There is Something About the Yuan/Dollar Rate

SERGIO DA SILVA
Department of Economics, Federal University of Rio Grande Do Sul, Porto Alegre RS 90040-000, Brazil
E-mail: SergioDaSilva@angelfire.com

ANNIBAL FIGUEIREDO
Department of Physics, University of Brasilia, Brasilia DF 70910-900, Brazil
E-mail: annibal@fis.unb.br

Price changes of the Chinese yuan/US dollar rate are found to display a Sierpinski triangle in an IFS clumpiness test. This fractal structure has been shown to appear in the 'chaos game', where randomness coexists with deterministic rules. This paper explains this finding by taking the relative deviations from the line that best fits the path of the yuan/dollar rate over time. Such a time series is discovered to be short-range autocorrelated noise and to follow an exponential decay.

I. INTRODUCTION

This paper assesses the behavior of the Chinese yuan/US dollar exchange rate over time. Price changes of this time series are found to clump together in a fractal structure easily recognizable as a Sierpinski triangle when an IFS (Iterated Function System) clumpiness test is carried out. This is in line with the recent research that suggests that some complex financial systems may hide simple underlying structures. This is suggestive, too, of some sort of global determinism in data. The nature of such determinism is investigated and a major finding is that the relative deviations from the line that best fits the series of the yuan/dollar rate are short-range autocorrelated.

Section II presents the data, analysis is undertaken in Section III, and Section IV concludes.

II. DATA

Two decades of daily Chinese yuan/US dollar rates were taken. The data set employed was collected by the Federal Reserve Bank of New York from a sample of market participants. Foreign exchange rates of the yuan/dollar refer to noon buying rates in New York from cable transfers payable in foreign currencies. The data set is available online from http://www.federalreserve.gov/releases/H10/hist/. The sample covers the time period ranging from 2 January 1981 to 29 December 2000 and constitutes a series of 4964 datapoints. As standard, 'holes' from weekends and holidays were ignored and analysis concentrates on trading days.

III. ANALYSIS

Price changes Y(t+)t)!Y(t) were initially taken (Y is a yuan/dollar rate, t stands for time, i.e. a trading day, and)t is considered to be one trading day). Table 1 shows that the distribution of data is markedly positively skew for the entire period. The distribution is not bell-shaped; there is a longer tail to the right. This means there is a small number of extremely high values which are not balanced by values on the other side of the distribution. Apart from the recent five-year period, such a markedly positively skew is not seen for sets of five-year data. Table 1 also shows the extremely high degree of peakedness (kurtosis) of the distribution relative to a normal distribution. The distribution of data is then leptukurtic. Apart from the first half of the eighties, subsets of the data also display leptukurtosis. This matches with the 'stylized fact' that historical financial returns show weak evidence of skewness and strong evidence of excess kurtosis at short horizons (Campbell *et al*., 1997, p. 16). Anyway there is a clear departure from normality.

Iterated Function Systems (IFS) have been developed for generating fractal shapes (Barnsley, 1988). In a picture displaying an IFS clumpiness test, white noise fills it uniformly whereas correlated noise or chaos generates localized clumps. Such a test for the data of price changes was performed using the *Chaos Data Analyzer* program by Sprott (1995), and clumps appeared for the entire period (Figure 1). Surprisingly, the data happen to idiosyncratically clump together and form a Sierpinski triangle. We checked for and found that a Sierpinski also emerges with the differences of [logY(t+)t)!logY(t)]×100, with the differences of [lnY(t+)t)!lnY(t)]×100, with the differences of

$[Y(t+\Delta t)-Y(t)]/\ln Y(t+\Delta t)$, and with percentage changes, i.e. $[Y(t+\Delta t)-Y(t)]/\{[Y(t+\Delta t)+Y(t)]/2\}$.

This finding is somewhat suggestive that the system is locally random and globally deterministic because the Sierpinski triangle is a fractal that results from a deterministic rule implemented in a random fashion. In literature, this feature has been dubbed the 'chaos game'. Even though points are plotted in a different order each time, the order depends on all the points plotted before it. Importantly the data may have a time memory and be autocorrelated. Here we take a closer look at the latter feature. An explanation tentatively advanced for the fact that all the datapoints of the price changes are attracted by the shape which is the Sierpinski triangle is that the Chinese authorities might have been playing the chaos game when pegging the yuan. Regarding this, we think of the fact that the yuan is convertible for current account transactions in the balance of payments but not convertible for capital transactions.

The statistical properties of price changes were also evaluated for $\Delta t \geq 2$. The Sierpinski triangle emerges for $\Delta t = 1$ and $\Delta t = 2$ alike. Hurst exponents were reckoned and they were found to grow systematically from 0.505 ($\Delta t = 1$) to 0.999 ($\Delta t = 1000$). A Hurst of 0.5 gives an indication of a random walk even if the series is not normally distributed, whereas a value of between 0.5 and 1.0 suggests a persistent, trend-reinforcing series. Thus, as Δt is increased, a series becomes more deterministic. Relative LZ (Lempel-Ziv) complexity indices were also calculated and they were found to decrease consistently from 0.611 ($\Delta t = 1$) to 0.024 ($\Delta t = 1000$). This means a series becomes more predictable as Δt is increased, since maximal complexity (randomness) have an LZ of 1 and perfect predictability is associated with an LZ of 0. As Δt is increased, the mean and standard deviation of a series increase steadily, and the skewness and kurtosis decrease (Table 2). Remarkably these changes are governed by power laws. Log-log plots of average values and standard deviations against Δt generate straight lines within the time window of 1 #)t # 1000, whereas power laws are present for skewness and kurtosis within the time window of 1 #)t # 100 (Figure 2). The slope 1.015 was calculated from the fitting line in Figure 2a. A non-Gaussian scaling behaviour (slope ¥ 0.5) is thus observed for the mean. The slope of the fitting line to the standard deviation (Figure 2b) is of 0.480. The slopes of the fitting lines to the skewness (Figure 2c) and kurtosis (Figure 2d) are -0.519 and -1.041 respectively. All these fitting lines adjust to data very well (> 0.99). As)t is increased, relative standard deviations (i.e. mean divided by standard deviation) also decrease according to a power law (not shown). This means a series becomes more deterministic as)t is increased, which is in line with the previous finding regarding Hurst exponents.

The possibility of either chaos or a scaling power law was considered in the series of price changes for)t = 1. Chaos is about dynamics that are apparently random but intrinsically deterministic. Chaotic systems display sensitivite dependence on changing initial conditions. This ‘butterfly effect’ can be confirmed by a largest Lyapunov exponent which is positive. We checked for the Lyapunovs by taking several embedding dimensions and found negative exponents throughout. The hypothesis that the series is chaotic can thus be discarded. We can still suspect of some sort of determinism (like correlated noise) in our time series, since negative largest Lyapunov exponents are common to periodical or quasi-periodical series. Correlation dimensions were also calculated and we found dimension values that fall short of five for several embedding dimensions. This gives, by itself, an indication of either chaos or quasi-periodic data. As chaos has been discarded, we think this finding reflects some other type of determinism.

The slope 1.015 of the power law found for the mean suggests the original data can be described by a linear model. Figure 3 displays the original yuan/dollar rate (Y) for the entire period together with the line that best fits the data. The regression line adjusts to data reasonably well (0.96). The underlying linear model for the growth path of Y is given by

$Y = 1.0528 + 0.0017t$.

Notice that the slope 0.0017 of the model lies in between the average values for)t = 1 and)t = 2 (Table 2), i.e. $0.0014 < 0.0017 < 0.0027$. It might be noted the Sierpinski triangle precisely emerges within these time lags, which suggests our linear model captures the causal process. Figure 4 plots the series built from the relative deviations from the regression line. This series seems to exhibit some structure. Analysis proceeded on this series. Figure 4 also plots price changes to show that the series lacks structure. Hurst exponents for these series were reckoned. The Hurst exponent for price changes equals 0.505, and the exponent for the relative deviations from the regression line is of 0.987. This gives another reason why we chose to evaluate the latter series rather than the seemingly structureless price changes.

The series of deviations from the line that best fits the yuan/dollar rate is discovered to be autocorrelated noise (Figure 5). A decreasing autocorrelation function is the typical shape for positively correlated stochastic variables. The finiteness of the area under the autocorrelation function in Figure 5 gives information about the typical time scale of the memory of the process. There exists a typical time memory called the correlation time of the process, and a correlation time of approximately 275 trading days was reckoned. Since there is a typical time memory this random process can be dubbed short-range correlated. Usually such stochastic processes have an exponential decaying autocorrelation function. An example is the statistical memory of the velocity of a Brownian particle.

Here the possibility of a power law can be ruled out because there is a typical time scale. If the area under the autocorrelation function were infinite, the process could possibly be long-range correlated and a power law be present. Indeed autocorrelated random variables may still be characterized by the lack of a typical temporal scale, an example being $1/f$ noise. We thus checked for a scaling power-law behaviour of the autocorrelation function and found none. A log-log plot of autocorrelation versus time (trading days) is not a straight line.

A straight line appears in a semi-log plot of autocorrelation against time within the time window of 60 trading days, though (Figure 6). The fitting line to the data is almost perfect, i.e. !0.9994. As expected for a short-range correlated random process, the decay of the autocorrelation function is thus exponential. A characteristic decay time of approximately 557 trading days was reckoned. This type of correlated noise can thus explain why the Sierpinski triangle emerged in the first place. (Further discussion about the autocorrelation function is presented by Mantegna and Stanley, 2000, Chapter 6.)

IV. CONCLUSION

The price changes of the Chinese yuan/US dollar rate display a Sierpinski triangle in an IFS clumpiness test. An explanation tentatively advanced for the presence of this type of determinism in data is that the Chinese authorities might have been playing the 'chaos game'. The presence of either chaos or a scaling power law is discarded, though. There exists a typical time memory in the data. The short-range autocorrelation in the series of relative deviations from the line that best fits the yuan/dollar rate path over time seems to follow an exponential decay with a characteristic decay time of 557 trading days.

REFERENCES

Barnsley, M. F. (1988) *Fractals Everywhere*, Academic Press, San Diego.

Campbell, J. Y., Lo, A. W. and MacKinlay, A. C. (1997) *The Econometrics of Financial Markets*, Princeton University Press, Princeton.

Mantegna, R. N. and Stanley, H. E. (2000) *An Introduction to Econophysics: Correlations and Complexity in Finance*, Cambridge University Press, Cambridge.

Sprott, J. C. (1995) *Chaos Data Analyzer: The Professional Version 2.1*, American Institute of Physics.

ACKNOWLEDGEMENTS. We thank Iram Gleria and Raul Matsushita for discussions.

Table 1. Statistics of the price changes of the Chinese yuan/US dollar rate

Increment	Approximate Dates	Skewness	Kurtosis
1-4963	2Jan81-29Dec00	56.1914	3486.0
1-2482	2Jan81-29Dec90	0.4566	400.4
2483-4963	2Jan91-29Dec00	0.0011	1211.7
1-1240	2Jan81-29Dec85	-0.0831	2.6
1241-2480	2Jan86-29Dec90	0.4748	329.5
2481-3720	2Jan91-29Dec95	0.0002	607.0
3721-4963	2Jan96-29Dec00	4.2450	72.5

Table 2. Statistics of the price changes Y(t+)t)-Y(t) of the Chinese yuan/US dollar rate as)t is increased

)t	MEAN	STANDARD DEVIATION	SKEWNESS	KURTOSIS
1	0.001358716	0.0452239	56.19205	3486.111
2	0.002716565	0.06395055	39.69156	1739.179
3	0.00477851	0.07834738	32.33437	1155.007
4	0.005439659	0.09043688	27.92416	862.2501
5	0.006802159	0.1010176	24.92554	687.2753
6	0.008162879	0.1106423	22.71841	571.0565
7	0.009524184	0.1194982	21.00071	488.0495
8	0.01088675	0.127753	19.6131	425.7462
9	0.01224548	0.1354754	18.46852	377.5243
10	0.01360681	0.1427989	17.49499	338.8131
20	0.02721369	0.2016542	12.27185	166.2569
30	0.04078981	0.246702	9.931661	108.6707
40	0.05439437	0.2848278	8.522133	79.85203
50	0.0600893	0.3183079	7.550436	62.54979
60	0.08172119	0.3483846	6.824158	50.98393
70	0.09544425	0.3758773	6.260847	42.79709
80	0.1091548	0.4011256	5.805501	36.67871
90	0.1228776	0.4242898	5.425252	31.92236
100	0.136587	0.4460424	5.097228	28.09047
200	0.2761396	0.6138916	3.258321	11.02717
300	0.4215286	0.7404115	2.308792	5.130466
400	0.5710534	0.8431156	1.73666	2.503621
500	0.7252226	0.9222706	1.300857	0.9718378
600	0.8866395	0.9883024	0.9414905	0.04323482
700	1.055644	1.035943	0.6455272	-0.4483174
800	1.232067	1.087487	0.4746937	-0.5007609
900	1.413405	1.126063	0.2955008	-0.5177348
1000	1.593572	1.176453	0.2349033	-0.1661569

Figure 1. *IFS clumpiness test for the price changes of the Chinese yuan/US dollar rate, 2Jan81—29Dec00: a Sierpinski triangle.*

Figure 2. *Price changes* Y(t+)t)-Y(t) *are governed by power laws as*)t *is increased (from Table 2). Figures 2a and 2b display the power laws for means and standard deviations, respectively, within the time window of* 1 #)t # 1000. *Figures 2c and 2d show the power laws for skewness and kurtosis, respectively, within the time window of* 1 #)t # 100.

Figure 3. *Chinese yuan/US dollar rate, 2Jan81—29Dec00 together with the line of best fit.*

Figure 4. *Relative price changes and deviations from the line that best fits the growth path of the Chinese yuan/US dollar rate, 2Jan81—29Dec00 (from Figure 3) While the price change series (Figure 4b) lacks structure (Hurst exponent of* 0.505*), the series of deviations from the regression line (Figure 4a) seems to be structured (Hurst of* 0.987*).*

Figure 5. *Autocorrelation function for the series of relative deviations from the line that best fits the yuan/dollar rate growth path (Figure 5a). The finiteness of the area under the decreasing autocorrelation function suggests there exists a typical time memory called the correlation time of the process, which was reckoned as* 274.4 *trading days. Since there is a typical time memory this random process is short-range correlated. Figure 5b displays a semi-log plot of the autocorrelation function (Figure 5a) within the time window of* 60 *trading days. The straight line corresponds to exponential decay with a characteristic decay time of* 557.53 *trading days. After about* 60 *days (not shown) the correlation vanishes.*

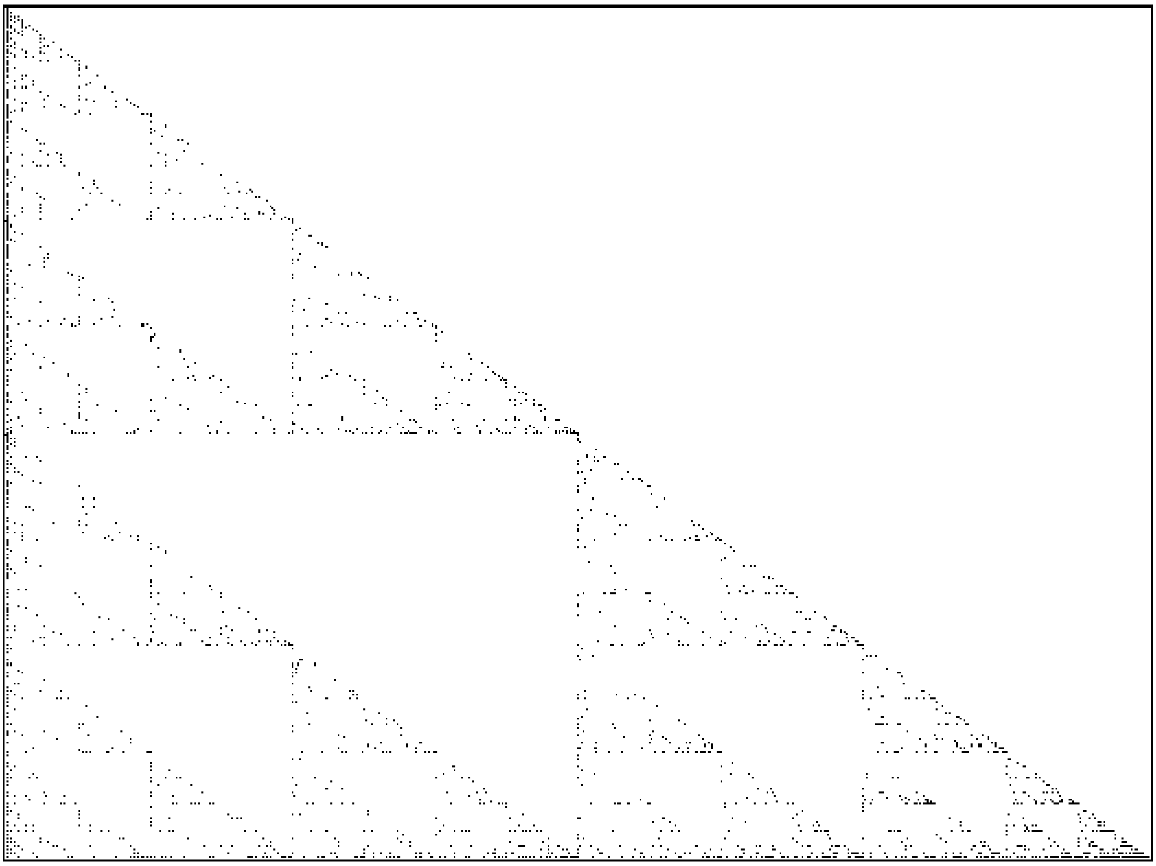
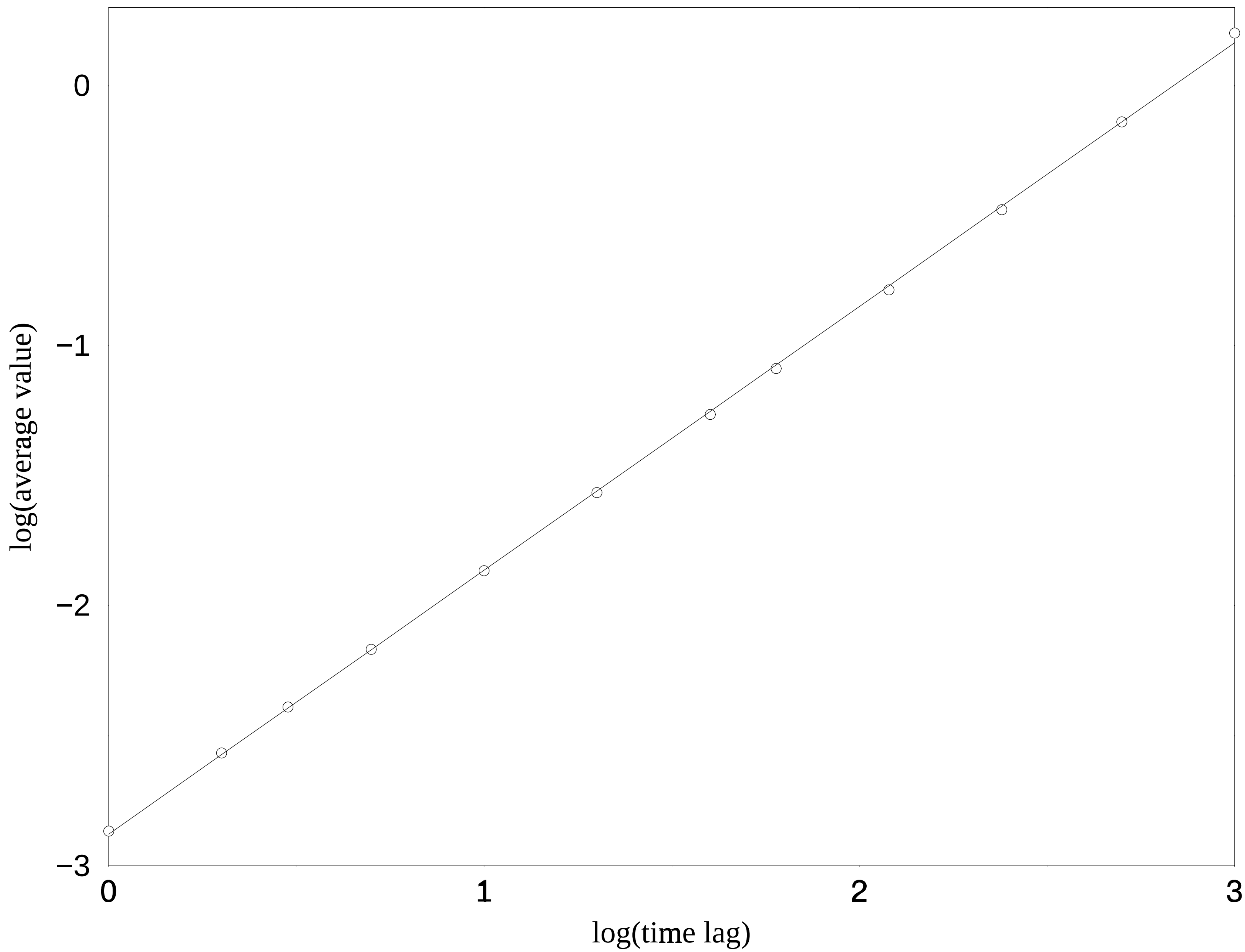
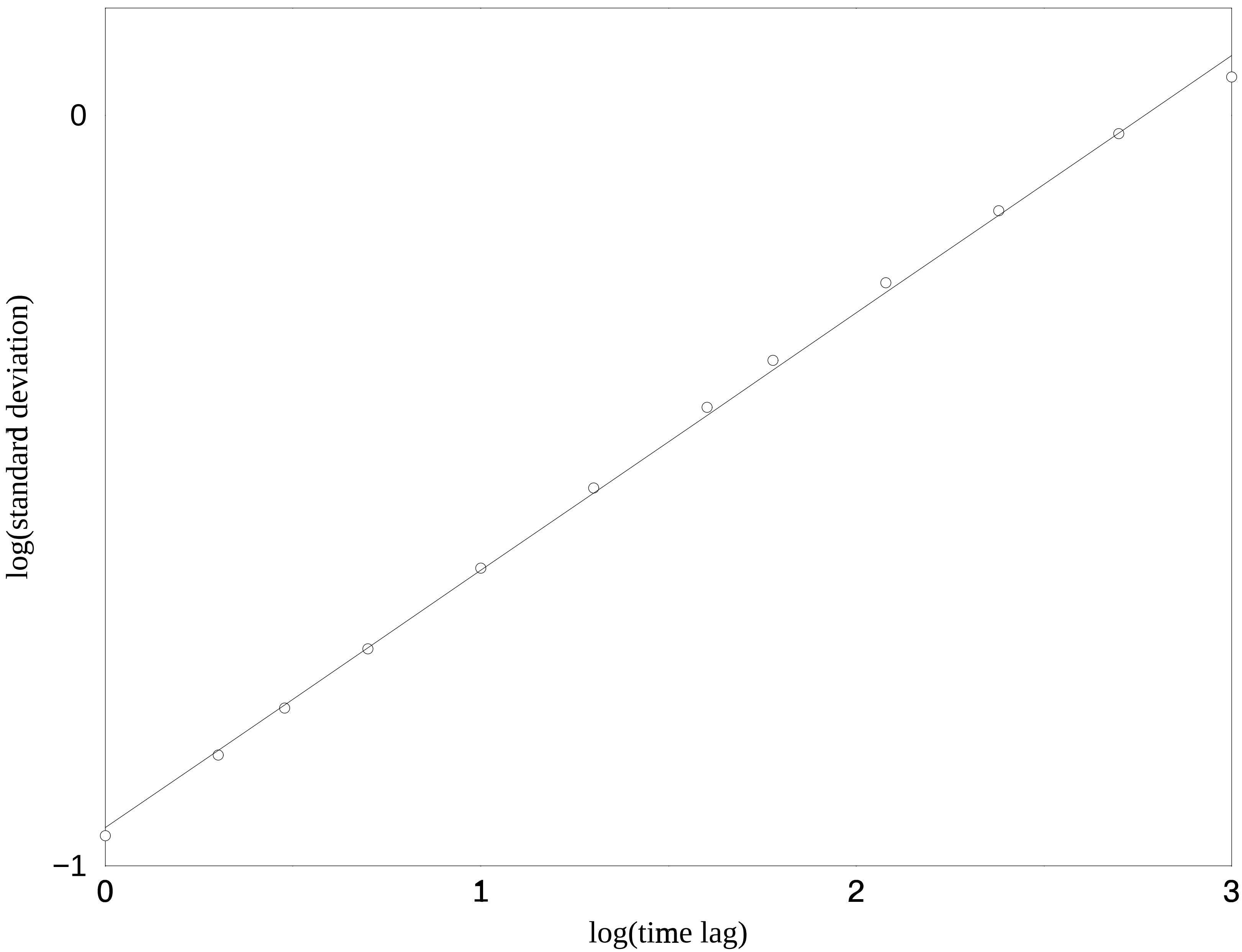
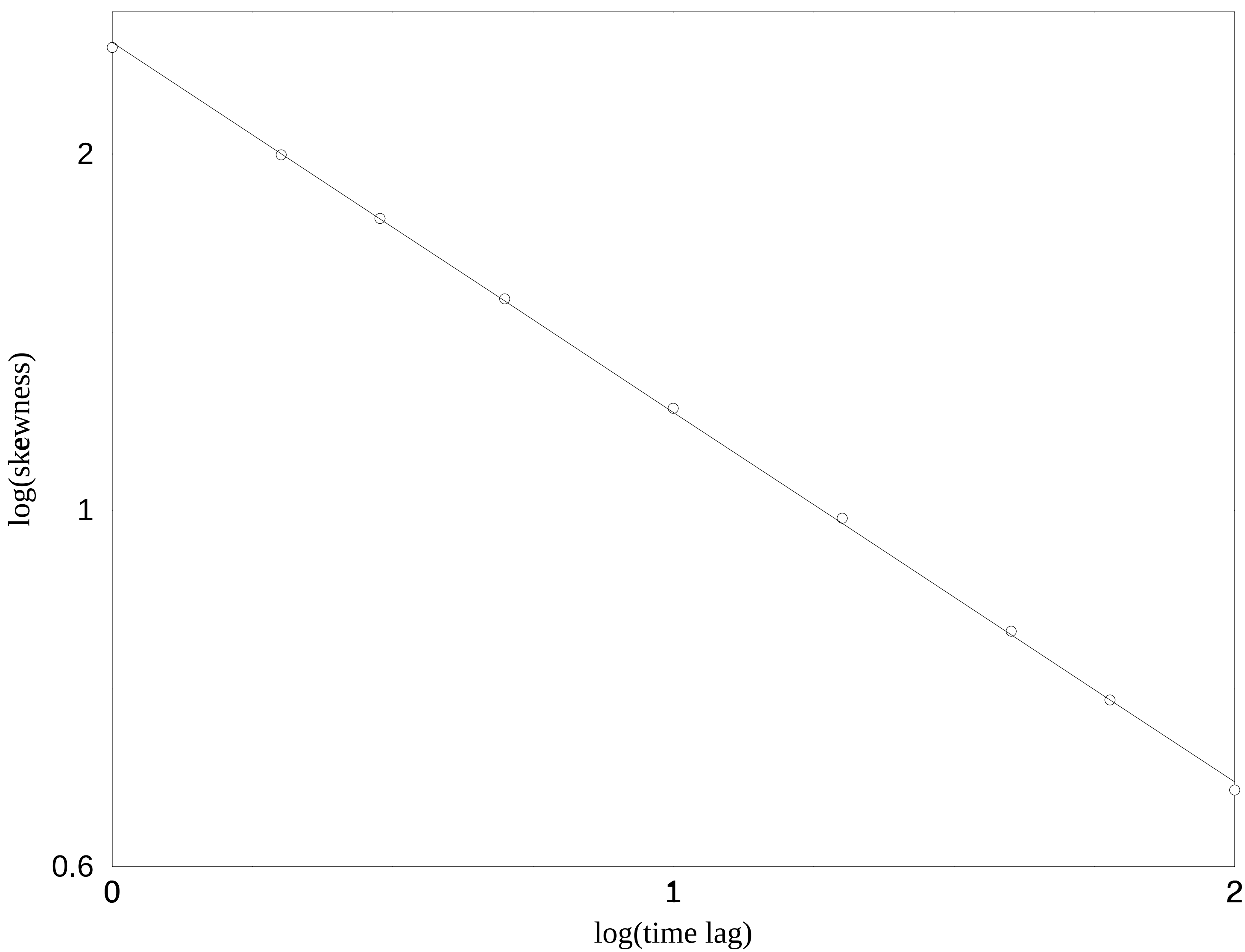


Figure 1

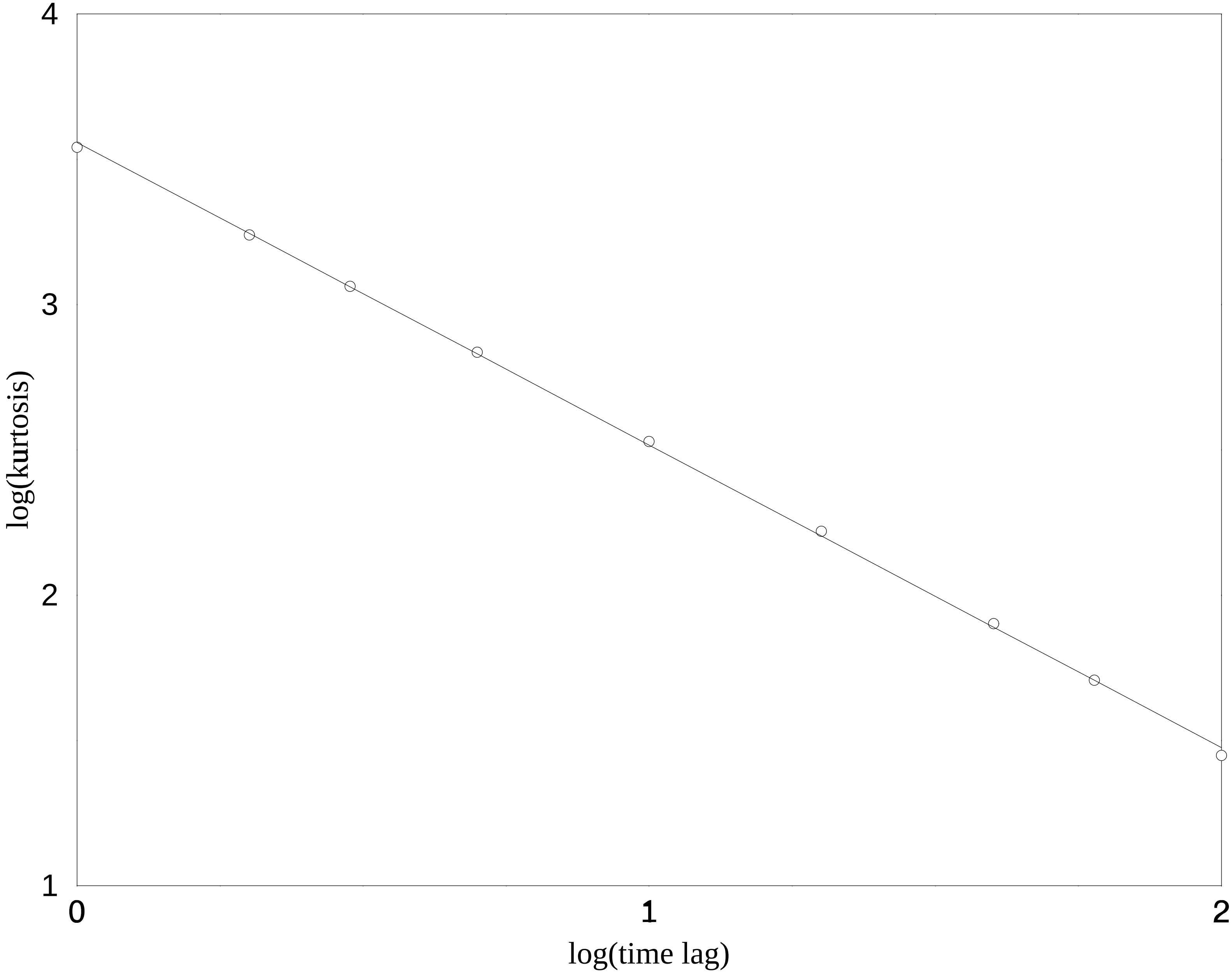


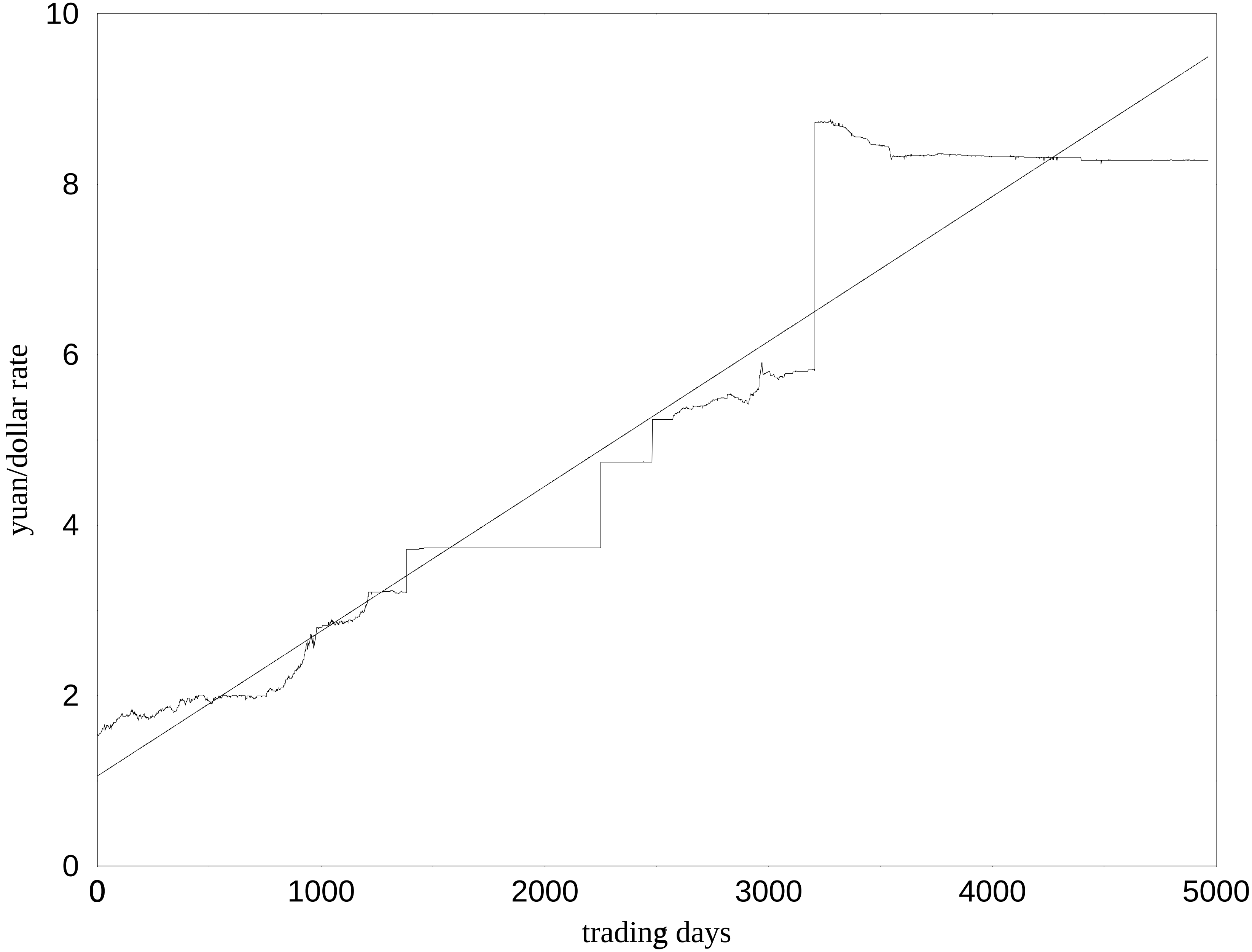
0
−1
−2
−3
0
1
2
3
log(average value)
log(time lag)


0
−1
0
1
2
3
log(time lag)
log(standard deviation)

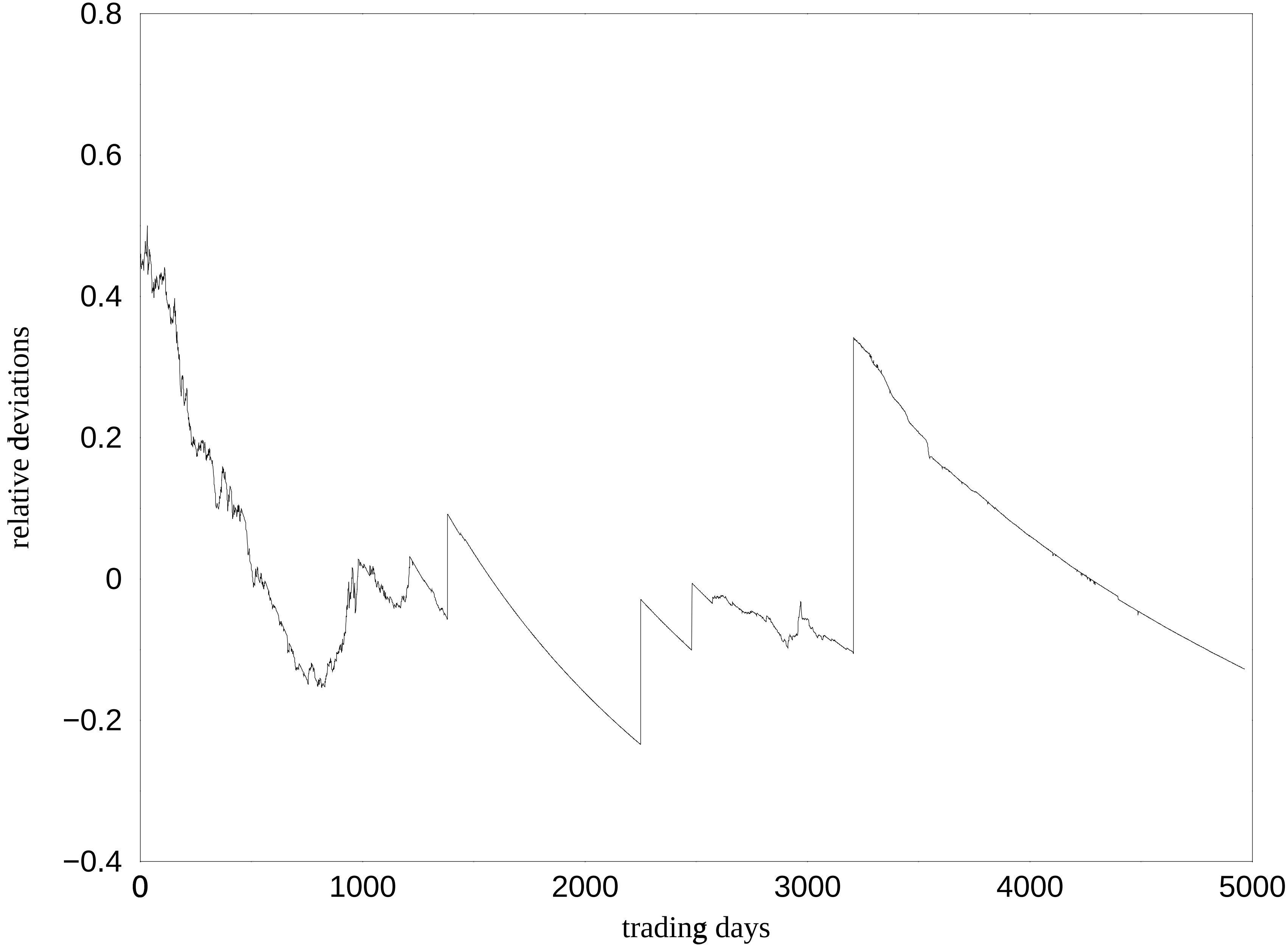


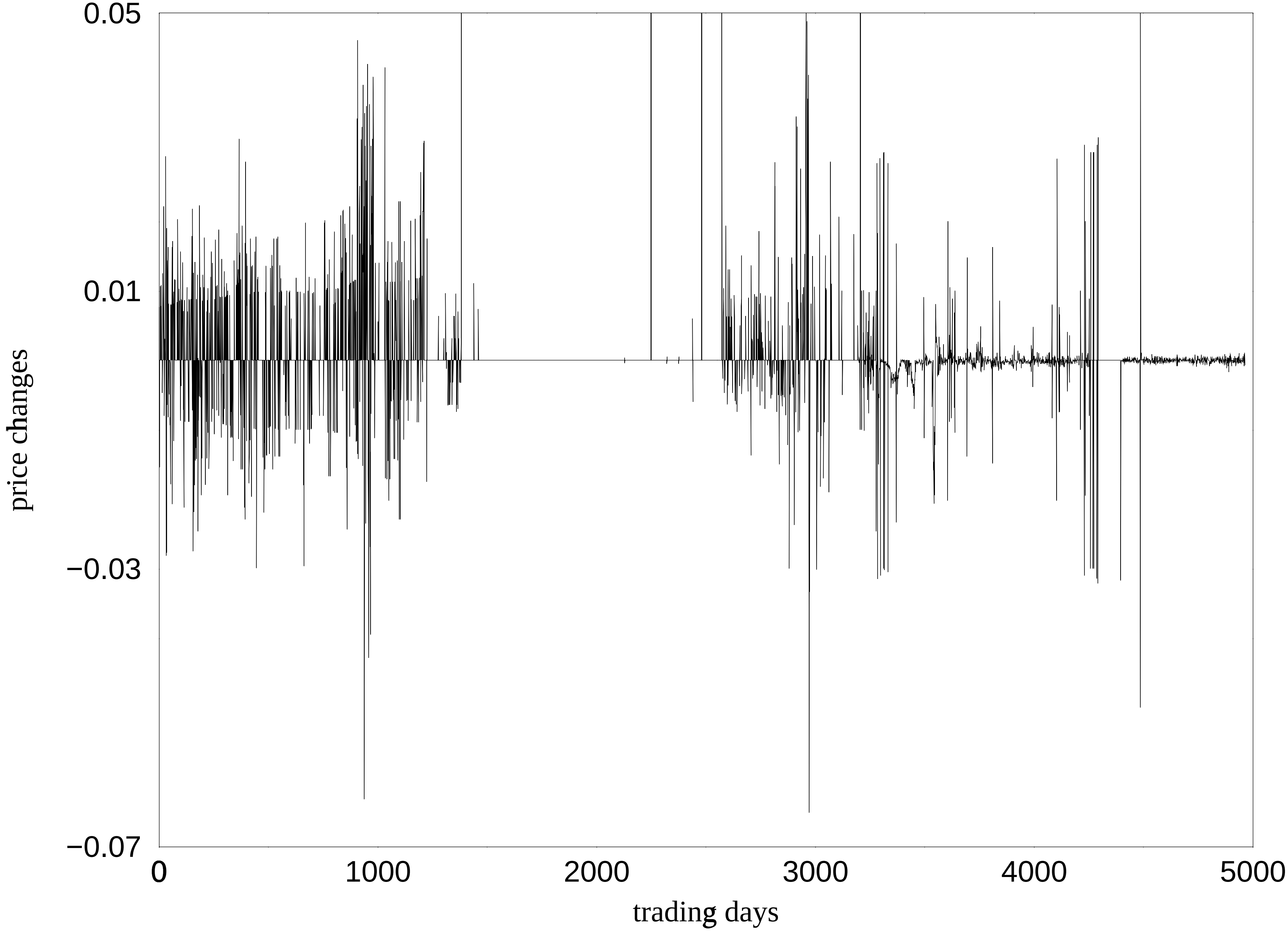
log(skewness)
2
1
0.6
0
1
2
log(time lag)


4
3
2
1
0
1
2
log(kurtosis)
log(time lag)

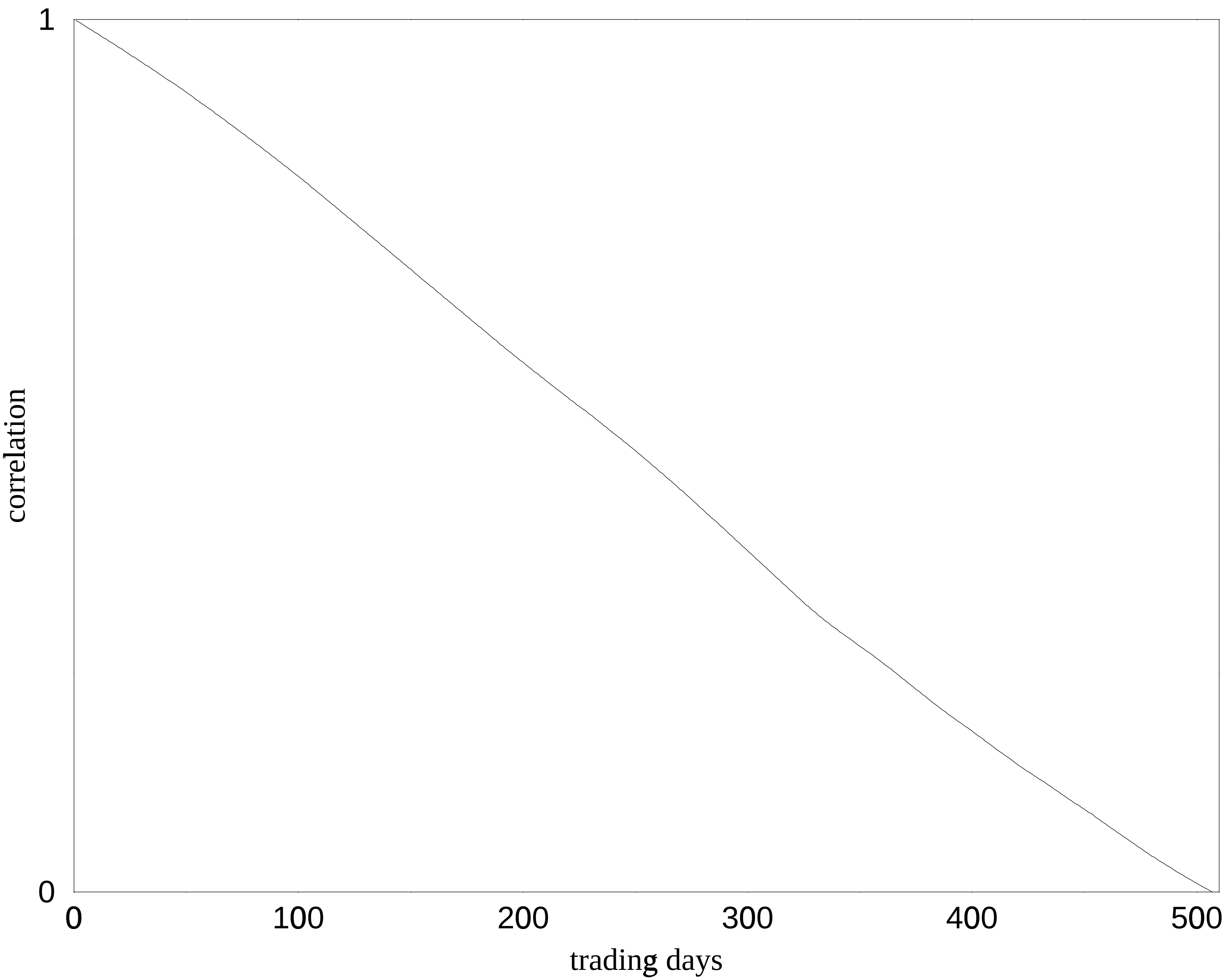


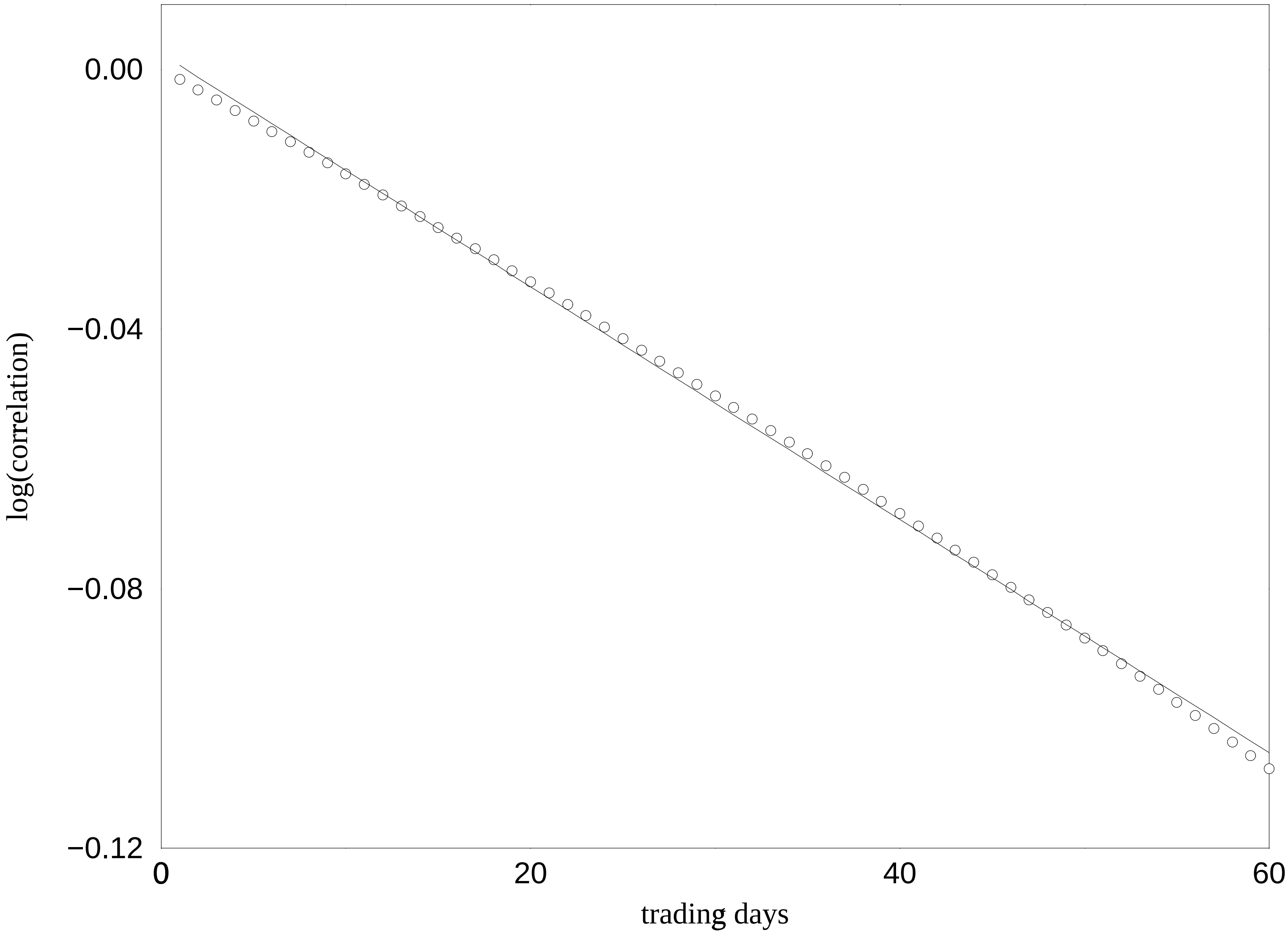
10
8
6
4
2
0
0
1000
2000
3000
4000
5000
yuan/dollar rate
trading days


0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
1000
2000
3000
4000
5000
relative deviations
trading days



0.05
0.01
−0.03
−0.07
0
1000
2000
3000
4000
5000
price changes
trading days


1
0
correlation
0
100
200
300
400
500
trading days


0.00
−0.04
−0.08
−0.12
log(correlation)
0
20
40
60
trading days