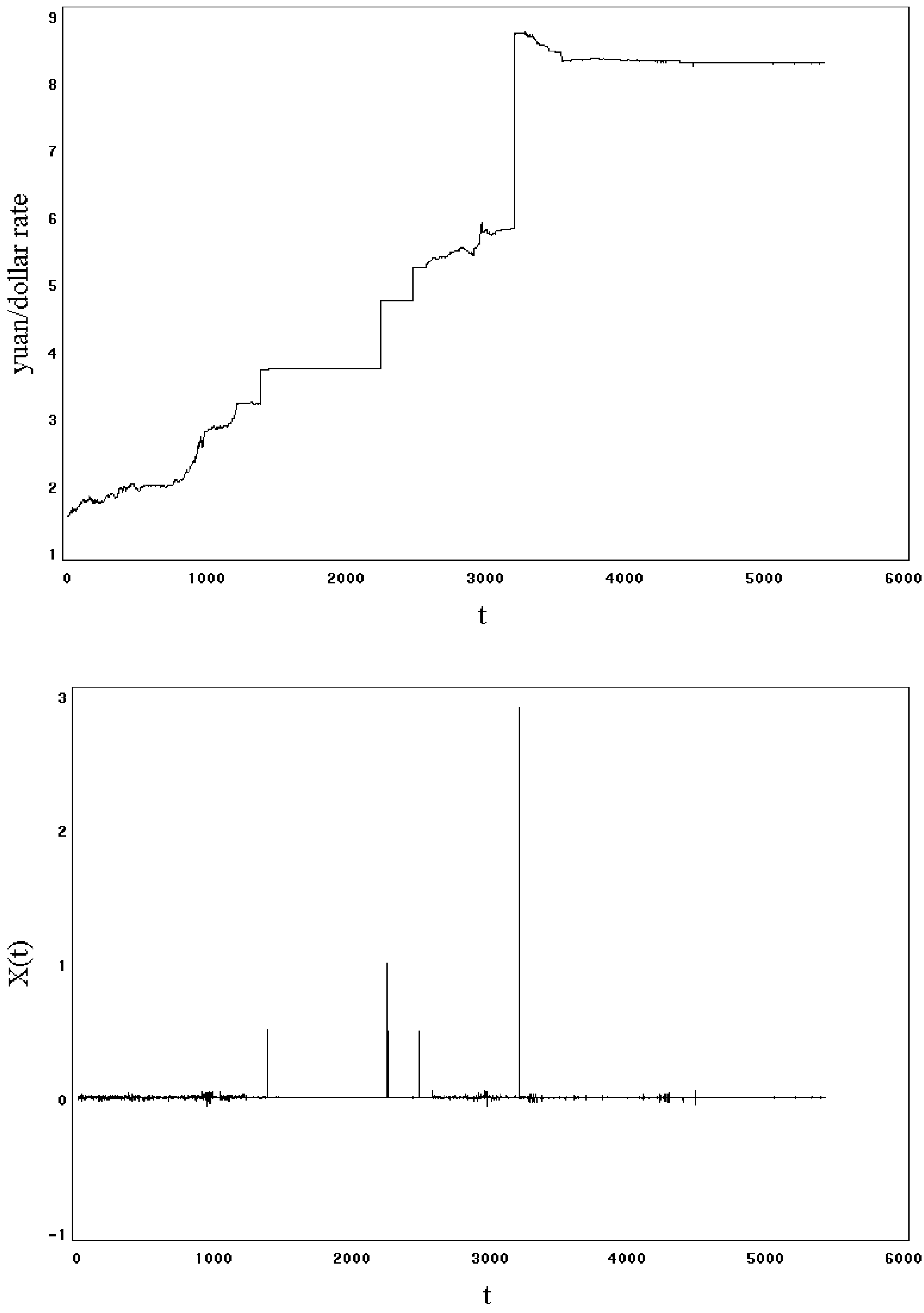


Figure 1. Daily observations of the yuan/dollar rate (upper panel) together with their price changes (lower panel) from 2 January 1981 to 1 November 2002.

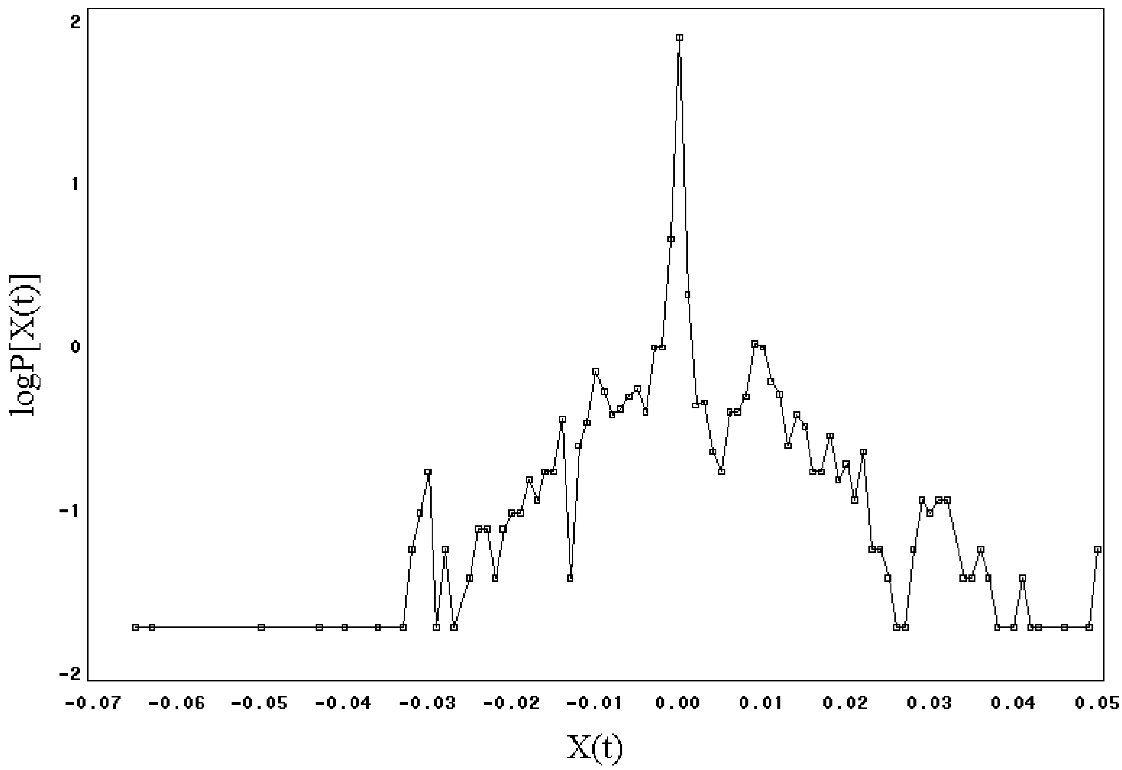


Figure 2. Logarithm of the probability density function of the yuan/dollar price changes with the four major episodes of intervention dropped.

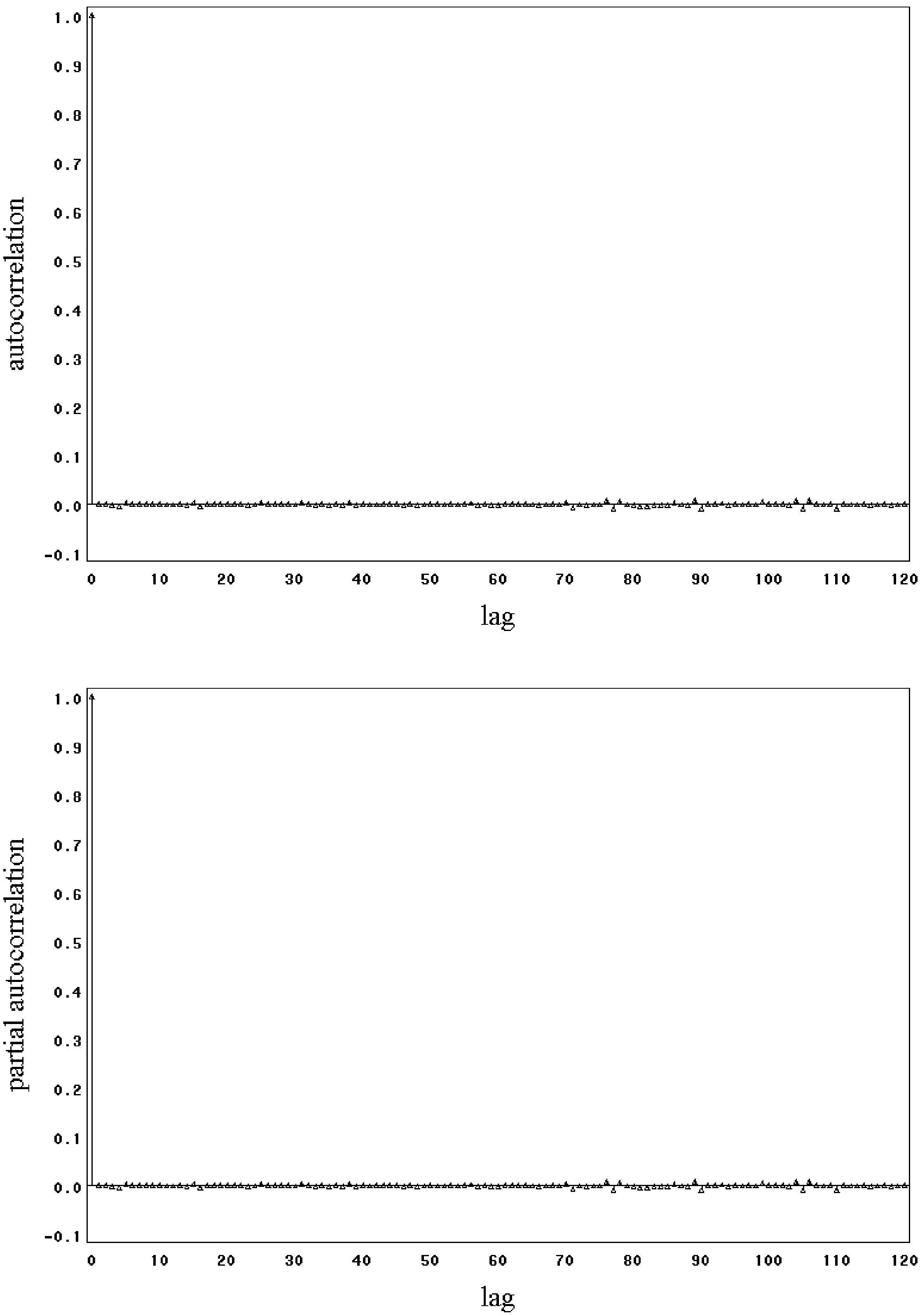


Figure 3. Autocorrelation function (upper panel) and partial autocorrelation function (lower panel) of the yuan/dollar price changes. Autocorrelations do not significantly departure from zero after the first lag.

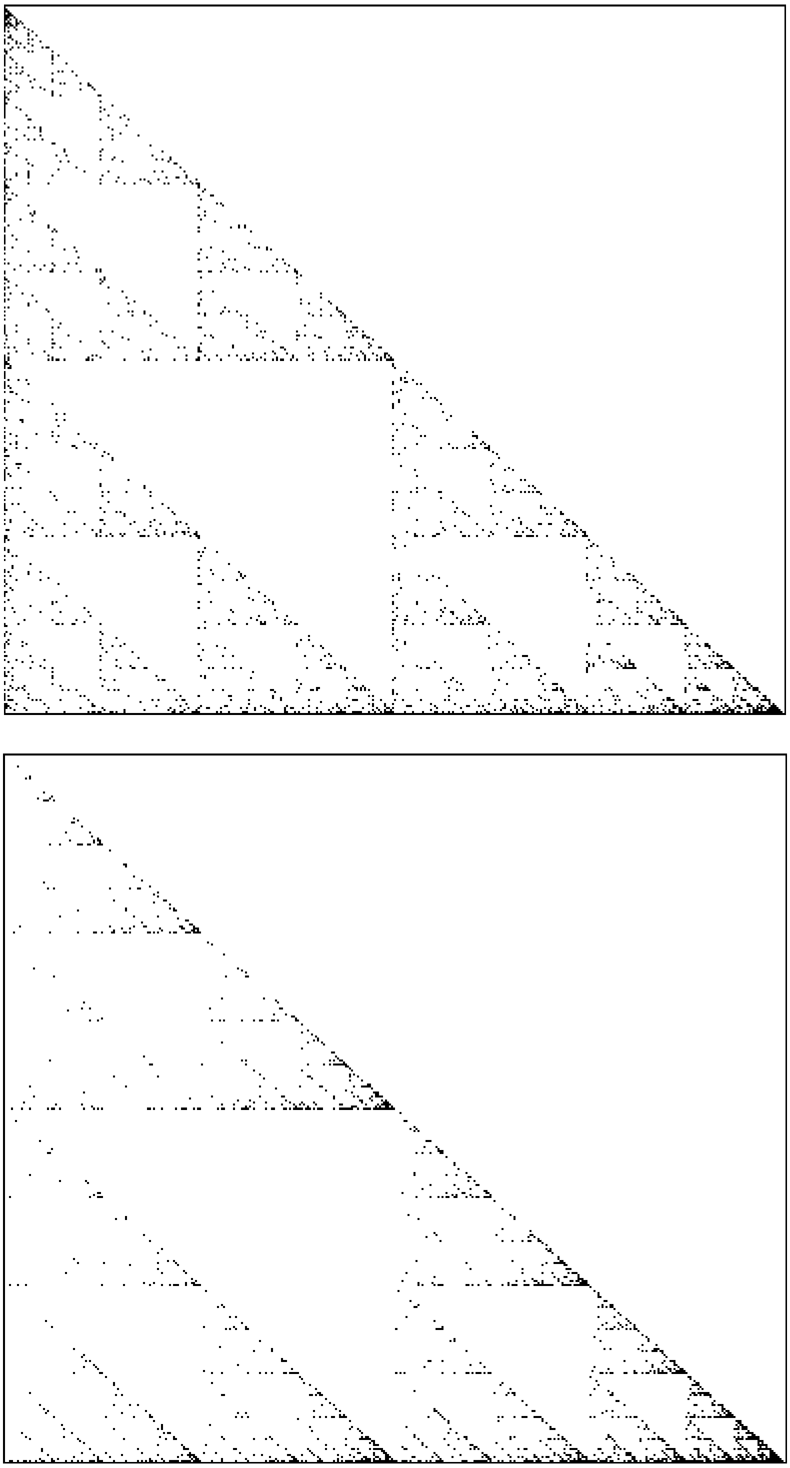
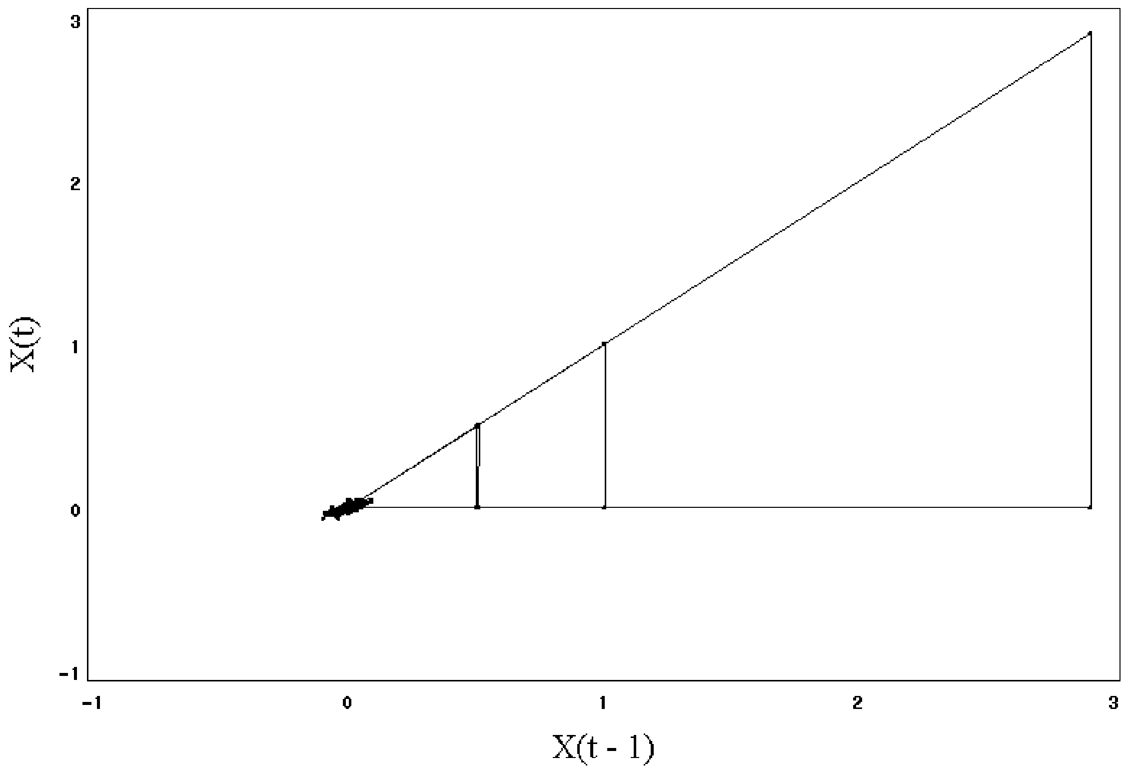


Figure 4. IFS clumpiness test for price changes of the Chinese yuan/US dollar rate (upper panel) together with the same test for realizations of our model (lower panel). A Sierpinski triangle emerges in both cases.



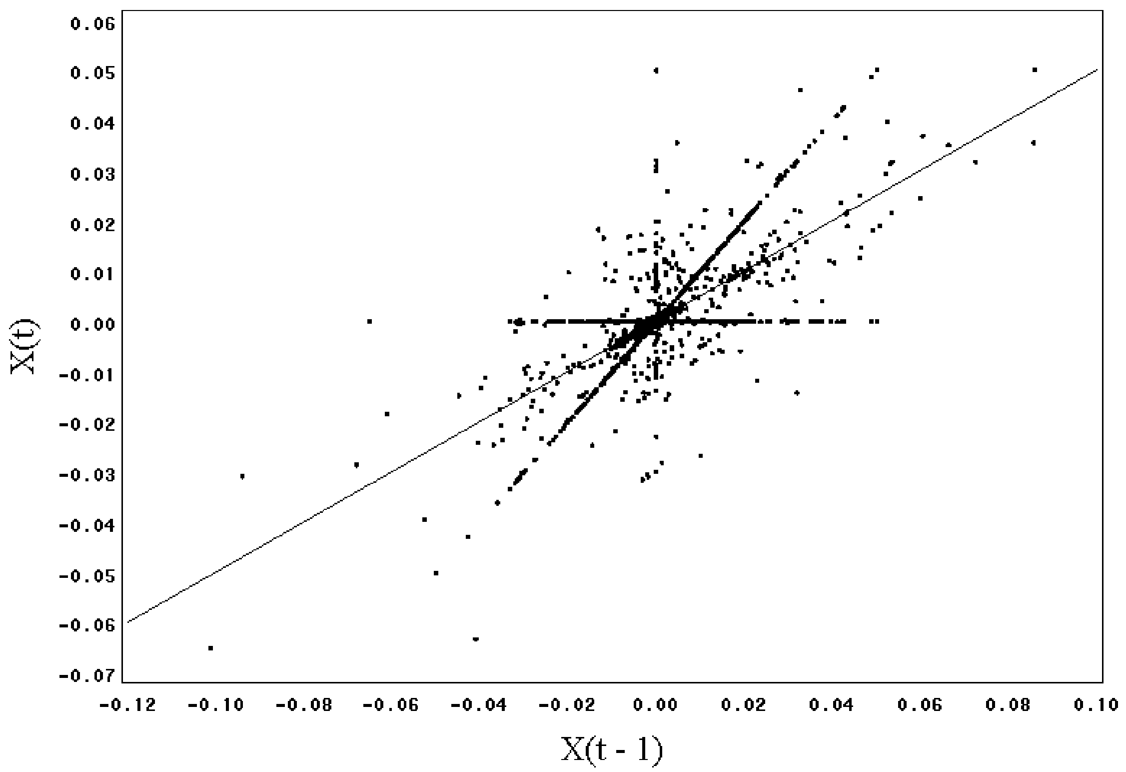


Figure 5. Phase diagram of the yuan/dollar rate at $X(t)$ plotted against $X(t-1)$ (upper panel) together with the same plot with the four major episodes of intervention ($X(t)$, $X(t-1) > 0.05$) dropped (lower panel).

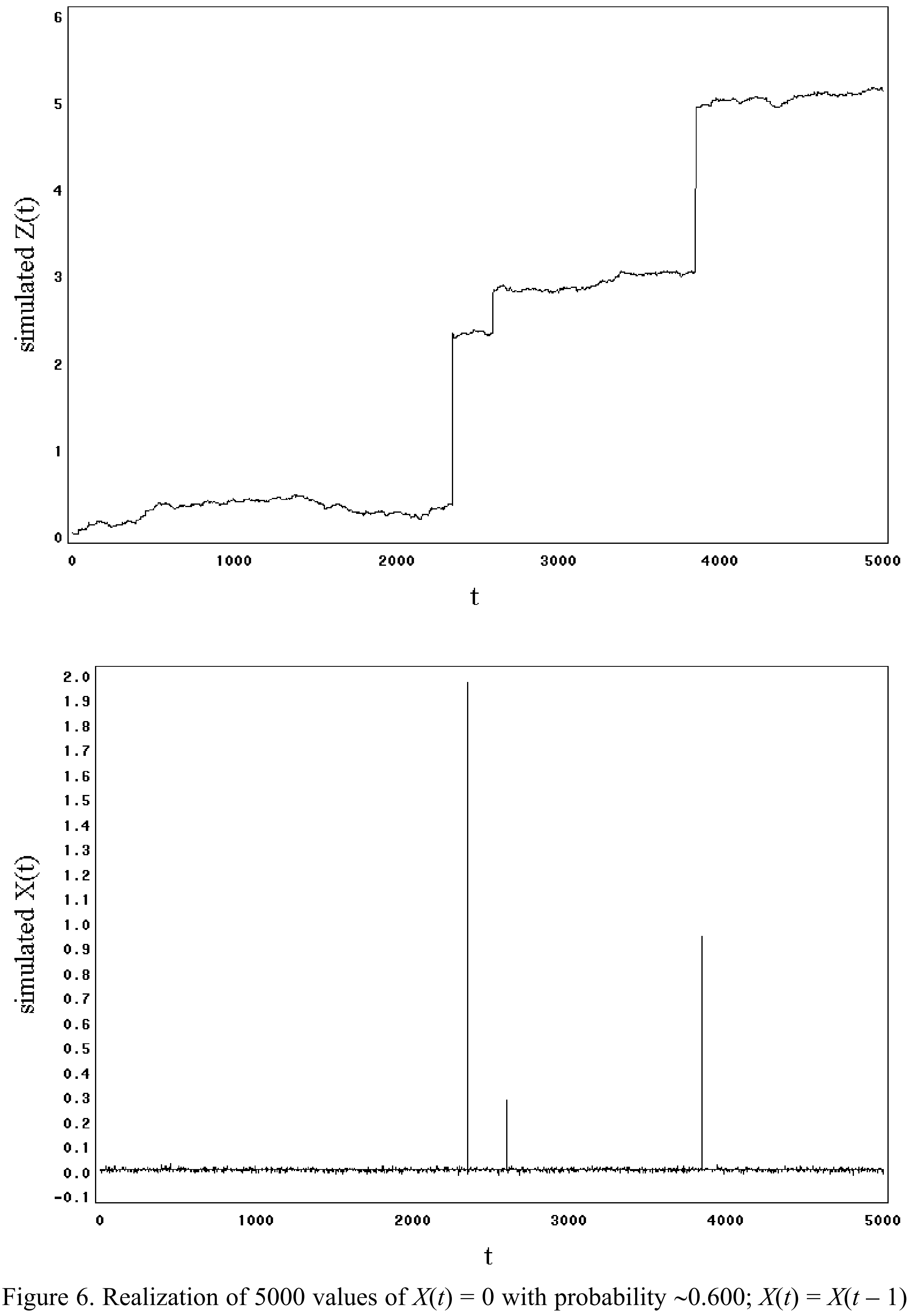


Figure 6. Realization of 5000 values of $X(t) = 0$ with probability ~0.600; $X(t) = X(t-1)$ with probability ~0.120; $X(t) = 0.5\,X(t-1) + 0.0003 + \varepsilon(t)$ with probability ~0.279; $X(t) = 1.5\,\eta(t)$ with probability ~0.001 (lower panel) together with the realization for $Z(t) = Z(t-1) + X(t)$ (upper panel).

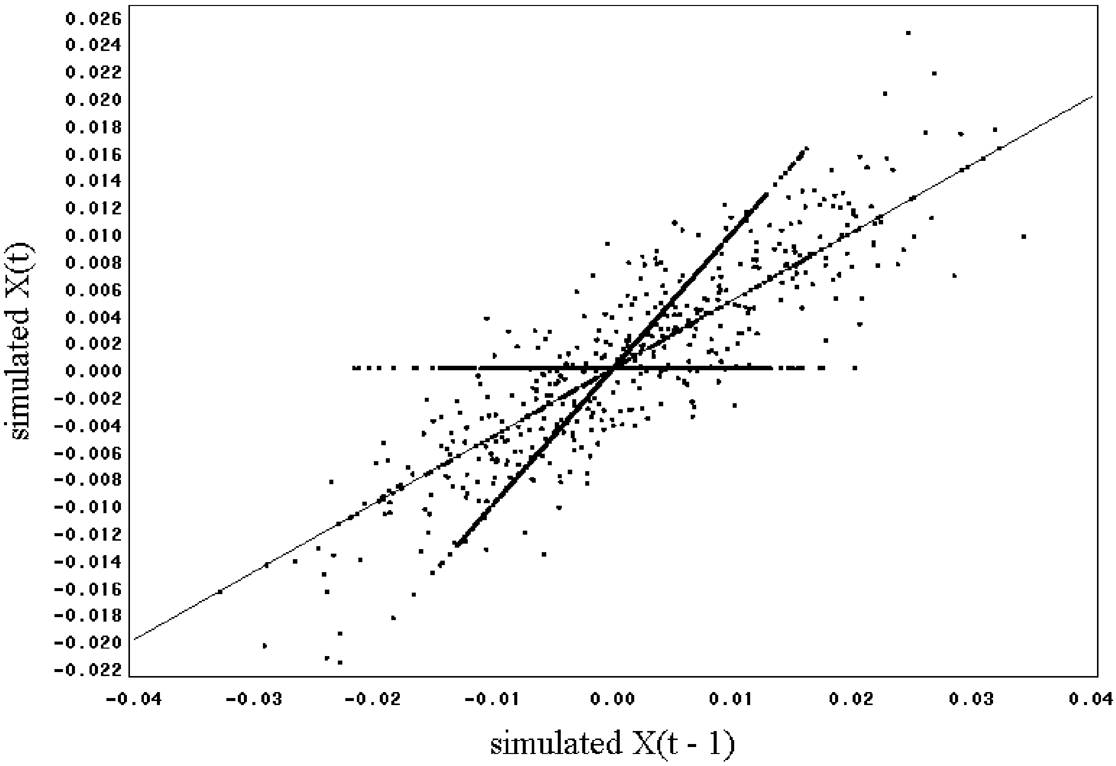


Figure 7. Phase diagrams for the simulated realization with the four major episodes of intervention dropped. The structure replicates that for the original data in Figure 5.

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