

BUSINESS CYCLE IN THE INDUSTRIAL PRODUCTION OF BRAZILIAN STATES

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Abstract

The present paper applies the Markov switching model with the aim of checking two industrial production features of six major Brazilian states. Firstly, we try to determine the date of business cycles and, soon afterwards, we verify the existence or not of an unobservable component that is common to all series from 1980 to 2001. The characteristics of this comovement are checked by multivariate models. The results from the univariate estimates allow us to infer about the existence of a similar behavior between the industrial production of the southeastern states and that observed in the southern states. The multivariate model efficiently captures the cyclic behavior of industrial production when used regionally, thus revealing a dynamics for growth and recession periods, which differs between these two regions.

Key words: *Markov-switching*, VAR, Business Cycle.

JEL Codes: C32; C50; E 32; N16; R11.

1. Introduction

The Brazilian economy has been put through large political and macroeconomic changes in the last two decades. By analyzing these two extremes throughout this period, we note that the country has shifted from a situation with high inflation rates, restricted trade openness, excessively controlled exchange rates, constant fiscal deficits and a strong presence of the state in the production sector in the 1980s to a scenario with greater trade openness, lower price indices in an inflation targeting regime, and a fluctuating exchange rate, in addition to an improved control of monetary and fiscal policy in the early 21st century.

Throughout this period, the Brazilian economy has faced several problematic situations, such as the financial crises in Mexico in 1994, in Asia in 1997, in Russia in 1998 and in Argentina in 2001. In the national scenario, some attention should be paid to the debt crisis in the early 1980s, several economic plans (Cruzado I and II, Summer I and II, Collor I and II and the Real Plan) and to the exchange rate regime shift in 1999, whose two macroeconomic measures exerted a significant impact not only upon the structure of the Brazilian industry but also upon its spatial allocation.

Firstly, the intention of the federal government of increasing trade openness in 1988 and in the early 1990s contributed towards enhancing imports, especially of intermediate goods, thus bringing about competition in some important sectors of the Brazilian industry. For an analysis of the changes to the country's output matrix, see Pinheiro and Almeida (1995), for foreign trade policies, see Portugal (1994), and for industrial policy adopted in the early 1990s, see Guimarães (1996). Secondly, the process of industrial decentralization of the Brazilian economy, both in terms of added value and employment, was quite intense until the mid-1980s and, later on, until the second half of the 1990s. The intensification of the fiscal war between

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Brazilian states contributed to the decision of companies to allocate new investments to regions far off from major centers, which resulted in the spatial reallocation of a few industrial investments (see Pacheco, 1999).

Therefore, the several structural breaks that have occurred in macroeconomic series throughout the last twenty years in Brazil are expected to generate parameter instability, which offers a good opportunity to experiment with the several econometric models that seek to explain the behavior of these variables, especially of economic cycles.

In the last few years, the investigation into the existence of a dynamic relationship between economic policy variables has been intensified by the economic theory. This way, the idea of comovement as an essentially macroeconomic phenomenon is firmly founded upon the definition of business cycle given by Burns and Mitchell (1946, p.3): *Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle.*³ Aside from this comovement characteristic, it is also possible to check the existence of nonlinearity in the evolution of business cycles. In other words, these cycles may be divided not only into phases, but also into different regimes. Also, it is possible to assess the behavior of these economic variables in these phases, that is, whether contractions are shorter and more violent than expansions (see Sichel, 1993).

The first approach to the modeling of business cycles was established by Burns and Mitchell (1946), in which the existence of a cycle is ascribed to the total decline in production. However, several other empirical methods may be used to explain this economic behavior.

An alternative to the classic method is to determine the deviations of the cycle from its long-term trend by means of a linear filter. One of the hypotheses used here to isolate the cyclic component by using a linear filter is that the series might be stationary around a trend. Nevertheless, this hypothesis is arguable when it comes to several macroeconomic series, such as GDP and industrial production.⁴ Another option, which could give the idea of comovement, is to obtain a single series by linearly combining a set of another series and by analyzing the cyclic behavior of the single series as representative of the other ones by means of main component analysis. Barros (1993) has used this method for Brazilian series and found some evidence of correlation in the frequency of business cycles, which supports the hypothesis of comovement.

Several formulations were suggested for assessing these relationships, but after Sims (1980) criticized the traditional macroeconomic modeling, the parametric analysis by VAR models began to be largely employed in macroeconometrics.⁵

Among these models, those with a multivariate stochastic structure are of special interest, since they can be used to model the economic relationships between variables, not only on the short run, but also on the long run, with the use of autoregressive vectors and with the concept of co-integration and error correction mechanism. Engle and Issler (1993) have applied this method to assess comovements of per capita GDP on the short and long run in Brazil, Argentina and Mexico in the postwar period, and have found evidence of common cycles and trends.

³ It is worth mentioning that in the definition of business cycle the authors described four different periods in economy: prosperity, crisis, depression and revitalization.

⁴ Due to the existence of unit root, and possibly of a linear trend, the use of a linear filter would result in a spurious cycle.

⁵ Nonparametric tools used for the analysis of comovement are the autocorrelation function and the spectral density function.

A proposed simplification of the VAR structure are the factor models. In this case, the comovements of contemporaneous macroeconomic variables occur since, to some extent, they are caused by common shocks. For instance, in a factor model, the behavior of a set of n variables is qualitatively similar to the behavior of only one variable, that is, the common factor. In the last decades, the analytical models that formalize the construction of economic indicators that allow identifying and foreseeing the moments in which an economy goes into or out of recession have gained momentum. One of the most recent and influential examples is the linear dynamic factor model by Stock and Watson (1989, 1991, 1993), where the comovements between economic variables are captured by a composite index. Picchetti and Toledo (2002) estimate a stochastic component common to the four series comprised by the industrial production index of IBGE (Brazilian Institute of Geography and Statistics) from the dynamic factor model proposed by Stock and Watson (1991).

Nonlinear models, such as those of regime shift, may also be used to characterize these comovements, in addition to dynamic factor models. Recently, after the publication of an article by Hamilton (1989) on GDP business cycle in the USA, the interest in nonlinear estimates has been aroused by the Markov switching model, both for applications in macroeconomic series and financial series. Krolzig (1997) made a generalization of Hamilton univariate model, characterizing the international business cycle as a common regime shift in the stochastic process of economic growth of six OCDE countries. In this case, for a given regime, the comovements of the growth rate are represented by an autoregressive vector model.

Krolzig and Sensier (2000) have used an error correction model with regime shift (MSI-VECM) to investigate the relationship between business cycles in the United Kingdom and the changes to the industrial structure of the country, and found evidence of a common cycle. An MS-VEC model was also used by Krolzig, Marcellino and Mizon (2000) to identify a common cyclic component of job market in the United Kingdom. Phillips (1991) estimated bivariate regime shift models for industrial production of the USA, Canada, United Kingdom and Germany, and found little evidence of transmission of business cycles between the industries of these countries.

Of note, however, these two characteristics of business cycle, comovements, and nonlinearity were treated separately until the publication by Diebold and Rudebusch (1996). These authors therefore proposed a multivariate dynamic factor model with regime shift that dominates these two key characteristics of business cycles, where the growth rate of each series considered as coincident indicators depends upon the current and past values of an unobserved common factor, which is then interpreted as a composite index of coincident indicators.

An extensive literature review was done with the intention of testing several characteristics of this class of models. One of them is that macroeconomic variables have a different behavior within the phases of the business cycles, causing asymmetries to appear. Clements and Krolzig (2000) analyzed the conditions in which MS-AR models are able to generate several types of asymmetry in the business cycle by suggesting the use of parametric tests to identify the asymmetries in these cycles. The three types of asymmetry assessed are the following: *deepness*, which tests the existence of asymmetry within the width of the series; *steepness*, which tests the asymmetry within the magnitude of the cycle; and *sharpness*, which tests the presence of asymmetry in the business cycle transitions. These tests were applied by the authors to a bivariate model of production and employment in the USA. They detected the presence of asymmetry. Later on, Clements and Krolzig (2001) used these tests in a regime shift model to check whether the international price of oil, inserted as an exogenous variable, would cause asymmetry in the production growth rate in the USA, but no evidence of this asymmetry was observed. Warne (2000) analyzed Granger noncausality hypothesis from an MS-VAR model, and applied the test for data obtained in the USA, but did not find any evidence of causality in the mean between currency and production.

These econometric techniques have been seldom applied to assess the behavior of Brazilian macroeconomic series, except for the studies conducted by Chauvet (2002), Chauvet, Lima and Vasquez (2002), Barros (1993), Engle and Issler (1993) and Picchetti and Toledo (2002). Just as underscored in Pinheiro and Almeida (1995) and Pacheco (1999), the Brazilian industry has a different production dynamics among Brazilian states, and the regime shift methodology proves adequate for characterizing not only these differences, but also for explaining the effects of an adverse shock to the regional industry. Thus the objective of this article is to apply such method to confirm a priori the stylized facts of business cycles in the series of industrial production of the states of São Paulo (SP), Minas Gerais (MG), Rio de Janeiro (RJ), Paraná (PR), Santa Catarina (SC) and Rio Grande do Sul (RS).

This way, we will be able to check the moments of expansion and recession in the regional industrial production by establishing a relationship with the several economic plans implemented in Brazil, in addition to identifying whether there is a common pattern in the economic growth dynamics of these states. It is important to emphasize that the existence of comovements indicates a narrow relationship between industries in different regions over time. On top of that, as there is a common movement of expansion and contraction in the business cycle in the industry, the modeling of economic growth as a joint stochastic process is encouraged, and the multivariate models, designated as MS-VAR, are used. In this case, the business cycle may be identified as a regime shift in the average growth rate, which occurs simultaneously among all the states or in a given region.

A lot of information may be obtained with this econometric tool, for instance, determining the industrial growth dynamics in these states, that is, the persistence of the cycle in growth periods or reduction in activity, checking whether a positive or negative shock, of whatever nature, to the Brazilian economy, produces a similar impact in different states or regions, and whether the regime shift caused by an unobservable variable occurs simultaneously among states. These results are of great worth as they qualify and quantify the economic cycle of industries in the major Brazilian states, helping with the prediction of sectoral impacts on the presence of economic shocks.

The article is organized as follows. Section 2 describes the process of parameter estimation of univariate and multivariate models. Section 3 presents the data and results of the statistical tests applied to the univariate and multivariate models. Section 4 concludes.

2. Methodology

When a time series is amenable to a structural break, which might occur in the variable coefficient, in the intercept and in the variance, the parameters of the static model vary over time. This way, the hypothesis of stationarity and normality is violated and nonlinearity results. In the last few years, the interest in the nonlinear modeling of economic time series, especially regime shift models, has gained momentum.

The regime shift in the time series may be characterized endogenously, by the own model, or exogenously, with an intervention from dummy variables, which implies a priori knowledge of the moment of regime shift. Among the several classifications presented in the literature, we will use Markov switching.⁶

A Markov process is a classic stochastic process in which random variable X_t is particularly time-dependent. This process will be discrete or continuous depending on the states (s_t) in which the variable is.

⁶ For discussion about other formulations, see Tsay(1989), Granger and Teräsvirta(1993), Dijk(1999), Teräsvirta and Anderson(1992) and Tsay(1998).

In the former case, we have $s = (1,2,3,\dots)$ and in the latter, $s = (-\infty, \infty)$. If a Markov process has a finite or numerable number of states, then it is called a Markov chain.

The special characteristic of these models is the hypothesis that the realization of unobserved state $s_t \in \{1, \dots, k\}$ is determined by a Markov stochastic process in discrete state and discrete time, defined by transition probabilities.

The probability that X_{t+1} is in state j , considering that X_t is in state i – called one-step transition probability– is then represented by:

$$P_{ij}^{t,t+1} = \Pr\{X_{t+1} = j / X_t = i\} \quad (2.1)$$

As we can observe, transition probability $P_{ij}^{t,t+1}$ is not only a function of the state, but also a function of the transition time. However, if $P_{ij}^{t,t+1}$ is independent of time, the Markov process has a stationary transition probability, and $P_{ij}^{t,t+1} = P_{ij}$.

Since k states might exist, the transition probabilities between these states may be represented by a matrix of transition probability $P = [p_{ij}] \in M(k \times k)$, exactly as in 2.2:

$$P = \begin{bmatrix} p_{11} & p_{21} & \dots & p_{k1} \\ p_{12} & p_{22} & \dots & p_{k2} \\ \vdots & \vdots & \vdots & \vdots \\ p_{1k} & p_{2k} & \dots & p_{kk} \end{bmatrix} \quad (2.2)$$

where: $\sum_{j=1}^k p_{ij} = 1$ for $i=1,2,\dots,k$ and even if, $p_{ij} \geq 0$ for $i,j=1,2,\dots,k$. The vector of Markov transition probability is given by $P = (P_{11}, \dots, P_{kk})'$, $(k^2 \times 1)$.

Take, for example, the reduced form with only two states. Thus, $s_t \in \{1,2\}$, and then 2.2 will be given by:

$$P = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix} \quad (2.3)$$

i.e., a first-order Markov chain that represents the transition between the two states and that can also be seen as;

$$\begin{aligned} p(s_t = 1 / s_{t-1} = 1) &= p_{11} & p(s_t = 1 / s_{t-1} = 2) &= p_{21} \\ p(s_t = 2 / s_{t-1} = 1) &= p_{12} & p(s_t = 2 / s_{t-1} = 2) &= p_{22} \end{aligned} \quad (2.4)$$

Finally, based on the values of transition probabilities obtained in 2.4, it is possible to calculate the length of each regime, that is, the persistency from $\frac{1}{1 - p_{ii}}$. The length of each regime may differ, but with the hypothesis that the matrix of transition probability is fixed, the length of regimes will be constant in time, that is, the expected conditional length does not vary with the cycle. Filardo and Gordon(1998) extend the model proposed by Filardo(1994) by

using a Bayesian methodology and consider that the evolution of the unobserved state depends on the available information in the time series⁷ y_t . Thus, 2.3 is given by:

$$P = \begin{bmatrix} p_{11}(y_t) & 1-p_{22}(y_t) \\ 1-p_{11}(y_t) & p_{22}(y_t) \end{bmatrix}$$

In the estimate of 2.4, we presume that the variable is known, but not the states. As illustration, consider vector $y_t = (y_{1t}, \dots, y_{nt})'$, $\{y \in \mathfrak{R}^n\}$ of observations with $t = 1, \dots, T$ and $s_t \in \{1, \dots, k\}$ the different unobserved states in which the variable might be. An occasional discrete shift is believed to occur in the level, variance, intercept, or in the autoregressive dynamics of y_t .

By assuming a distribution function for variable y , then $y_t \sim N(\mu_1, \sigma_1^2)$ if the process is in regime 1, $y_t \sim N(\mu_2, \sigma_2^2)$ if the process is in regime 2, and so on and so forth, until regime k with $y_t \sim N(\mu_k, \sigma_k^2)$. The parameter vector of the model is; $\theta = (\mu_1, \mu_2, \dots, \mu_k, \sigma_1^2, \sigma_2^2, \dots, \sigma_k^2)'$ and the density function of y_t is given by:

$$f(y_t / s_t = j; \theta) = \frac{1}{\sqrt{2\pi\sigma_j}} e^{\left\{ \frac{-(y_t - \mu_j)^2}{2\sigma_j^2} \right\}} \quad j=1, 2, \dots, k \quad (2.5)$$

The unconditional density of y_t , based on the sum of all k states that may possibly occur in t , is described by 2.6,

$$f(y_t; \theta) = \sum_{j=1}^k P(y_t, s_t = j; \theta) \quad (2.6)$$

This equation represents the sum of distributions produced by a density that depends on $P(s_t = j; \theta)$. As there are T observations, the log-likelihood of 2.6 is given by:

$$L = \sum_{t=1}^T \log f(y_t; \theta) \quad (2.7)$$

The objective is to maximize the likelihood function of the observed data $(y_T, y_{T-1}, \dots, y_1; \rho, \theta)$ where $\rho = (\rho_{11}, \dots, \rho_{kk})'$ and θ as defined above, based on the choice of population parameters (ρ, θ) , that is, transition probabilities, mean, and variance.

This maximization is achieved by an iterative process, in which it is necessary to have the initial values of the parameter vector θ for the model so that it can be implemented. We should underscore that convergence takes place when the variation between θ in iteration $m+1$ and θ in iteration m is less than any specified value ε , or when the first-order condition for maximum likelihood is met within a tolerance interval. The estimate of maximum likelihood is

⁷ The authors showed that the time-varying regime shift model, by means of Bayesian priors produce estimates of recession probabilities that are more aligned with the data of the business cycles found by the NBER for the American economy than the ones that were generated by the model with fixed transition probability, which uses initial information about the parameters.

then given by $\hat{\theta}$, and therefore, it is possible to make inferences about the regimes associated with each observation y_t in time.

Although maximum likelihood estimation is a method that has optimal asymptotic properties, we do not have a theoretical solution to the likelihood equation in some applications.

In this case, it is necessary to use some numerical optimization technique applied to the likelihood in order to obtain the parameters for the model. One of the alternatives proposed by Hamilton (1990) to the use of Newton-Raphson or David-Fletcher-Powell methods is the EM (Expectation-Maximization) algorithm, which was initially introduced by Dempster, Laird and Rubin (1977)⁸.

The EM algorithm is an iterative technique for the estimation of maximum likelihood designed for a general class of models where the observed time series depends on some stochastic variable that is not observed.

The application of the EM algorithm in econometrics occurs through the likelihood function. Each iteration of this algorithm consists of two steps, expectation (E) and maximization (M). Initially, the unknown parameters of the model, means, and variances for the different states, vector θ , Markov transition probabilities $\rho = (\rho_{11}, \dots, \rho_{kk})'$ and the probabilities of initial state ρ_0 are chosen. After that, also considering the vector of observations y_t , the smoothed probabilities are estimated.

The next step consists of maximization. Here, the parameter vector (mean and variance) is derived from the first-order condition of the maximum likelihood estimation. To find its value, the smoothed probabilities obtained in the previous step have to be replaced.

Thus, the mean of each state μ_k^l is obtained, and it may be used to obtain the covariance-variance matrix Ω_k^l , transition probabilities, and the probability of state ρ_l . Later, ρ_l is used to obtain μ_k^k and Ω_k^k and so on and so forth until μ_k^k and Ω_k^k is obtained.

This way, each EM iteration involves one step of the filter and one of the smoothing, followed by an updating of the first-order condition and of the estimated parameters, which guarantees the increase in the value of the likelihood function. For further details about the EM algorithm and its use in the maximum likelihood estimation, see Hamilton(1990) and Ruud(1991).

Therefore, if we know θ , then the probability that the process is in some regime s_t based on the information available up to t , $\rho(s_t / y_1, \dots, y_t; \theta)$, is called filtered probability. On the other hand, if all the information is used to determine s_t , $\rho(s_t / y_1, \dots, y_T; \theta)$, then we have a smoothed probability.

The univariate regime shift models can be extended to the multivariate case, where the aim is to check the existence of some similar behavior of unobservable components throughout time. In the models with MS-VAR regime, we assume that regime s_t is generated by an ergodic Markov chain and with homogeneous discrete states defined by transition probabilities 2.2.

MS-VAR models are considered a generalization of finite order autoregression for the vector of time series $y_t = (y_{1t}, \dots, y_{kt})'$ of order k , with $t=1, \dots, T$, given by:

$$y_t = A_0 + A_1(y_{t-1}) + \dots + A_p(y_{t-p}) + \varepsilon_t \quad (2.8)$$

⁸ One of the problems that might appear in this process of maximization is related to the fact that as there is a sum of distributions in $f(y_t; \theta)$, local instead of global maximums may be found in several applications. Thus, since the quality of the initial estimates may strongly influence the final result, it is advisable to have the maximization for different initial values for parameter vector θ .

assuming that $\varepsilon_t \sim NID(0, \Sigma)$.

The idea behind the class of models with multivariate regime shift is that the parameters of process 2.8 (intercept, coefficient and variance) depend on a regime variable that is not observed:

$$y_t = A_0(s_t) + A_1(s_t)(y_{t-1}) + \dots + A_p(s_t)(y_{t-p}) + \varepsilon_t \quad (2.9)$$

whereby $\varepsilon_t \sim NID(0, \Sigma_{s_t})$.

It is important to emphasize that 2.9 does not wholly describe the data generating process; however, we still lack the formulation of a regime generating process s_t that may also be given by 2.2. In this case, the intercept is not a simple parameter, and it is actually generated by a stochastic process.⁹ The mechanism of dynamic propagation of impulses in the MS-VAR model for the system consists of linear autoregression, which characterizes the transmission of shocks and the regime shift that represents common shocks.

The basic finite VAR model is given by a change in intercept, that is, MSI – Markov switching intercepts:

$$y_t = A_0(s_t) + A_1 y_{t-1} + \dots + A_p y_{t-p} + \varepsilon_t \quad (2.10)$$

that is equivalent to $y_t = A_0 + A_1 y_{t-1} + \dots + A_p y_{t-p} + \varepsilon_t$ in a structural VAR model without regime shift. By subtracting y_{t-1} on each side, we obtain the model in the form of vector error correction (VEC)

$$\Delta y_t = A_0(s_t) + \pi y_{t-1} + \sum_{i=1}^p A_i \Delta y_{t-i} + \varepsilon_t \quad (2.11)$$

This model is called MSCI(k, r)-VAR(p), autoregressive vector of order p with regime shift with k states and cointegration rank r . Aside from this formulation, another way to implement the regime shift is in the parameter of the mean $\mu(S_t)$ such that:

$$\Delta y_t - \mu(s_t) = \pi(\beta^1 y_{t-1} - \delta) + A_1(\Delta y_{t-1} - \mu(s_t)) + \dots + A_p(\Delta y_{t-p} - \mu(s_t)) + \varepsilon_t \quad (2.12)$$

where $\beta^1 y_{t-1} - \delta$ determines the correction of the long-term equilibrium. The shift may occur only in the long-term equilibrium $\delta(s_t)$, in the long-term equilibrium and in the joint mean, in coefficients A_i , or in the variance of residuals Σ_{s_t} .

For example, consider vector $z_t = (y_t, c_t)$ where the relation of co-integration is given by $\delta = c_t - y_t$ in which δ is the drift of the system. Therefore, the MS-VECM model can be represented by 2.13;

$$\begin{bmatrix} d_{11}(L) & d_{12}(L) \\ d_{21}(L) & d_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta c_t - \mu(s_t) \\ \Delta y_t - \mu(s_t) \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} (c_{t-p} - y_{t-p} - \delta(s_t)) + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \quad (2.13)$$

with $\varepsilon_t \sim NID(0, \Sigma)$.

⁹ For a detailed specification of the properties of this model, see Krolzig (1996).

Just as described in Krolzig (1997), the process of estimation of $MSCI(k,r)\text{-VAR}(p)$ models consists of two stages. First, we have an analysis of maximum likelihood, Johansen procedure, to determine the number of cointegration vector r . After that, the EM algorithm is used to obtain the model parameters, also by means of maximum likelihood estimation.¹⁰

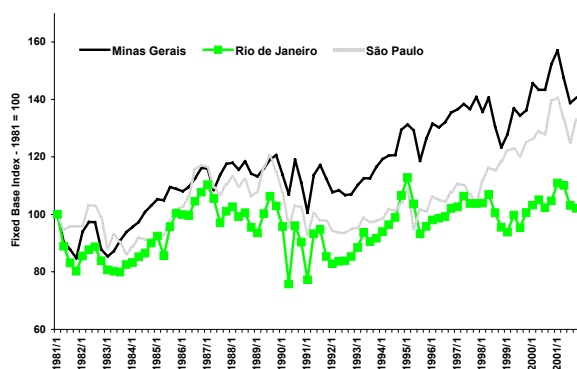
3. Empirical results

The data series used in the present paper are the indices of physical production calculated by IBGE for the states of SP, MG, RJ, PR, SC and RS, each of them consisting of 84 quarterly observations, seasonally adjusted, in the period between 1981:1 and 2001:4.¹¹ We opted for using quarterly data, instead of monthly ones, in order to reduce the problems with the dynamic specification of the model.

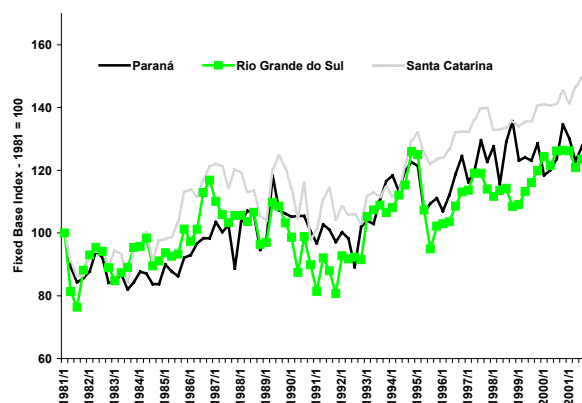
The behavior of the industrial production of these states can be seen in graphs 3.1 and 3.2, where the existence of common breaks with a similar movement between the series throughout the sample period seems clear. In the first case, the joint break may mean a nonlinear dynamics, whereas the second characteristic suggests some investigation into the existence of comovement.

As can be observed, the tendency towards the growth of industrial production was too often interrupted by sudden breaks during this period. In the early 1980s, the country was recovering from the negative effects of the second oil price shock, and the debt crisis in 1982 overwhelmed the economic activity in all Brazilian states. The process of macroeconomic adjustment in the subsequent years yielded quite positive results for the national industry; however, its effects were better in southeastern states than in the southern ones.¹²

Graph 3.1 – Physical production
Southeastern states



Graph 3.2 – Physical production
Southern states



The Cruzado Plan, a heterodox attempt to fight inflation, implemented in February 1986, and the Cruzado II Plan implemented in June in that same year, did not cause the desired effect since they produced an illusive situation of price and exchange levels that led the country into a serious crisis at the beginning of 1987. The sequence of mistakes in the economic policy,

¹⁰ Of note, for the same infinite-order VAR model, its finite-order representation still preserves the asymptotic results for the cointegration relations. See Saikkonen(1992) and Lütkepohl and Saikkonen(1995) for a detailed specification. Under the hypothesis that the data generating process is an $MSCI(M,r)\text{-VAR}(p)$ process, we may use Johansen framework for cointegration, just as in Krolzig(1996).

¹¹ The raw data of the indices were deseasonalized by the multiplicative moving average ratio, and the quarterly index was obtained through the simple mean of the monthly index.

¹² Between the first quarter of 1984 and the last quarter of 1986, the industry of São Paulo, Rio de Janeiro and Minas Gerais grew respectively, 34.8%, 28.7% and 23.4%, while the industry of Paraná, Santa Catarina and Rio Grande do Sul grew 15.8%, 26.6% and 20.5%, respectively.

Summer Plans I and II in January and May of 1989, the Collor Plan in March of 1990 and Collor II Plan in January of 1991, stagnated the Brazilian economy until the end of 1992. Between 1990 and 1992 the industrial production of Rio de Janeiro and São Paulo decreased by 24.8% and 18.9%, respectively.

The year 1994 was marked by the changes to the monetary and exchange rate regimes which contributed towards the increased growth rate of the national industry, spearheaded by the aggregate demand, a process of growth that was only cut short because of the Mexican crisis at the end of that year. From then onwards and until the end of 1997, industrial production entered a new growth cycle, especially due to the advancements of the industry in the states of SC, MG and RS.¹³ A brief interruption to this growth at the end of 1998 and the change to the exchange rate policy in early 1999 characterized a new scenario of industrial expansion until the end of year 2000.

Although the rate of industrial production indicates the existence of a comovement through these years, we note that the average annual growth rate of these states and its variability were quite different. Among southeastern states, MG was the one with the highest average growth rate (2.16%), followed by SP with 1.23% and RJ with 0.39%. Nevertheless, the production of Rio de Janeiro showed to have the greatest variability, with the highest growth within one single quarter, 23.7% in 90/III, and also the largest reduction, 23.4% in 90/II.

Of the southern states, SC had the highest average growth rate (2.29%), followed by PR with 1.44% and RS with 1%. The largest growth rate in a quarter was observed in the state of PR, 18.9% in 89/II and the greatest reduction occurred in RS, 20.6% in 81/II.

The analysis of the indices and of the rate of quarterly variation shows that all the series have a high correlation, especially the correlation between the rate of industrial production of the states of MG and SC, which is 0.95, and 0.78 for the correlation between the growth rates of the states of MG and RJ.

A series of statistical tests was carried out to check the characteristics of the data. The ADF unit root test for level series shows that it is not possible to reject the hypothesis H_0 of unit root. When the growth rate test $y_t = \ln(y_{1t} - y_{1t-1})$ is performed, the rejection of H_0 indicates that $y_t \sim I(0)$, where y_t is the vector given by $y_t = [y_{1t}^{SP} y_{2t}^{MG} y_{3t}^{RJ} y_{4t}^{PR} y_{5t}^{SC} y_{6t}^{RS}]'$, as shown in table 1 in the appendix.

The test proposed by Hylleberg, Engle, Granger and Yoo(1990), whose aim is to check the existence of seasonal unit root, was carried out for zero, biennial, and annual frequencies in the level series, and its results show that $y_t \sim I_0(1)$, $y_t \sim I_{1/2}(0)$ and $y_t \sim I_{1/4}(1)$. In other words, there is unit root at frequency zero, as confirmed by the ADF test, absence of unit root in the biennial cycle and, it is not possible to reject the null hypothesis of annual frequency for all states, as shown in table 2 in the appendix.

We should not forget that a structural break in a data series might bias the results yielded by usual unit root tests towards the rejection of the null hypothesis. Therefore, a unit root test that incorporates this structure should be used. Since the series of regional industrial production apparently has these breaks due to the several economic plans implemented, Perron's test (1997) is applied to three possible breaks, one in the intercept, in the intercept and in the slope, and the last one where the trend is associated with the movement of structural break. The results showed that, for the level series, it is not possible to reject the unit root null hypothesis in any of the states, as shown in table 3 in the appendix.

In addition, the test shows the moment in which this break occurred for each of the methods. Of note is the change observed in the intercept for all states between 1989-I and 1992-I and in the slope in 1989-III. Therefore, based on the results of these tests, it is possible to conclude that the series of industrial production of the six states has a unit root and, this way,

¹³ About the Economic Policy adopted for this period, see Pastore and Pinotti(1999).

there might be a linear combination between them that is stationary. Moreover, if these series are cointegrated, then there is a common stochastic trend.

Akaike and Schwartz criteria, used to select the number of lags in VAR (p), indicate the presence of only one autoregressive component,¹⁴ and Granger causality tests show the existence of a strong relationship between the industrial production of the states of SP and MG and the southern Brazilian states. Among southeastern states, the states of SP and MG stand out as the major promoters of industrial economic activities in Brazil, especially between them and in causality, from MG to PR and SC and from SP to RS.

The cointegration properties of data are investigated within a linear VAR representation using Johansen maximum likelihood procedure for the entire sample period, with five different alternatives,¹⁵ the most important of which is that of the model without deterministic trend and with the cointegration equation in the intercept. The results of this test are shown in table 3.1.

As observed, the model accepts the hypothesis that there is only one cointegration vector for y_t which, normalized for São Paulo, is given by equation 3.1:¹⁶

$$\beta_{SP} = (1 \quad 12.82 \quad 1.30 \quad -3.22 \quad -13.34 \quad 3.83 \quad -4.94c) \quad (3.1)$$

(14.361) (2.538) (3.854) (14.656) (4.703) (5.464)

Table 3.1 – Johansen Likelihood Ratio Test

Eigenvalue	0.432	0.247	0.191	0.172	0.076	0.042
LR	112.73	66.331	43.012	25.642	10.135	3.603
H ₀ : Rank = r	r=0	r≤1	r≤2	r≤3	r≤4	r≤5

Interestingly enough, the deviation in the equilibrium caused by a positive shock to the industrial production of Paraná and Santa Catarina produces a negative impact on the entire system.

3.1. Analysis of Business Cycle in Univariate Series

The next step is to obtain the nonlinear univariate formulations of regime shift, which will allow collecting the stylized facts of business cycles for each state. Initially, models with only one state variable in the mean, of the MSM(2)-AR(p) type with $0 \leq p \leq 4$, with later addition of a state shift in the variance, that is, MSMH(2)-AR(p) formulations, with $0 \leq p \leq 4$. In a state, we have a low or negative production growth rate, called recession; in the other state, we have a positive average growth rate associated with the expansion phase.¹⁷

In addition to the analysis of significance of parameters in each structure, the Akaike, Schwartz and Hannan-Quinn information criteria were used to select the number of lags in the models and the specification tests proposed by Hamilton(1996) were also applied to check the adaptation of these models to the data.¹⁸

¹⁴ Thus, the dynamics of the model for the equilibrium correction mechanism is given by $\Delta y_t = \pi y_{t-1} + v_t + \varepsilon_t$.

¹⁵ Namely they are: a model without deterministic trend and with no intercept in the cointegration equation, without deterministic trend, but with an intercept in the cointegration equation; model with linear trend and cointegration equation with an intercept, a model with linear trend in the series and in the cointegration equation and, finally, a model with a series containing a quadratic trend and an cointegration equation with linear trend.

¹⁶ The order of variables SP and MG was modified, and only one cointegration vector was observed. Equation 3.1 can be represented as $y_t^{SP} = 4.94 - 12.82y_t^{MG} - 1.30y_t^{RJ} + 3.22y_t^{PR} + 13.34y_t^{SC} - 3.83y_t^{RS}$.

¹⁷ MSM(k)-AR(p) models with $k > 2$ and $p > 4$ were tested, but they had convergence problems.

¹⁸ The univariate estimations and these tests were performed in Gauss 3.2.

For models in which only the mean is governed by an unobserved variable, the information criteria select the model with 4 lags for São Paulo, while for the other states, the AR(0) model is chosen. Now considering the model with regime switching in variance, the model with 4 lags is chosen for SP, the AR(0) model for MG, PR, RJ and RS, and the model with 2 lags, for SC.

Based on Hamilton (1996) tests, the MSM(2)-AR(p) formulations have problems with convergence, average autocorrelation, presence of ARCH effect and of poor nonlinear specification in all series, even after trying to implement the iterative process with different initial values. Furthermore, in several formulations, the value of transition probabilities was zero.

Therefore, the hypothesis that the residuals are homoskedastic is rejected by testing out the MSMH(2)-AR(p) models.¹⁹ For the formulations that consider the presence of autoregressive components, the results of Hamilton(1996) specification tests did not prove satisfactory. The MSMH(2)-AR(0) formulation, however, had few specification problems. Chauvet(2002), for instance, also used the MSM(2)-AR(0) model to estimate the business cycle of Brazilian GDP in order to overcome the problem with the structural breaks in the series.²⁰

The standard likelihood ratio test carried out to determine the functional form of univariate models, that is, regarding MSMH(2)-AR(p) as unrestricted model, and MSM(2)-AR(p) as restricted model, indicates that for SP, in AR(1) and AR(0) formulations, and for PR in AR(3) and AR(2) formulations, it is not possible to reject the null hypothesis in favor of the constant variance model. In all other states and for all other formulations of SP and PR, the hypothesis of presence of a Markov process in the variance is accepted, a result that was already expected due to the large difference found in σ_1 and σ_2 . This way, we may conclude that the variance affects the regime shift in the mean, thus improving the characterization of the business cycle.

In addition, while for the MSM(k)-AR(p) formulations we have absorbent states in at least one data series, that is, the transition matrix is reducible, in the formulations with state-variance, this characteristic only exists for MG in MSMH(2)-AR(4) and for SC in MSMH(2)-AR(2). By means of the eigenvalue, we verify that all the transition matrices are ergodic and, therefore, the regimes converge towards an unconditional probability distribution.

The estimation of the trend model, given that the variability of the series follows two states, produced very different results. First of all, the reduction of the value of σ_1 in relation to that obtained from the model without state-variance. Another important aspect is the significant reduction of the value of the mean in the second state, μ_2 , found in MSMH(k) models, in relation to the value of μ_2 found in the specification of the constant variance.

In general, transition probabilities p_{11} decreased and p_{22} increased when the hypothesis of state-variance was inserted. The values of p_{11} for Rio de Janeiro and Santa Catarina that were zero in the first model became positive and this rendered the observed persistencies closer to reality.

We should not forget that nothing guarantees that the model proposed by Hamilton(1989) describes the behavior of data better than linear autoregressive models. This way, in addition to the specification tests proposed by Hamilton(1996), the standard likelihood ratio test developed by Hansen(1992) was used. This test compares the linear AR(p) model with the nonlinear model proposed by Hamilton, despite the fact that these models are

¹⁹ Even if the residuals are homoskedastic, the data series Δy_t may be heteroskedastic. See Krolzig(1997).

²⁰ Note that in MSM(2)-AR(0), the series y_t under question is the function of an integrated process that follows a Markov chain and a white noise process, that is, $y_t = \mu_{s_t} + \varepsilon_t$. Krolzig(1997) found broad similarity between business cycle estimations for Germany's GNP between MSM(k) and MSI(k) models.

estimated through unobservable parameters, such as transition probabilities and the means in each state.

The test may be applied to regime shift models, both in the mean and in the intercept or in the regression parameters and in the variance. While this test regards the regime shift in the mean as an alternative model, it is not possible to reject the null hypothesis in favor of the linear model in any of the series at any interval used for the mean and for the transition probabilities.

On the other hand, when the regime shift occurred in the intercept, the results were somewhat ambiguous. In the series of physical production of SP, the null hypothesis is rejected in favor of the nonlinear model in all of the lags considered. However, for MG, RJ, and SC, the null hypothesis is not rejected when the model with only one lag is considered. For the series of PR and RS, it is not possible to reject the linear model.

Somehow, we can say that for southeastern states the test signals the acceptance of a nonlinear model, whereas for southern states, an autoregressive model would fit the data better. Table 3.2 shows the results for the MSMH(2)-AR(0) formulation, and table 3.3, the results of the specification tests proposed by Hamilton(1996) for the parameters in table 3.2.

As we can see, the results displayed in table 3.3 show the good adaptation of the MSMH(2)-AR(0) formulation to the data. There is neither autocorrelation in the average growth rate for the two states, μ_1 and μ_2 , nor autocorrelation in regime 1 and between the regimes in all the series. However, there is incidence of the ARCH effect and of Markov specification in the series of PR and RS. The industrial production of RJ continues to have autocorrelation in regime 2, just as observed in previous formulations, where autoregressive components were present.

Table 3.2 – MSMH(2)-AR(0)

	SP	MG	RJ	PR	SC	RS
μ_1	1.245 (0.497)	1.647 (0.356)	1.696 (0.569)	2.586 (1.206)	0.768 (0.427)	0.881 (0.596)
μ_2	-0.248 (0.844)	-0.385 (0.806)	-1.600 (1.562)	-0.657 (1.090)	0.373 (0.784)	-0.405 (1.380)
σ_1	4.335 (2.095)	2.715 (1.084)	5.439 (4.504)	5.957 (4.275)	4.289 (1.380)	8.654 (3.784)
σ_2	29.984 (7.230)	29.646 (6.581)	69.872 (21.985)	48.722 (12.340)	35.562 (6.703)	73.818 (20.911)
P_{11}	0.787 (0.113)	0.820 (0.092)	0.776 (0.109)	0.159 (0.294)	0.980 (0.027)	0.835 (0.113)
P_{22}	0.869 (0.090)	0.891 (0.072)	0.790 (0.121)	0.629 (0.206)	0.988 (0.014)	0.830 (0.116)
P_1 Length	5	6	5	2	50	7
P_2 Length	8	10	5	3	84	6
Eigenvalue λ_2	0.656	0.711	0.566	-0.212	0.968	0.665
$\bar{\xi}_1$	0.381	0.377	0.484	0.306	0.375	0.507
$\bar{\xi}_2$	0.619	0.622	0.516	0.694	0.625	0.492
Log-Likelihood	-162.79	-157.69	-183.94	-190.14	-167.98	-188.82

*The priors are given by (a=0.2 b=1 c=0.1). The vector of initial parameters $(\mu_1^0 \mu_2^0 \sigma_1^0 \rho_{11}^0 \rho_{22}^0)$, for each series is given by: SP=MG=RJ=PR=SC=RS=(1 -1 1 6 1.5 1.5).

Table 3.3 – Specification Test – MSMH(2)-AR(0)

Test	SP	MG	RJ	PR	SC	RS
White	2.19	5.34	7.14	7.77	3.05	1.62
Autocorrelation	(0.70)	(0.25)	(0.13)	(0.10)	(0.55)	(0.81)
White	5.50	2.48	9.60	11.4	6.67	14.9
ARCH	(0.24)	(0.65)	(0.05)	(0.02)	(0.15)	(0.00)
White	11.9	6.86	8.13	10.1	3.03	14.2
Markov Specification	(0.02)	(0.14)	(0.09)	(0.04)	(0.55)	(0.01)
LM	0.15	0.22	0.18	0.64	0.22	0.73
Autocorrelation in regime 1	(0.70)	(0.64)	(0.67)	(0.42)	(0.64)	(0.39)
LM	1.44	0.24	4.54	5.40	2.17	0.15
Autocorrelation in regime 2	(0.23)	(0.62)	(0.03)	(0.02)	(0.14)	(0.70)
LM	1.74	0.02	0.54	1.70	1.64	0.10
Autocorrelation between regimes	(0.19)	(0.89)	(0.46)	(0.19)	(0.20)	(0.75)
LM	1.36	0.44	3.93	0.03	0.07	3.52
ARCH	(0.24)	(0.51)	(0.05)	(0.86)	(0.79)	(0.06)

Note: The asymptotic p-value is shown between parentheses. $\chi^2(1) = 3.84$ to 5%.

Thus, by comparing the results obtained from the specification tests in table 3.3 and from Hansen(1992) tests, the best univariate estimate for each series of the physical production of the states is the model with shift in the mean without autoregressive components.

An alternative to the model described in table 3.3 is the simple random walk. To verify whether this hypothesis is valid or not for the rate of variation of the physical production, we use Wald test, with two hypotheses.²¹ One hypothesis that checks the independence of states, that is, if s_t is independent of s_{t-1} , based on the null hypothesis ($P_{11}=1-p_{22}$), and the other one which verifies whether the average rates are the same, that is, $\mu_1=\mu_2$. Table 3.4 shows the results for these two hypotheses.

Table 3.4 –Wald Test

H ₀	SP	MG	RJ	PR	SC	RS
$P_{11}=1-p_{22}$	14.94	26.33	13.24	0.61	720.8	10.89
	(0.00)	(0.00)	(0.00)	(0.43)	(0.00)	(0.00)
$\mu_1=\mu_2$	2.06	5.20	4.35	3.97	0.19	0.67
	(0.15)	(0.02)	(0.04)	(0.05)	(0.66)	(0.41)

Note: The asymptotic p-value is shown between parentheses. $\chi^2(1) = 3.84$ to 5%.

Notably, it is not possible to reject the null hypothesis of random walk from $P_{11}=1-p_{22}$ only for the series of Paraná. For all the other series, the null hypothesis is strongly rejected. Due to the low values of the average rates in the second state, μ_1 and μ_2 were very close, and the means equality test reveals that they are the same for the series of SP at 15%, SC at 66% and RS at 41%. Therefore, we may conclude that after the results shown in table 3.4, the movements in the industrial production of the states do not seem to be described by long swings, with a quick alternation between expansion and recession periods.

The first prominent characteristic of table 3.2 involves the business cycles of the states, where a growth period and a recession period in industrial production are not defined only for Santa Catarina. In expansion periods, the production of Paraná grows 10.7% p.a., while the

²¹ See Engel and Hamilton(1990).

lowest rate is that of SC, with a growth of 3.11% p.a. Periods of reduced activity are more intense in RJ, (-6.2%) p.a., whereas SC has the positive rate of 1.5% p.a.

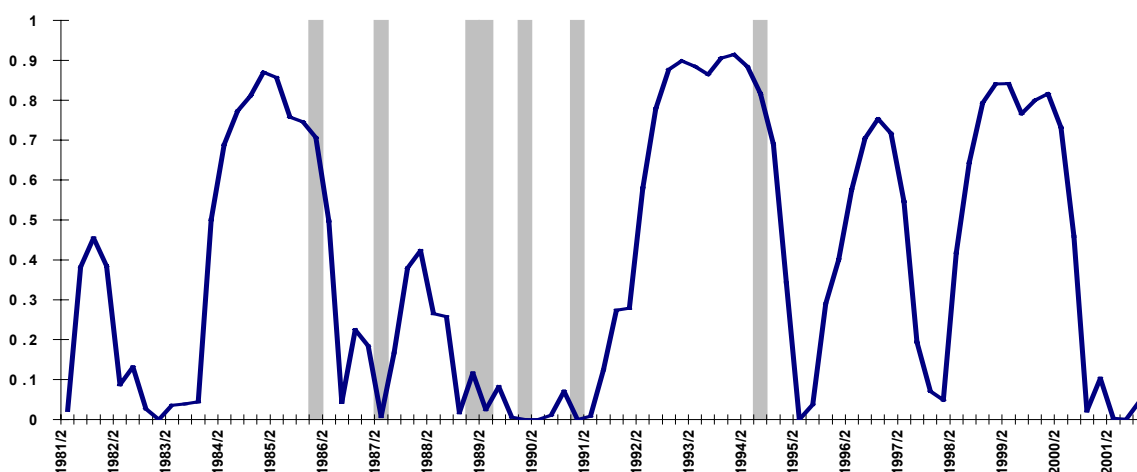
This result is in agreement with the finding that all the series have a high variance and, due to the fact that there is a more often shift in regime in the industrial production of PR, the growth rate is harder to predict. In fact, the industrial production is an indicator of activity that responds more intensely and immediately to variations in economic conditions. With mean and variance following a Markov process, we can see that high growth rates are associated with low variance, while in the periods of reduced activity, we have high variability, especially in Rio de Janeiro and in Rio Grande do Sul.

The asymmetry of cycles, measured by the average duration of growth and recession periods, is of great importance as well. Except for RJ, which seems to have business cycle symmetry, the model allows capturing the asymmetry in the phases of business cycles in the physical production of the industry of other states. The industrial production in Santa Catarina has the largest persistency for the average growth rate, 50 quarters, and also the largest rate of reduction, 84 quarters. In the other states, these rates oscillate between a maximum of 7 quarters for growth, in the case of RS, and of 10 quarters for reduction, observed in MG, which is consistent with the traditional definition of business cycle.

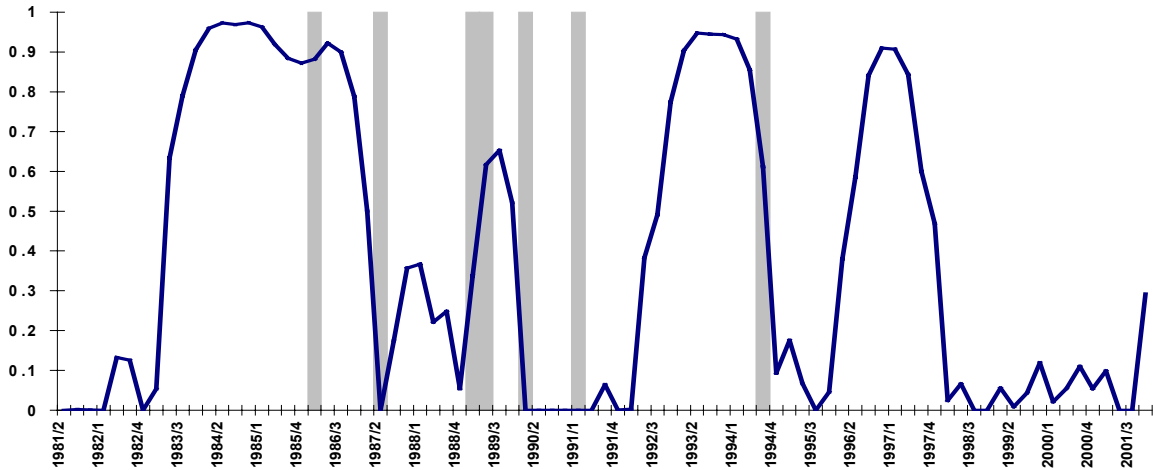
Graphs 3.3 to 3.8 show the smoothed probabilities of being in a growth period $P(s_t=1)$, for MSMH(2)-AR(0) model, that is, $\rho(s_t = 1/\psi_\tau) \geq 0,5$, and the dark bars represent the dates of the economic plans.

As we can see, the growth cycles of the southeastern states seem to have a similar behavior over time. Among the southern states, only the industrial production of RS seems to have a common relationship with the southeastern states, differing from the cycle of SC and PR. This coherence in the regime shift between the periods of expansion and recession suggests the investigation of a business cycle from a common regime shift.

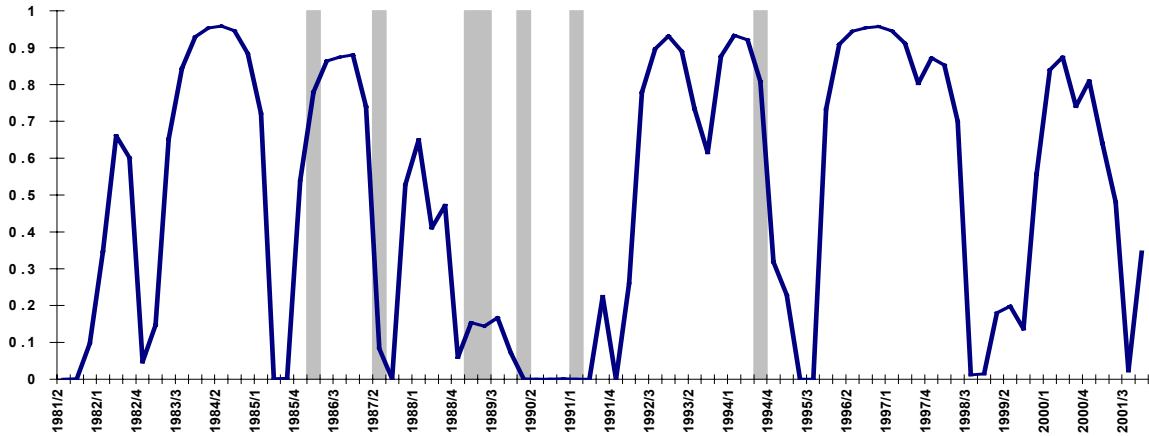
Graph 3.3 – $P(s_t=1)$ for São Paulo



Graph 3.4 – $P(s_t=1)$ for Minas Gerais



Graph 3.5 – $P(s_t=1)$ for Rio de Janeiro



Graph 3.6 - $P(s_t=1)$ for Paraná

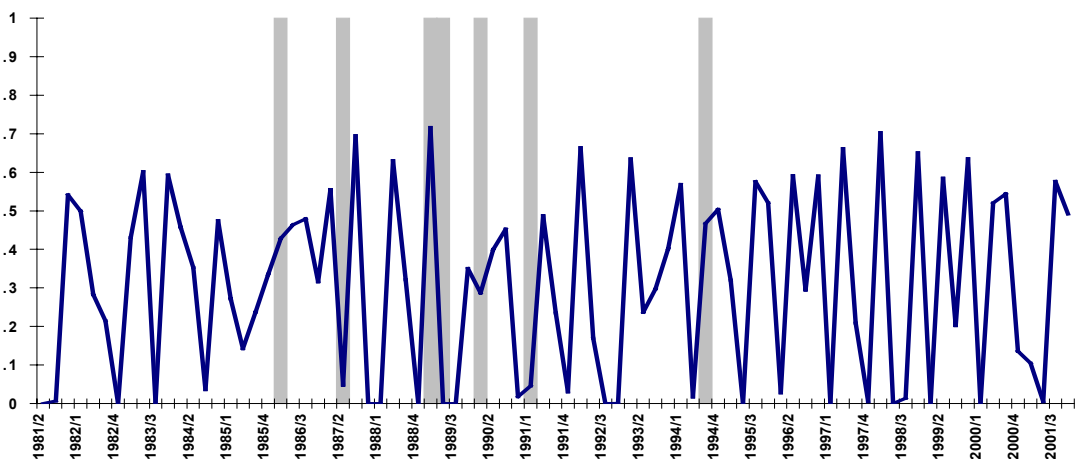
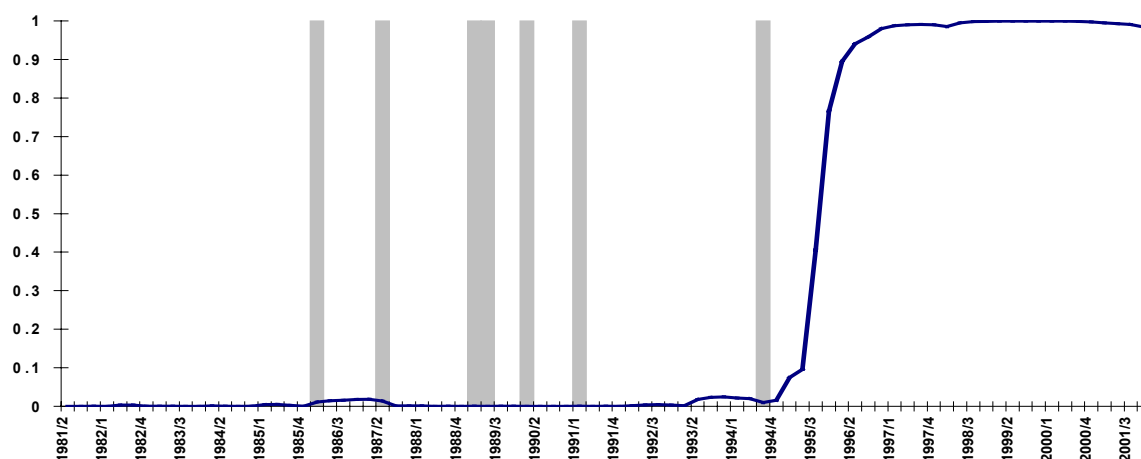


Table 3.5 shows the intervals that characterize the beginning and end of recessions in the industrial production for the five states.²² In the absence of an official determination of the

²² The criterion adopted here to characterize an economic cycle was based on the proposal of the NBER and the same used in Chauvet(2002), that is, a recession (expansion) corresponds to a reduction (increase) in activity with

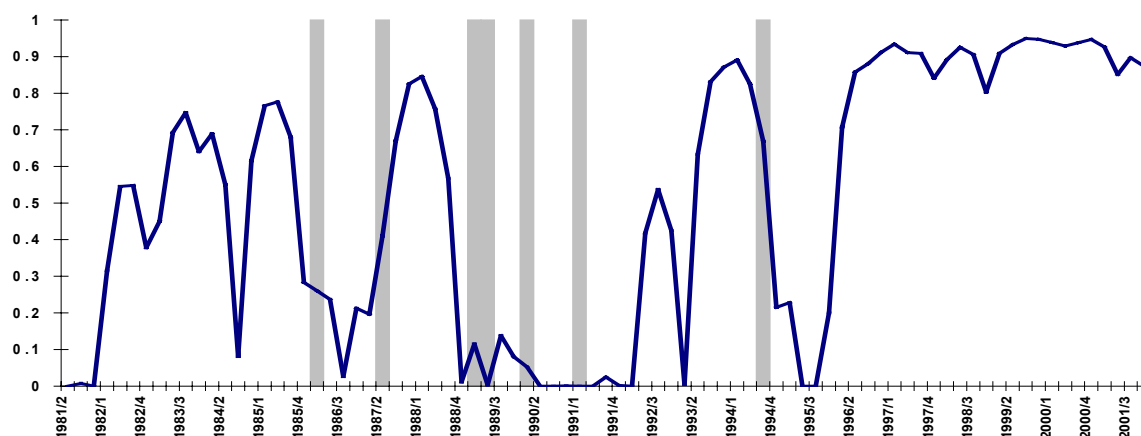
business cycle for the Brazilian economy, the periods of expansion and recession observed for the industrial production of the states are compared with the economic plans implemented in Brazil in the last twenty years. As we can see, the industry of SP and MG have 5 recessions, RS has 6, RJ presents 8 and PR has 11 recessions, where special attention should be paid to the early 1980s, which was marked by a generalized reduction in the industrial activity, and to the economic plans implemented in the second half of that decade, which stagnated production until mid-1992.

Graph 3.7 - $P(s_t=1)$ for Santa Catarina



The states that most suffered from the implementation of these plans were RJ and PR. The industry that had the longest expansion period was that of RS with 18 quarters, beginning in 87/IV and ending in 93/III, and that of MG, between 83/II and 87/I. The recovery of the Brazilian economy by the end of 1992, with the impeachment of president Collor, may be seen from the expansion of production in the industry of all states, especially in the southeastern ones, where SP grew 11 quarters.²³

Graph 3.8 - $P(s_t=1)$ for Rio Grande do Sul



After the Real Plan we can perceive the impact of the Mexican crisis on the national industry at the end of 1994, which, in the case of SP, MG and RS, extended until the beginning

a minimum length of 2 quarters. The author found that Brazil's GDP had 8 recessions and 9 expansions between the second quarter of 1980 and the first quarter of 2000, which were caused by external shocks.

²³ Chauvet(2002) also found out that the Brazilian economy started the 1980s in recession and that between 1983 and 1987 it experienced 16 quarters of expansion.

of 1996. However, the second half of the 1990s was characterized by smaller recessions, except for the industry of RS, which showed a drop in production between 96/I and 01/IV.

Table 3.5 – Dates of Business Cycles

São Paulo		Minas Gerais		Rio de Janeiro		Paraná		Rio Grande do Sul	
Peak	Troughs	Peak	Troughs	Peak	Troughs	Peak	Troughs	Peak	Troughs
81:II	83:IV	81:II	83:I	81:II	82:I	81:II	81:III	82:II	82:III
86:II	92:I	87:II	89:I	82:IV	83:I	82:I	83:I	83:II	84:II
95:I	96:I	90:I	92:III	85:II	85:III	84:I	86:IV	84:IV	85:III
97:III	98:II	94:IV	96:I	87:II	87:III	87:IV	88:I	87:III	88:III
00:III	01:IV	97:IV	01:IV	88:II	92:I	88:III	88:IV	93:II	94:III
				94:IV	95:III	89:II	91:IV	96:I	01:IV
				98:III	99:III	92:II	92:IV		
				01:II	01:IV	93:II	93:IV		
						94:II	94:III		
						95:I	95:II		
						97:III	97:IV		
						00:IV	01:II		

Note: Peaks are the end of an expansion period and the beginning of a recession, while troughs are characterized as the beginning of an expansion and the end of a recession.

3.2. Common Regime Shift

If the business cycle is a common characteristic of several macroeconomic time series, then the use of a system may improve the statistical inference about the common component of this cycle. Therefore, the multivariate regime shift model is expected to produce better results than the univariate model, in terms of capturing the characteristics of the business cycle.

So, the next step is to jointly estimate the short-run and long-run dynamics of the quarterly growth rate $\Delta y_t = [\Delta y_t^{SP}, \Delta y_t^{MG}, \Delta y_t^{RJ}, \Delta y_t^{PR}, \Delta y_t^{SC}, \Delta y_t^{RS}]'$, in a vector error correction model with shifts in the k regimes. These common characteristics seem to occur, for the series used herein, in function of the shocks caused by the economic plans implemented in Brazil, especially in the 1980s, in addition to international crises.

Assuming that there is a perfect correlation between regime shifts, the mechanism of dynamic propagation of impulses represents not only the transmission of shocks between states or regions, but also the common shocks. This mechanism consists of the transition matrix specified in 2.2. Initially, we estimated models in order to select the lag of VAR by using the regime shift in the mean, in the intercept, and in the variance, also considering the hypothesis of $k > 2$; however, it is important to underscore that, although it is possible to estimate models with more than two states, the convergence of the model is more difficult the larger the k value.

The likelihood ratio test carried out to select the lag of VAR, indicates only one lag for all the variables with $\chi^2_{(36)} = 0.000$, just as obtained in the linear VAR, and the results for the MSMH(2)-VAR(1) model for all the series together, with regime shift in the average rate of growth μ and in the error variance Σ , showed satisfactory results. The resulting model is given by equation 3.2:

$$\Delta y_t - \mu(s_t) = A_1(\Delta y_{t-1} - \mu(s_{t-1})) + \varepsilon_t \quad \varepsilon_t \sim (0, \Sigma_{(s_t)}) \quad (3.2)$$

whereas table 3.6 shows the estimated parameters of interest.

Table 3.6 – Results for the MSMH(2)-VAR(1) Model – All states

	SP	MG	RJ	PR	SC	RS
μ_1	1.508 (0.403)	1.379 (0.360)	0.968 (0.358)	0.738 (0.735)	1.094 (0.357)	1.007 (0.570)
μ_2	-0.979 (0.694)	-0.539 (0.701)	-0.715 (1.119)	0.239 (0.906)	0.325 (1.038)	-0.103 (1.171)
SP_{t-1}	-0.097 (0.098)	0.211 (0.080)	0.326 (0.080)	-0.209 (0.154)	0.090 (0.078)	0.178 (0.128)
MG_{t-1}	0.363 (0.122)	0.078 (0.104)	0.466 (0.104)	-0.310 (0.181)	0.018 (0.101)	0.081 (0.163)
RJ_{t-1}	-0.444 (0.125)	-0.487 (0.110)	-0.744 (0.131)	-0.113 (0.174)	-0.357 (0.119)	-0.378 (0.176)
PR_{t-1}	-0.013 (0.060)	0.013 (0.050)	-0.052 (0.048)	-0.335 (0.092)	-0.007 (0.047)	0.023 (0.078)
SC_{t-1}	0.339 (0.110)	0.148 (0.095)	0.205 (0.104)	0.288 (0.155)	-0.116 (0.097)	-0.052 (0.150)
RS_{t-1}	0.025 (0.080)	0.221 (0.067)	0.311 (0.066)	0.277 (0.121)	0.409 (0.065)	0.198 (0.106)
σ_1	2.611	2.052	1.826	5.231	1.845	3.100
σ_2	4.559	4.760	7.394	6.085	6.717	7.574
P_{11}		0.632			Length: 2.7 quarters	
P_{22}		0.550			Length: 2.2 quarters	
Eigenvalue		$\lambda_1=1$			$\lambda_2=0.182$	

As we can see, the estimates for the mean in state 1, except for PR, were all significant and, for SP, SC and RS its value is greater than that obtained by the univariate MSMH(2)-AR(0) model. For the second state, the parameters are not significant and were larger than the univariate estimates for SP, MG and PR. The variances in state 1 had closer values, but remarkably decreased for the second state.²⁴

We can also note that there is some asymmetry in the growth rate of industrial production in all states, both for the expansion and recession periods; and PR and SC show a positive average growth rate associated with the second regime. São Paulo is the state with the largest growth rate in the expansion and recession periods. From the results of $P_{11}>0$, $P_{22}>0$ and $0<\lambda_2<2$, we may conclude that the transition matrix is irreducible and ergodic.

The probability that all states have in $t-1$ an expansion of the average production and keep it in t , that is, p_{11} , is 0.632, resulting in a length of 2.7 quarters for the expansion period. On the other hand, the period of joint reduction in the production of states lasts 2.2 quarters. Graph 3.9 shows the cycles of expansion in the joint industrial production between the six states. The definition of the periods of the recession cycle for the MSMH(2)-VAR(1) model is shown in table 3.7.

Table 3.7 – Dates of Business Cycles

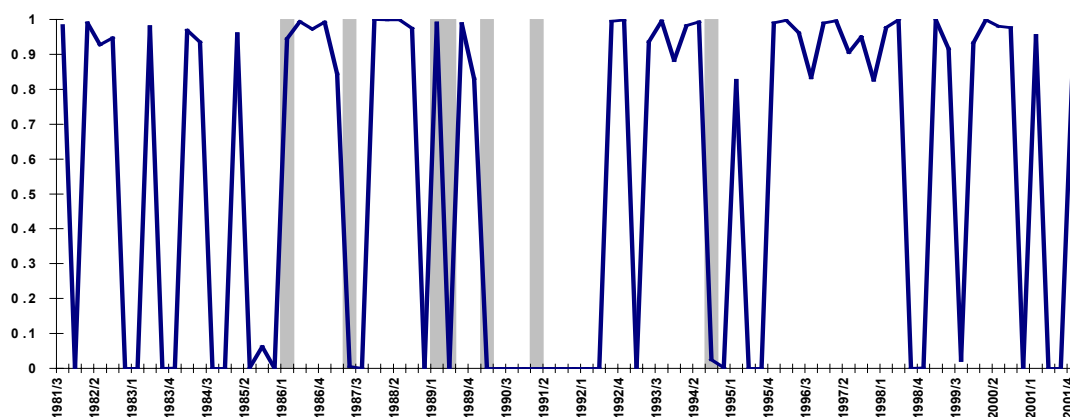
Troughs	82:I	84:I	86:I	87:IV	89:III	92:III	93:II	95:IV	99:I	99:IV
Peaks	82:III	84:II	87:I	88:III	89:IV	92:IV	94:II	98:II	99:II	00:III

In the first half of the 1980s, industrial activity had two long periods of recession, which did not occur in the second half. Between 84/II and 86/I, industrial activity reduced during 7 quarters in a row; however, the longest recession observed in the series occurred

²⁴ For the estimation of parameters for the multivariate models, we used the Ox 3.0 statistical package.

before the Real Plan, between 89/IV and 92/III. The longest expansion period was observed between 95/IV and 98/II, with 10 quarters of increasing the economic activity.

Graph 3.9 – Probability of Expansion



The Brazilian industry also exhibited some signs of recession at the end of 2000, which was not reversed due to electric power rationing, high real interest rates, the Argentinean economic crisis and the reduction of the economic activity of the USA. The model estimated here determined that the industrial production of Brazilian states, altogether, oscillated between a period of growth and reduced activity at the end of 2000. The estimated reduction in the average growth rate of industrial production of states, measured by $\mu_1 - \mu_2$, shows that a larger reduction occurs in the southeastern states, [2.487 1.918 1.683 0.499 0.769 -1.110] that is, when one goes from an expansion period to a recession period, the industrial activity of SP has the greatest impact with 10.32% p.a., and that of PR the lowest with 2.01% p.a.

In addition, two other characteristics motivate the estimation of a regional multivariate model. Firstly, the behavior of the autocorrelation matrix of table 3.6. By analyzing these coefficients, we note that innovations in the growth rate of the industrial production of SP in $t-1$ do not produce an effect on the production of the state in t . The same is valid for the autocorrelation of the states of MG and RS.

Moreover, there seems to exist a larger autocorrelation between the growth rates of the industrial production of the southeastern states than between these states and the southern states, which also seem to have a poor relation between themselves. Finally, it is possible to see that the autocorrelation of the growth rate of the industrial production of RJ with all the other states is negative, where we note that the positive shocks in $t-1$ to the growth of the industry of RJ produce more significant effects, and inversely in t , in the southeastern states than in the southern states.

Secondly, the fact that, by analyzing the nonlinear univariate estimates, the southeastern states seem to have a cyclic behavior that is more clearly defined than the industrial production of the southern states. This way a multivariate model is estimated for the southeastern states and another one just for the southern states.

For the first case, the autoregressive coefficients of lag 2 in MSMH(2)-VAR(2) did not show to be significant. With the likelihood ratio test we have $\chi^2_{(9)} = 0.000$, in which the restriction of VAR(2) for VAR(1) and of VAR(1) for VAR(0) is not accepted. However, the H-Q criterion selects VAR(1), and most of the autoregressive coefficients are significant. The results for the estimates of the multivariate model of the southeastern region are shown in table 3.8.

Table 3.8 –MSMH(2)-VAR(1) – Southeastern Region

	SP	MG	RJ
μ_1	1.340 (0.531)	1.367 (0.444)	1.758 (0.447)
μ_2	-0.362 (0.799)	-0.215 (0.883)	-1.222 (1.047)
SP_{t-1}	-0.058 (0.152)	0.175 (0.100)	0.247 (0.130)
MG_{t-1}	0.379 (0.168)	0.183 (0.119)	0.535 (0.186)
RJ_{t-1}	-0.285 (0.135)	-0.341 (0.095)	-0.541 (0.172)
σ_1	2.678	1.476	2.585
σ_2	5.060	5.386	7.221
P_{11}	0.730	Length: 3.7 quarters	
P_{22}	0.755	Length: 4 quarters	
Eigenvalue	$\lambda_1=1$	$\lambda_2=0.485$	

As we can observe, the average growth rate of SP is smaller in the regional formulation than in the formulation in table 3.6, but greater than the rate observed in the univariate formulation. For MG, the value found for μ_1 is smaller than the previous formulations. For RJ, the average rate is greater in the regional formulation and the effects of a regime shift, measured by $\mu_1 - \mu_2$, are also greater in the industry of RJ.

In general, the variances in the two states remained the same, but the length of recession and expansion is longer than that estimated in the complete VAR. Here, the transition matrix is also irreducible and ergodic. On top of that, there does not seem to have autocorrelation in the innovations of SP and also for MG. The effects of a shock to the growth rate of the industry of RJ on the other two states continues to be negative; however, with lower coefficients than those obtained in the general formulation.

Graph 3.11 shows the behavior of probabilities of expansion for MSMH(2)-VAR(1) model of the southeastern region, and table 3.9 shows the correspondent dates.

Graph 3.11 – Probability of Expansion

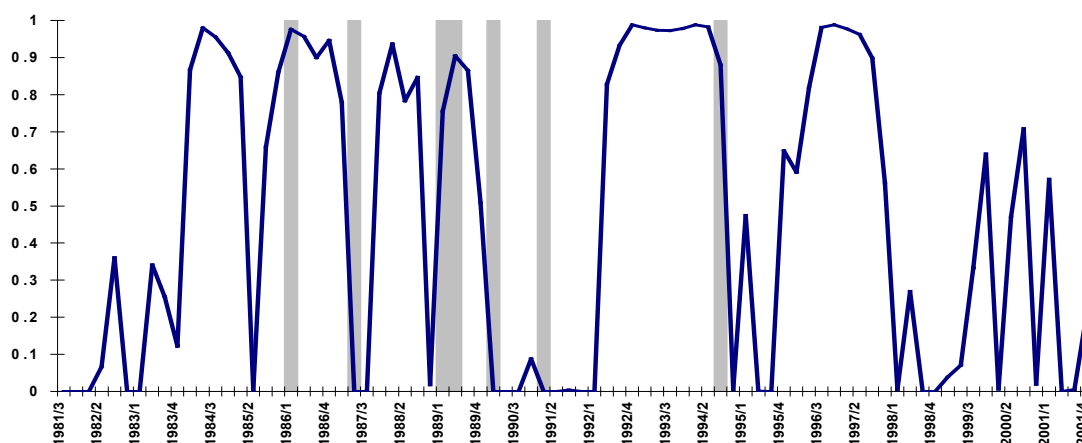


Table 3.9 – Dates of Expansion Cycle

84/I – 85/I	85/III – 87/I	87/IV - 880/III
89/I – 89/IV	92/II – 94/III	95/IV -97/IV

Finally, a model for the industrial production of southern states was estimated. The likelihood ratio test rejects the hypothesis of restriction of 2 lags in VAR for 1 and from 1 to 0. The Akaike and H-Q criteria select VAR(0), but Schwartz criterion selects VAR(2). Thus, the MSMH(2)-VAR(2) formulation is selected for the southern region.

When we compare the results of table 3.10 with the ones obtained in table 3.6, we note that the growth and reduction rate increased, that is, in μ_1 and μ_2 for PR and RS. The length of the growth and recession period also increased remarkably, going to 8.6 and 8.4 quarters, respectively. From the values of P_{11} , P_{22} and λ_2 , we see that the transition matrix is irreducible and ergodic.

The autocorrelation matrix shows that innovations in period $t-2$ and $t-1$ for all three states cause changes to the industrial production of SC only. The growth rate of the industrial production of PR is affected only by innovations in the industry of the state one and two quarters before. On the other hand, the production of RS does not seem to be affected by shocks in the other states. Graph 3.12 shows the probability associated with the regime of expansions, where the dark bars describe the economic plans, and table 3.11 shows the expansion periods for the joint industrial production of the three southern states.

Table 3.10 – Results for MSMH(2)-VAR(2) estimates – Southern Region

	PR	SC	RS
μ_1	1.221 (0.509)	0.816 (0.270)	1.458 (0.544)
μ_2	-0.305 (0.540)	0.407 (0.793)	-0.565 (0.836)
PR_{t-1}	-0.406 (0.107)	0.121 (0.064)	0.136 (0.096)
PR_{t-2}	-0.435 (0.100)	-0.125 (0.058)	-0.073 (0.094)
SC_{t-1}	0.117 (0.142)	-0.346 (0.123)	-0.175 (0.157)
SC_{t-2}	-0.121 (0.142)	-0.234 (0.117)	-0.284 (0.157)
RS_{t-1}	0.121 (0.119)	0.266 (0.079)	0.048 (0.116)
RS_{t-2}	0.076 (0.119)	0.109 (0.060)	-0.122 (0.107)
σ_1	5.417	1.853	3.697
σ_2	4.874	6.463	6.608
P_{11}	0.884	Length: 8.6 quarters	
P_{22}	0.882	Length: 8.4 quarters	
Eigenvalue	$\lambda_1=1$	$\lambda_2= 0.766$	

By comparing the multivariate model obtained for the southeastern region with the estimates for the southern region, we conclude that the dynamics of expansion and recession between the industries of these two regions are quite different, although there is some similarity in the cycle in some periods.

Graph 3.12 – Probability of Expansion

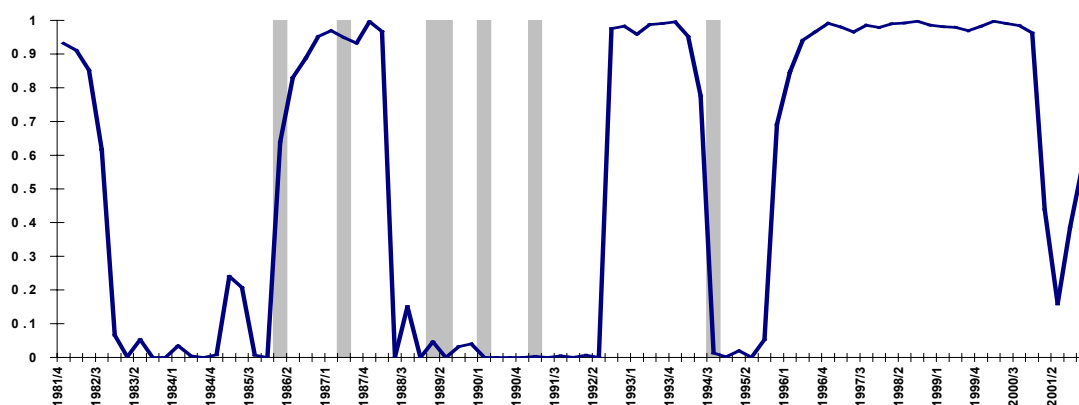


Table 3.11 – Dates of Expansion Cycle

81/IV – 82/III	86/I - 88/I	92/III – 94/II	95/IV – 00/IV
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The parameter μ_1 for the southeastern states is larger than that observed for the southern states, except for RS. This means that, when one is in a cycle of expansion, the southeastern industry has a larger and less variable average growth. However, according to the transition probabilities, this expansion has a shorter length in the southeast, 3.7 quarters, than that observed in the southern region, 8.6 quarters.

By analyzing the parameter μ_2 it is also possible to infer that the reduction in activity is larger for the southeastern states than for the southern ones, except for RS, which also has a large average rate of reduction. The variability of this regime is, in general, larger for southeastern states than for the southern ones. Nevertheless, the cyclic period of recession is shorter for southeastern states, 4 quarters, against 8.4 of the southern states.

4. Conclusion

The several shocks that the Brazilian economy has been put through in the last few years have remarkably affected the growth dynamics of the national industry. By using nonlinear univariate regime shift models, we were able to determine the date of the periods in which the industrial production of the states of São Paulo, Minas Gerais, Rio de Janeiro, Paraná, Santa Catarina and Rio Grande do Sul were in expansion or recession.

Due to the similarity between the cycles of this production, especially between the southeastern states, a multivariate MSMH(2)-VAR(p) model was separately estimated for the set of the six states and for the two regions.

The results showed that the growth dynamics and reduction in activity presented by the southeastern states differs from that observed in the southern states. When one is in a period of industrial expansion, the southeastern states tend to have average growth rates greater than those found in the southern states, except for RS. However, this expansion cycle of the southeastern region has a shorter length than that observed in the southern region, 3.7 and 8.6 quarters, respectively.

In periods of recession, the reduction in industrial activity is also larger in the southeastern states, but this recession period tends to last longer in the southern states (8.4 quarters), in comparison to the production of the southeastern region, (4 quarters). These results indicate that the macroeconomic shocks will have different magnitude and dynamics between these two regions.

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Appendix

Table 1. Unit root test – with shifts in level and differences - (1981:1 to 2001:4)

	τ	τ_{μ}	τ_{τ}	I(.)		τ	τ_{μ}	τ_{τ}	I(.)
SP	0.657	-1.299	-2.360	I(1)	Δ SP	-9.951 ^a	-9.974 ^a	-9.948 ^a	I(0)
MG	1.651	-1.178	-2.497	I(1)	Δ MG	-9.403 ^a	-9.660 ^a	-9.599 ^a	I(0)
RJ	0.470	-1.946	-2.433	I(1)	Δ RJ	-10.013 ^a	-9.978 ^a	-9.901 ^a	I(0)
PR	1.217	-0.901	-3.834	I(1)	Δ PR	-3.893 ^a	-4.095 ^a	-4.088 ^a	I(0)
SC	1.966	-1.202	-3.677	I(1)	Δ SC	-8.481 ^a	-8.737 ^a	-8.672 ^a	I(0)
RS	1.033	-2.241	-3.877	I(1)	Δ RS	-9.252 ^a	-9.320 ^a	-9.258 ^a	I(0)

Note: τ means with no constant, τ_{μ} is the test using a constant and τ_{τ} is the test with constant and trend. I(.) is the order of integration. Rejects at 1%, b 5% and c above 10%.

Table 2 – Seasonal unit root test

States	Aux. Reg.	$t: \pi_1$	$t: \pi_2$	$t: \pi_3 \cap \pi_4$
SP	-	1.302	-4.992	27.221
	I	-0.792	-5.018	27.432
	I, SD	-0.785	-4.941	26.223
	I, Tr	-1.412	-4.974	26.683
	I, SD, Tr	-1.378	-4.896	25.488
MG	-	1.890	-6.494	16.522
	I	-1.838	-6.558	17.561
	I, SD	-1.832	-6.470	16.984
	I, Tr	-2.605	-6.699	18.576
	I, SD, Tr	-2.564	-6.607	17.960
RJ	-	0.644	-6.084	17.680
	I	-2.302	-6.258	19.478
	I, SD	-2.264	-6.133	18.680
	I, Tr	-2.481	-6.261	19.437
	I, SD, Tr	-2.432	-6.133	18.620
PR	-	1.722	-4.504	13.681
	I	-0.864	-4.506	13.766
	I, SD	-0.858	-4.409	13.202
	I, Tr	-2.563	-4.638	14.753
	I, SD, Tr	-2.498	-4.535	14.125
SC	-	1.682	-4.691	21.140
	I	-0.886	-4.672	21.161
	I, SD	-0.872	-4.582	20.332
	I, Tr	-2.101	-4.761	22.146
	I, SD, Tr	-2.058	-4.668	21.265
RS	-	0.828	-6.066	22.223
	I	-2.024	-6.247	23.750
	I, SD	-1.983	-6.146	22.656
	I, Tr	-2.873	-6.353	24.803
	I, SD, Tr	-2.815	-6.248	23.669

Note: The deterministic term is zero (-), one intercept (I), one seasonal dummy variable (SD) and one trend (Tr). a – significance level at 1% and b at 5%. The table can be obtained in Hylleberg et. alli(1990).

Table 3 – Unit root test in the presence of structural break

<i>Test</i>	<i>Model</i>	<i>SP</i>	<i>MG</i>	<i>RJ</i>	<i>PR</i>	<i>SC</i>	<i>RS</i>
UR $t_{\alpha}^*(i)$	1	-4.625	-4.886	-4.755	-6.537	-5.547	-4.847
	2	-4.592	-4.796	-4.726	-6.370	-5.786	-4.771
	3	-3.116	-2.794	-2.655	-5.706	-4.067	-4.035
STUD $t_{\alpha,\hat{\theta}}^* \text{ e } t_{\alpha,\gamma}^*$	1	-4.625	-4.886	-3.895	-6.409	-5.533	-4.620
	2	-3.205	-3.037	-3.156	-3.012	-5.306	-2.471
	3	-3.111	-2.789	-2.645	-3.386	-4.066	-4.035
STUDABS $t_{\alpha, \hat{\theta} }^* \text{ e } t_{\alpha, \gamma }^*$	1	-4.625	-4.886	-3.895	-6.409	-5.533	-4.620
	2	-3.205	-3.037	-3.156	-3.012	-5.306	-2.471
	3	-3.111	-2.789	-2.645	-3.386	-4.066	-4.035