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# FEAR OF DISRUPTION: A MODEL OF MARKOV SWITCHING REGIMES FOR THE BRAZILIAN COUNTRY RISK CONDITIONAL VOLATILITY<sup>1</sup>

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## Abstract

In the literature, little role is attributed to the country risk conditional volatility in the determination of the macroeconomic equilibrium in a developing small open economy (DSOE). This paper posits the prime hypothesis that, in the presence of multiple equilibria and self-fulfilling prophecies, one of the reasons why investors prefer to speculate in a determined country's sovereign bonds, raising its country risk levels, is the switch of the expected macroeconomic fundamentals' conditional variance towards a higher regime. Non-linear GARCH models are applied to monitor different switching regimes of the Brazilian country risk conditional volatility, with special emphasis on Markov switching regimes. Results indicate that the high volatility regime periods, better identified by the latter, coincide with all the severe liquidity crisis episodes suffered by Brazil from May 1994 through September 2002. Thus, although not free of limitations, the country risk's high conditional volatility regime might determine a bad equilibrium and its monitoring might work as a practical tool to assess the duration of liquidity crises in a DSOE highly dependent on foreign capital inflows such as Brazil.

**Keywords:** Markov switching, non-linear GARCH, conditional volatility, country risk, multiple equilibria, self-fulfilling prophecies, liquidity crisis.

**JEL Codes:** C22, E44, F41, G15

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## 1. Introduction

The country risk literature has grown considerably over the last decade due to the massive rescheduling of emerging market foreign debts in the early nineties and the necessity of monitoring the creditworthiness degree of such economies, especially after the Asian crisis in 1997. Such literature is primarily concerned about the country risk role in the determination of the macroeconomic equilibrium in a developing small open economy (DSOE) which is highly dependent on foreign capital inflows. Examples of such vast literature abound: Ahumada and Garegnani (2000), Avila (2000), Kehoe and Perri (2000), Hamann (2001), Neumeyer and Perri (2001), Caballero and Krishnamurthy (2001, 2002), Céspedes, Chang and Velasco (2002), Aizenman and Turnovsky (2002), Uribe (2002), among others.

Country risk levels, as measured by the spread between bonds issued by sovereign governments and comparable bonds issued by the U.S. Treasury (assumed to be of *zero* risk), are correlated with macroeconomic fundamentals but cannot be fully explained only by them. Empirical work indicates that large part of country risk spreads are accounted for market sentiments or *global effects* (Larraín, Reisen and von Maltzan, 1997, Eichengreen and Mody, 1998, Mauro, Sussman and Yafeh, 2000, Kaminsky and Schmukler, 2001, Blass, Peled and Yafeh, 2002, Lazrak and Leroux, 2002, Fiess, 2003). The sudden deterioration of market feelings about the expected macroeconomic fundamentals of the debtor country, due to a coordination failure problem, causes drastic impacts on the perceived future wealth of such country (Avila, 2000), immediately raising the country risk premium, and complicating its debt repayment schedule. Since such DSOE is highly dependent on heavy capital inflows to clear balances, foreign investors demand an even higher risk premia to keep investments in such a country turning it into a vicious circle. This explosive self-fulfilling prophecy might lead to what is known as a *sudden stop* (Calvo, 1998) of capital inflows. When capital inflows interrupt, countries are forced to liquidate international reserves, reduce the current account deficit and consequently lower aggregate demand and social spending.

This takes place because of the high correlation that exists between country risk and currency risk (Garcia and Didier, 2000, Schmukler and Servén, 2001). Thus, in order to accommodate the country risk's high

sensitivity to market psychology, macroeconomic country risk models had to incorporate a mechanism typical of second generation currency crisis models: the multiple equilibria. In this kind of models, the result is determined by the market, being one out of the two (or even more) equilibria sustainable indefinitely. Two examples of such literature consist of Razin and Sadka (2002) and Agénor (1997). According to these authors, financial crisis in DSOEs occur when a country abruptly shifts from a "good" equilibrium with a low country risk premium and low public-debt service towards a "bad" equilibrium with sudden stops and capital reversals. Agénor (1997) extends the analysis by incorporating the notion of shock temporality. But none of these models explains why certain shocks are perceived to be temporary while others are permanent.

The answer to this question might be rooted in the process in which investors make their decision and form expectations and beliefs. Different empirical results obtained by Caballero (2001) and Aizenmann (2002) suggest that the uncertainty about a country's macroeconomic fundamentals plays a definite role in the determination of the emerging market macroeconomic equilibrium since they influence the foreign and domestic decision process of investment allocation. Consequently, countries that display greater macroeconomic volatility, mainly if politically induced, present greater probability of being potential targets of speculative attacks and chances are they are likelier to default (Catão and Sutton, 2002). Thus, the volatility of investors' expectations, or country risk variance, might indicate how sensitive investors are to expected fundamentals and foreign debt rollovers. If investors are too sensitive, certain shocks to country risk levels might be considered severe enough to trigger a sudden stop process and determine a new unique equilibrium instead of the multiple equilibria that existed before. In other words, second moments should no longer be relegated to second thoughts as it has been in the traditional country risk literature.

The remainder of this paper is structured as follows. Section 2 adapts a currency crisis game to the country risk literature where the uncertainty regime switch is one of the reasons to determine a new unique equilibrium and trigger a sudden stop process. It also mentions the empirical methodology that is best indicated to capture volatility switches. Section 3 describes the data analysed. Section 4 presents the main

results and section 5 concludes and offers some final remarks.

## 2. Theoretical Framework

Following Morris and Shin (1998), Sbracia and Zaghini (2000) present a currency crisis game that incorporates uncertainty about macroeconomic fundamentals as one of the sources of a speculative attack against the exchange peg. As a result of the known high correlation between currency and country risk, such game is adapted to explain how and why investors raise and reduce country risk spreads. The structure of this two-stage game goes as follows.

As a result of its foreign debt rescheduling, the DSOE government commits itself before creditors to implement a comprehensive structural reform programme. Such programme aims to improve the DSOE's macroeconomic fundamentals so as to honour future repayments to creditors. So, supposing there is a proportion  $l \in [0,1]$  of speculators on bonds issued by such economy which, in turn, is characterized by a space of states  $\theta$  of macroeconomic fundamentals. The price of such titles is an increasing function  $f(\theta)$  of the expected states  $\theta$ , which can take values over the interval  $[0,1]$ , where  $\theta = 1$  corresponds to a state of "sound fundamentals". In this game, there are two types of players: speculators and the DSOE government.

If investors doubt the speed of the reform programme advancement, the speculators attack: they hold a short position in a put option on the DSOE's bonds in a repurchase agreement and immediately pour them into the secondary market. This reduces the nominal price of the bonds and raises both the spreads and the country risk levels. If the reform advancements eventually prove to be slow, deteriorating fundamentals and bond prices, the speculative attack is said to be successful. The speculators' payoff in this case is  $d - f(\theta) - t$ , with  $d - f(\theta) > t$ , where  $d$  stands for the level of current bond prices;  $d - f(\theta)$  represents a continuous gross speculation gain function which is differentiable and increasing in  $\theta$ , and  $t$  stands for the transaction costs. If the attack is not successful, the speculators lose the transaction costs  $t$ . Nevertheless, if speculators are satisfied with the pace of the reforms, they refrain from attacking the DSOE and get 0.

The structural reform program advancements yields  $v(\theta) > 0$  to the government but it politically costs it  $c$ , due to the internal opposition. Such cost function  $c$  is continuous, differentiable and decreasing in the state  $\theta$  of fundamentals but increasing in the ratio of  $l$

speculators, i.e.  $\partial c(\theta, l) / \partial \theta < 0$  and  $\partial c(\theta, l) / \partial l > 0$ . Thus, whenever the speed of reform advancements is fast, the government receives  $v(\theta) - c(\theta, l)$ ; if the speed is not fast, its payoff is zero.

In order to spice this game, two further hypotheses might be assumed without further complications:  $\lim_{\theta \rightarrow 0} c(\theta, 0) > v(\theta)$  and  $\lim_{\theta \rightarrow 1} c(\theta, 1) > v(\theta)$ . The first hypothesis indicates that the cost of speeding up reforms in the worst state of fundamentals, even in the absence of speculators, is greater than the resulting gains for the government. This means that the internal political opposition is enough to deter reform advancements. The second hypothesis indicates that the cost of speeding up reforms when there is a large number of speculators, even in the best state of fundamentals, is greater than the resulting gains for the government. This might happen since speculation and volatility walk side by side, interfering in the per capita consumption smoothing and causing greater political pressure so as to revise the economic model of the country in practice.

Still, let  $\theta_1$  be the value of the state of fundamentals that solves  $v(\theta_1) - c(\theta_1, 0) = 0$ , i.e.,  $\theta_1$  is the value of  $\theta$  for which, in the absence of any speculators, the government is indifferent between speeding up reforms and slackening off (leaving the reforms to the following government or term). Whenever  $\theta < \theta_1$ , the government prefers not to speed up reforms, even if no speculator has poured the DSOE's sovereign bonds into the secondary market, since the gains do not compensate for the costs. Likewise, let  $\theta_2$  be the value of  $\theta$  that solves  $d(\theta_2) - t = 0$ . Whenever  $\theta_2 < \theta$ , speculators prefer not to attack since the speculative gain is less than the cost of transaction if the attack is unsuccessful.

### *The game with complete information*

In the first stage, speculators observe  $\theta$  and simultaneously decide whether to attack. In the second state, assuming that  $\theta_1 < \theta_2$ , the government which knows  $\theta$ , observes the share of speculators attacking the currency and decides whether to speed up the programme of structural reforms.

The game is solved backward, by finding the government's optimal strategy, represented by the following function:

$$\psi(\theta, l) = \begin{cases} \text{speeds\_up}, & v(\theta) > c(\theta, l) \\ \text{slackens\_off}, & v(\theta) \leq c(\theta, l). \end{cases}$$

According to the previous hypothesis, when  $l = 1$ , the government's optimal strategy function indicates that the government does not speed up its structural reform programme for any  $\theta$  in  $[0,1]$ . Likewise, the government speeds up reforms whenever  $\theta \in (\theta_1,1]$ . For a given  $\psi$ , the reduced form solution to the speculators' game provides the tripartition of the space of fundamentals which also characterises the second generation currency crisis models (Flood and Marion, 1996). Thus, if the expected state of fundamentals belongs to:

- $[0, \theta_1) \Rightarrow$  unique equilibrium: all agents speculate and the government cannot speed up reforms;
- $[\theta_1, \theta_2] \Rightarrow$  multiple equilibria: agents can either attack by holding short positions and pouring bonds into the secondary market so that the government faces difficulties trying to advance the reform program or not attack and let the government speed up reforms;
- $(\theta_2, 1] \Rightarrow$  unique equilibria: all agents refrain from attacking and the government speeds up reforms.

Hence, outside the interval  $[\theta_1, \theta_2]$ , speeding up reforms is only a function of  $\theta$ , whereas inside such interval, the result of the game depends on which self-fulfilling equilibrium speculators are coordinating. This interval is known as the "ripe for attack" zone. Here, if speculators believe structural reforms will not be speeded up, implying macroeconomic fundamentals will not improve and increasing the likelihood of the DSOE insolvency, they attack. They pour DSOE sovereign bonds into the secondary market, the country risk rises forcing interest rates up and causing recession, unemployment and capital flight. The opposition puts more pressure on the revision of the economic model, augmenting the political risk, and the government ends up not being able to speed up reforms turning at last into a self-fulfilling prophecy.

### ***The game with incomplete information***

However, in reality, speculators cannot really observe the true state  $\theta$  of macroeconomic fundamentals. Instead, they infer such state, forming expectations about them under the form of a probability distribution. Such distribution is continuous with total support over  $[0,1]$  and density function  $\eta^4$ . If  $\eta$  is common knowledge to speculators, such probability distribution represents the public information available to them, their set of beliefs.

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<sup>4</sup>  $\tilde{\Theta}$  denotes the random variable that represents the distribution of the state of fundamentals and takes value on  $[0,1]$  with density function  $\eta$ .

As before, as the government knows  $\theta$  and observes  $l$ , its optimal strategy is the same function  $\psi$  of the complete information setting. If  $\theta$  belongs to  $[0, \theta_1)$ , the government slackens off and if  $\theta$  belongs to  $[\theta_1, \theta_2]$ , it is the speculators expectations that will determine the outcome of the game.

For a given  $\psi$ , it is necessary to compute the payoff  $u(a_i, a_{-i})$  of a speculator that pours bonds when all other speculators also attack and the payoff  $u(a_i, d_{-i})$  of a speculator that attacks when all other speculators refrain from attacking. Analytically, the expected payoffs are:

$$u(a_i, a_{-i}) = \int_0^1 (d - f(\theta) - t) \eta(\theta) d\theta$$

$$u(a_i, d_{-i}) = \int_0^{\theta_1} (d - f(\theta) - t) \eta(\theta) d\theta - \int_{\theta_1}^1 t \eta(\theta) d\theta.$$

The payoff  $u(a_i, a_{-i})$  assumes such form following the initial hypotheses, since the government is not able to speed up reforms when all speculators attack, even at the best state of fundamentals. However, for the payoff  $u(a_i, d_{-i})$ , it is worth noting that in the interval  $[0, \theta_1)$ , the government is not able to speed up the reforms whereas in the interval  $[\theta_1, 1]$  it manages to do so. The equilibria of the reduced-form game is better specified by the following proposition:

**Proposition (Prati and Sbracia, 2002)** *The ("attack") strategy profile in which all investors pour bonds is an equilibrium if and only if  $u(a_i, a_{-i}) \geq 0$ . The ("don't attack") strategy profile in which all investors refrain from attacking is an equilibrium if and only if  $u(a_i, d_{-i}) \leq 0$ .*

If  $p$  denotes the probability that the state of fundamentals is not greater than  $\theta_1$ , it is possible to redefine the payoffs as follows:

$$u(a_i, a_{-i}) = d - E[f(\tilde{\Theta})] - t \geq 0$$

$$u(a_i, d_{-i}) = \left( d - E[f(\tilde{\Theta}) | \tilde{\Theta} \leq \theta_1] \right) p - t \leq 0.$$

The first inequality establishes that when a speculator expects all other investors to speculate together, she attacks while the net gain is positive. The second inequality establishes that when a speculator expects to attack alone, she attacks up to the point where the transaction costs are equal to the probability of gain while the expected state of fundamentals is less than  $\theta_1$ . Therefore,  $p$  represents the probability of a forced

reduction in the pace of reforms, since the government cannot speed up reforms even when there are no speculators.

The condition  $d - E[f(\tilde{\Theta}) | \tilde{\Theta} > \theta_1] \geq 0$  is always satisfied because  $d - f(\theta) \geq 0$  for all  $\theta$ . Thus,  $u(a_i, a_{-i}) \geq u(a_i, d_{-i})$  and the attack and the don't-attack strategy profiles, or both, are equilibria of this game. Since both  $u(a_i, a_{-i}) \geq 0$  and  $u(a_i, d_{-i}) \leq 0$  are valid for the same  $\eta$ , the following three situations are identified:

- if  $u(a_i, d_{-i}) > 0$  and  $u(a_i, a_{-i}) > 0$ , then, unique equilibrium: all speculators attack and the government is forced to slacken off;
- if  $u(a_i, a_{-i}) \geq 0$  and  $u(a_i, d_{-i}) \leq 0$ , then, multiple equilibria: investors might either pour bonds (and cause the government to slacken off) or refrain from attacking (allowing the government to speed up reforms, given  $\theta > \theta_1$ );
- if  $u(a_i, a_{-i}) < 0$  and  $u(a_i, d_{-i}) < 0$ , then, unique equilibrium: all speculators refrain from attacking and the government decides whether it speeds up reforms depending on either  $\theta \leq \theta_1$  or  $\theta > \theta_1$ .

Rewriting the investors' expected payoffs in the proposition, it is possible to obtain the necessary and sufficient condition to generate multiple equilibria in this game:

$$d \in \left[ E[f(\tilde{\Theta})] + t, E[f(\tilde{\Theta}) | \tilde{\Theta} \leq \theta_1] + t/p \right].$$

If the interval above is denoted by  $\Delta \equiv [d_1, d_2]$ , it is easy to verify that one necessary condition to generate multiple equilibria is when  $\Delta$  is nonempty, i.e.,  $d_1 \leq d_2$ . This condition is satisfied when:

$$p \leq \frac{t}{E[f(\tilde{\Theta})] + t - E[f(\tilde{\Theta}) | \tilde{\Theta} \leq \theta_1]} \equiv s,$$

where  $s \in (0,1)$ .

### Uncertainty revision

But, what is the exact role of uncertainty in this game? Why the revision of investors' expected risk aversion might immediately trigger a speculative attack even though the DSOE's expected macroeconomic fundamentals have not changed yet? These points are certainly intriguing and the answer to them is not trivial.

In general, it is observed that some specific episodes generate much noise amongst investors, and this generates disparate and multiple information about the soundness degree of a DSOE's expected fundamentals, concomitantly raising agents' risk aversion towards investing in such economy. If the previous observation is assumed to be fairly reasonable, the analysis formalisation might continue as follows:

Let  $E[f(\tilde{\Theta})] = \bar{d}$ ,  $E[f(\tilde{\Theta}) | \tilde{\Theta} \leq \theta_1] = \bar{d}_{\theta_1}$  and  $p$  and  $s$  be the computed parameters of a given probability density function  $\eta$  that generates multiple equilibria. Analogously, let  $\bar{d}'$ ,  $\bar{d}'_{\theta_1}$ ,  $p'$  and  $s'$  be the computed corresponding parameters of another density function  $\eta'$  that represents a new set of beliefs assumed by the investor when the revision of expectations takes place.

When investors' uncertainty increases, this is reflected in the augmentation of the distribution variance, consequently raising  $p$  so that investors assume a new probability density function  $\eta'$  over  $[0,1]$ , where  $p' > s > p$ <sup>5</sup>. The increase in  $p$  means that some probability has shifted towards the left tail of the density function making  $u(a_i, d_{-i}) \geq 0$ . Moreover, it is not hard to check that the upper limit of the interval  $\Delta$  has decreased. This causes agents to believe that a positive payoff is achievable, even if only one speculator attacks, producing a single equilibrium: the bad equilibrium. Thus, this paper's prime hypothesis is that a higher variance regime of the country risk, which represents the way investors perceive a DSOE's fundamentals, is one source of a bad equilibrium generation.

Sbracia and Zaghini (2000) also evaluate two other cases when only  $\bar{d}$  or  $\bar{d}_{\theta_1}$  varies, also producing bad equilibria, since they directly move the interval  $\Delta$ 's lower and upper limits. An extension to such analysis would be to evaluate how these different parameters behave together in order to detect if a parameter – and in the affirmative case, which parameter – is the most decisive in the determination of good and bad equilibria. This could empirically test President Lula's declaration upon winning the Brazilian 2002 national elections: "hope has beaten fear". According to the

<sup>5</sup> In order to better understand the issue, let us focus on the case where only  $p$  increases whereas  $\bar{d}$  and  $\bar{d}_{\theta_1}$  remain the same, i.e.,  $\bar{d}' = \bar{d}$  and  $\bar{d}'_{\theta_1} = \bar{d}_{\theta_1}$ , without damage to the analysis. According to Sbracia and Zaghini (2000), for many classes of probability density functions, when  $\bar{d}$  is constant, both a decrease in  $\bar{d}_{\theta_1}$  and an increase in  $p$  are associated with an increase in the distribution variance.

theoretical framework just analysed, such declaration would mean that despite the larger uncertainty amongst investors, reflected in a higher country risk volatility regime, the expected fundamentals' levels have produced a good equilibrium, lowering country risk levels. Conversely, fear might beat hope when a higher country risk volatility regime might lead to a bad equilibrium, notwithstanding preserving the mean of expected fundamentals.

### Methodology

A testable implication of the prime hypothesis is that one of the sources of bad equilibria is when country risk conditional volatility switches to a higher regime; conversely, one of the sources of good equilibria is when country risk conditional volatility switches to a lower regime. Thus, liquidity crises suffered by various DSOE's might be better monitored by means of regime switching conditional volatility models.

In order to capture different volatility regimes, various non-linear GARCH based models, with observable components and non-observable components, are applied on the Brazilian country risk daily data. Among the GARCH-based models with observable components are: AGARCH (Engle, 1990), TGARCH (Glosten et al., 1993) and EGARCH (Nelson, 1991). The GARCH-based models with non-observable components are the Markov Switching TGARCH (MS-TGARCH), or SWGARCH-L models, (Almeida and Valls Pereira, 1999). The original GARCH model (Bollerslev, 1986) is also estimated for illustrative purposes.

If  $\varepsilon_t$  are the residuals of a white noise process that follows

$$\varepsilon_t | \Omega_{t-1} \sim D(0, h_t),$$

where  $D(\cdot)$  is a zero mean heteroscedastic parametric conditional distribution conditioned on  $\Omega_{t-1}$ , the information set that consists of all relevant information up to and including time  $t-1$ , then, AGARCH (1,1) and TGARCH (1,1) models follow the next structure:

$$h_t = \alpha_0 + \alpha_1 (\varepsilon_{t-1} - \kappa_1)^2 + \kappa_2 (\varepsilon_{t-1} - \kappa_1)^2 I[\varepsilon_{t-1} \leq 0] + \beta_1 h_{t-1},$$

where  $I[\cdot]$  is an indicator function, not correlated with  $\varepsilon_t^2$  and  $E [ I [ \varepsilon_t \leq 0 ] ] = P ( \varepsilon_t \leq 0 ) = 0.5$ ;  $\kappa_1$  is the asymmetric parameter and  $\kappa_2$  is the threshold parameter. Either  $\kappa_1$  or  $\kappa_2$  can equal zero.

EGARCH (1,1) models follow the next structure:

$$\ln(h_t) = \alpha_0 + \alpha_1 \left( \varepsilon_{t-1} / \sqrt{h_{t-1}} \right) + \gamma_1 \left( \left| \varepsilon_{t-1} / \sqrt{h_{t-1}} \right| - E \left[ \left| \varepsilon_{t-1} / \sqrt{h_{t-1}} \right| \right] \right) + \beta_1 \ln(h_{t-1}),$$

where  $E \left[ \left| \varepsilon_{t-1} / \sqrt{h_{t-1}} \right| \right]$  depends on the error distribution assumed for  $\left( \varepsilon_{t-1} / \sqrt{h_{t-1}} \right)$ .

Finally, MS-TGARCH ( $k,1,1$ ) follows the next structure:

$$h_t = g_{s_t} \left\{ \alpha_0 + \alpha_1 \frac{\varepsilon_{t-1}^2}{g_{s_{t-1}}} + \kappa_2 \frac{\varepsilon_{t-1}^2}{g_{s_{t-1}}} I[\varepsilon_{t-1} \leq 0] + \frac{\beta_1 h_{t-1}}{g_{s_{t-1}}} \right\},$$

where  $\kappa_2$  is a threshold parameter,  $g_{s_{t-1}}$  is a multiplying factor of each unobservable conditional volatility state  $s_t$  which, in turn, follows a first-order Markov process with the following transition probability matrix:

$$P = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{k1} \\ p_{12} & p_{22} & \cdots & p_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1k} & p_{2k} & \cdots & p_{kk} \end{pmatrix},$$

where  $k$  determines the total number of regimes and  $p_{k-1k}$  determines the probability of switching from regime  $k-1$  to regime  $k$ .

The estimation procedure and further details can be found in the previously indicated literature.

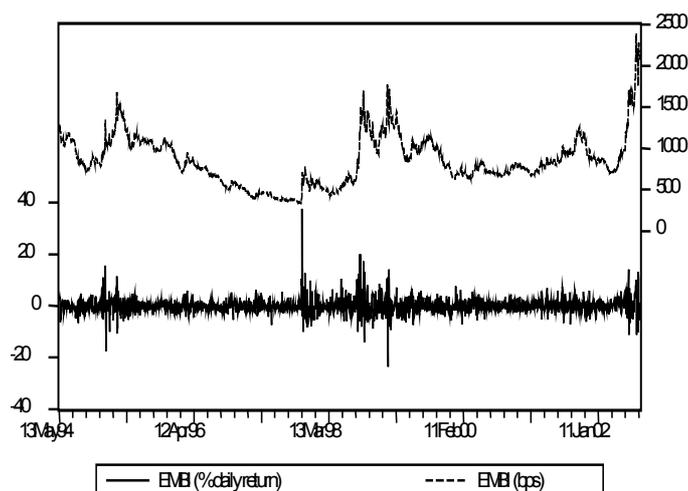
### 3. The Data

The data used in the estimation of the regime switching conditional volatility models are the daily percentage differences of the logarithmic values of the EMBI Global Brazil Sovereign Spread, computed in basis points (bps), and kindly ceded by J.P. Morgan-Chase Bank<sup>6</sup>. From the econometric viewpoint, such intuitive transformation is practical and efficient and it empirically coincides with the rough concept of an

<sup>6</sup> Other country risk proxies might be used (Garcia and Didier, 2000). The EMBI was chosen because of its unique computing methodology and widespread use among market and academic practitioners, being one of the variables used to form investors' expectations and beliefs.

asset return<sup>7</sup>. Although such transformation strictly denotes the speculator gain or rather the gain or return on a bundle of Brazilian sovereign bonds, the authors preferred to adopt here the use of the free concept of *return*.

The index sample period is comprised between 13 May 1994 and 27 September 2002, having the last 30 observations been left out for out-of-sample forecast exercises. Thus, conditional volatility models made use of a total of 2156 observations.<sup>8</sup> The original and transformed data can be seen in figure 1.



**Figure 1:** Daily observations in the level (dotted line) and daily percent log-returns (continuous line) of the EMBI Global Brazil Sovereign Spread, from 13 May 1994 until 16 August 2002.

The presence of outliers and volatility clusters strongly suggests that the transformed series present conditional heteroscedasticity, a common characteristic among high frequency finance and macroeconomic series. There are three big visual volatility clusters which denote periods of peaks in the original data in levels. The first cluster corresponds to the first three months of 1995 (Mexico crisis), the second starts at the end of October 1997 and ends in July 1999 (Asia, Russia, LTCM and Brazil crises) and the last one starts in May 2002 and lasts until the end of the series (Brazilian electoral crisis).

The summary statistics of the transformed series in table 1 confirm the pronounced kurtosis, much greater than the normal, as the distribution tails of this series are thicker than those of the Gaussian distribution. This reflects the fact that observations of great

<sup>7</sup> The return of an asset is computed this way, discounting the distribution of dividends. However, in applied econometric research this paper's return computation is widely used.

<sup>8</sup> To keep a uniform frequency of the data, Issler (1999) completes out the missing days, except for the weekends, with the last observation. Nevertheless, Valls Pereira (1999) warns against the consequences of such procedure.

magnitude occur more often than those expected of a normally distributed random variable.<sup>9</sup>

**Table 1** EMBI Global Brazil Sovereign Spread return summary descriptive statistics

Mean	0.02658
Minimum	-23.59369
Maximum	37.62355
Standard Deviation	3.03335
Skewness	1.38462
Kurtosis	20.58114
Jarque-Bera ( $p$ -value)	0.00000
Correlation ( $y_t^2, y_{t-1}$ )	0.13620
Total number T of observations	2156

Notes: EMBI Global Brazil Sovereign Spread return sample: 13 May 1994 through 16 August 2002.

Not only are the tails of the series distribution thicker than those of the normal distribution but the series also features positive skewness, meaning that the right tail is thicker than the left one. This might be due to the fact that great positive returns tend to occur more often than negative ones. The positive correlation between a one-period-lagged return and the squared return supports the fact that such great positive returns, in turn, may give rise to periods of great volatility. Both phenomena are also very common in some exchange markets (van Dijk and Franses, 2000). If, on the one hand, this is strong indication of non-linear conditional heteroscedasticity, on the other hand, it signals the absence of leverage effects. In other words, the asymmetry of the shocks that impact on the subsequent conditional volatility seems to be mostly explained by positive rather than negative lagged returns.

The low autocorrelation coefficients in table 2 demonstrate, in turn, that there is no obvious conditional predictability, without arbitrage opportunities for the average investor in Brazilian sovereign bonds. The same cannot be said of the squared returns, corroborating the visually supposed signs of the presence of conditional heteroscedasticity in the series.

<sup>9</sup> Like other Brazilian finance series, the greatest absolute return observed is on 27 October 1997 (see Almeida and Valls Pereira, 1999, Issler, 1999, and Valls Pereira et al., 1999), the Monday that follows the outbreak of the financial crisis in Hong Kong, started on the preceding Thursday. The least return is on 14 January 1999, one day after Gustavo Franco resigned from the presidency of the Banco Central do Brasil and one day before the Brazilian government floated the exchange for not having been successful in protecting the Brazilian *real* against speculative attacks.

**Table 2** Autocorrelation (partial autocorrelation)

EMBI Global Brazil Sovereign Spread			
Return		Return <sup>2</sup>	
$\rho_1$ ( $\mathbf{a}_1$ )	0.142 ( 0.142)	$\rho_1$ ( $\mathbf{a}_1$ )	0.143 ( 0.143)
$\rho_2$ ( $\mathbf{a}_2$ )	0.003 (-0.018)	$\rho_2$ ( $\mathbf{a}_2$ )	0.227 ( 0.211)
$\rho_3$ ( $\mathbf{a}_3$ )	-0.034 (-0.033)	$\rho_3$ ( $\mathbf{a}_3$ )	0.148 ( 0.099)
$\rho_4$ ( $\mathbf{a}_4$ )	0.000 ( 0.010)	$\rho_4$ ( $\mathbf{a}_4$ )	0.153 ( 0.087)
$\rho_5$ ( $\mathbf{a}_5$ )	0.004 ( 0.003)	$\rho_5$ ( $\mathbf{a}_5$ )	0.108 ( 0.038)

Notes: EMBI Global Brazil Sovereign Spread return sample: 13 May 1994 until 16 August 2002.

The presence of conditional heteroscedasticity was checked by Lagrange Multiplier tests, developed by Engle (1982) and the non-linear GARCH effects were checked by bias tests developed by Engle and Ng (1993). Such tests were performed on the residuals from an AR(1) model, with the order of such model determined by the AIC. The results displayed in table 3 confirm the presence of GARCH effects (LM tests) as well as shock asymmetry (bias tests). Comparing the  $p$ -values of the tests for non-linear effects, the shock asymmetry is very likely to be generated by the size of positive shocks rather than by the size of negative shocks.

**Table 3** Testing for (G)ARCH effects in daily EMBI Global Brazil Sovereign Spread log-returns

Tests	Test statistic	( $p$ -value)
LM test for ARCH (1) ( $p$ -value)	59.382	(0.0000) **
LM test for ARCH (5) ( $p$ -value)	42.050	(0.0000) **
LM test for ARCH (10) ( $p$ -value)	21.786	(0.0000) **
SB test	-2.918724	(0.0036)
NSB test	-2.001714	(0.0454)
PSB test	10.82087	(0.0000) **
General Asymmetry test	135.8539	(0.0000) **

Notes: (1) EMBI Global Brazil Sovereign Spread return sample: 13 May 1994 through 16 August 2002, (2) LM= Lagrange Multiplier, SB = Sign Bias, NSB = Negative Size Bias, PSB = Positive Sign Bias, (3) Tests applied to residuals from an AR( $k$ ), with  $k$  determined by the AIC.

## 4. Main Results

After having detected the presence of conditional heteroscedasticity in the series to be studied, various GARCH-based models that capture regime switches were then estimated with observable (AGARCH, TGARCH and EGARCH) and non-observable components (MS-TGARCH)<sup>10</sup>, assuming either a

<sup>10</sup> GARCH models with observable components were estimated using the GiveWin2 software's PC-Give 10.2 package. GARCH with non-observable components were estimated mixing codes by Nuno Miguel C.G. de Almeida and Raúl Susmel and the OPTMUM library with Gausswin 3.2 software.

Normal or a Student t error distribution<sup>11</sup>. Various specifications were estimated with  $k$  AR terms (with  $k = 1$  to 3),  $q$  ARCH components (with  $q = 1$  to 3) and  $p$  GARCH components (with  $p = 1$  to 5). Regarding the MS-TGARCH models, models with  $\kappa$  non-observable regimes were estimated, (with  $\kappa = 2$  and 3)<sup>12</sup>. All models were estimated by maximum likelihood and made use of the BFGS algorithm.

With the exception of the MS-TGARCH models, no other estimated GARCH specification managed to eliminate alone the autocorrelation of the residuals and of the squared residuals, even when assuming an alternative distribution to the Gaussian one. The residuals still displayed statistical properties very much similar to those described by the transformed series of study. Even the residuals of the GARCH (1,1) specification – the best specification according to the AIC and BIC – did not present white noise properties or absence of heteroscedasticity. This contradicts the empirical observation made by Bollerslev et al (1992) that in general such specification seems to suffice in practice. The quality of the fit and the diagnostic tests of such estimated models are shown in tables 4, 5, 6 and 7. The estimated parameters of such models are shown in tables 11 and 12.

The estimation of the Gaussian distributed MS-TGARCH models made use of more than 75 different vectors of starting values for the two-regime (78 different vectors) and three-regime (83 different vectors) specifications. Despite this, either the positive definite hessian was not obtained, due to its quasi-singularity, making it difficult to compute the standard deviation of the estimated parameters, or the estimated  $\theta_{ij}$  parameters caused the transition probabilities to approach the boundaries  $p_{ij} = 0$ , yielding very unstable smoothed probabilities. Therefore, total convergence was not reached with the normal distribution. Issler (1999a) faced the same difficulties when estimating Gaussian specifications of MS-ARCH models with three regimes for the cocoa spot price series. As a result, only Student t specifications of the MS-TGARCH models with two and three regimes were estimated. The quality of the fit and the diagnostic tests for these estimated models are shown in table 8. The estimated parameters can be found in table 13.

<sup>11</sup> In the case of the EGARCH specification, the alternative conditional distribution used was GED.

<sup>12</sup> A model with two states denotes a low volatility regime and a high volatility regime. A model with three regimes denotes a low volatility regime, a medium volatility regime and a high volatility regime.

**Table 4** *GARCH and EGARCH models for EMBI Global Brazil Sovereign Spread returns*

	GARCH (1,1)	GARCH (1,1)	GARCH (1,1)	GARCH (1,1)	EGARCH (1,1)	EGARCH (1,1)	EGARCH (1,1)	EGARCH (1,1)
Distribution	Gaussian	Student t	Gaussian	Student t	Gaussian	GED	Gaussian	GED
Exog. Var.	-	-	dummy	dummy	-	-	dummy	dummy
Parameters	5	6	6	7	6	7	7	8
LL	-4,890.5	-4,802.4	-4,859.1	-4,796.4	-4,874.6	-4,798.6	-4,866.5	-4,798.0
AIC	4.5413	4.4604	4.5131	4.4559	4.5274	4.4579	4.5208	4.4582
BIC	4.5545	4.4762	4.5289	4.4743	4.5432	4.4763	4.5393	4.4793
AIC.T	9791.0859	9616.7143	9730.2945	9606.8474	9761.1295	9611.1302	9746.9047	9611.9194
$\lambda$	0.96397	0.98544	0.97109	0.98594	0.92783	0.93824	0.94286	0.946352
Mean $h_t$	9.1139	9.4049	8.5762	9.0333	8.2890	8.4768	8.0133	8.1797
Var $h_t$	299.6630	304.3670	181.8980	214.8710	201.5900	215.4040	115.4250	130.0220
Res: Mean	0.12541	0.15279	0.11338	0.14033	0.03064	0.08377	0.05644	0.07432
Res: SD	1.92749	1.92789	1.91854	1.91602	1.92800	1.92752	1.91904	1.91792
Res: Sk	1.23659	1.21438	0.52270	0.71799	1.21113	1.23351	0.49997	0.54499
Res: Exc.K.	14.08231	13.98643	5.69164	7.40458	13.97226	14.06916	5.59497	5.72934
Res: Min	-23.93283	-23.96031	-23.97566	-23.94444	-24.09032	-23.98218	-24.04936	-23.96147
Res: Max	36.91450	36.84068	20.24902	26.53623	36.70403	36.85862	19.58574	20.39342

Notes: (1) The estimation sample covers 2156 observations, from 13 May 1994 to 16 August 2002, (2)  $\varepsilon_t = y_t - \phi_0 - y_{t-1}$ ,  $y_t = 100 * [\ln(EMBI)_t - \ln(EMBI)_{t-1}]$ ,  $E[y_t] = 0.026579$ ,  $\text{Var}[y_t] = 9.196970$ , (3) Exogenous variable in mean: vector of dummy variables for the 909th, 910th and 911th observations corresponding to 23, 24 and 27 October 1997, (4) LL = Log-Likelihood,  $\lambda$  = persistence (GARCH:  $\alpha_1 + \beta_1$ , EGARCH:  $\beta_1$ ),  $h_t$  = conditional variance, Res = residuals, SD = standard deviation, Sk = skewness, Exc.K. = excess kurtosis, Min = minimum, Max = maximum, (5)  $p$ -value of all  $\chi^2$  asymptotic and normality tests: 0.0000.

**Table 5** *TGARCH and AGARCH models for EMBI Global Brazil Sovereign Spread returns*

	TGARCH (1,1)	TGARCH (1,1)	TGARCH (1,1)	TGARCH (1,1)	AGARCH (1,1)	AGARCH (1,1)	AGARCH (1,1)	AGARCH (1,1)
Distribution	Gaussian	Student t						
Exog. Var.	-	-	dummy	dummy	-	-	dummy	dummy
Parameters	6	7	7	8	6	7	7	8
LL	-4,859.6	-4,789.2	-4,845.2	-4,784.4	-4,877.0	-4,795.3	-4,848.4	-4,790.0
AIC	4.5136	4.4492	4.5011	4.4456	4.5297	4.4548	4.5041	4.4508
BIC	4.5294	4.4676	4.5195	4.4667	4.5455	4.4733	4.5225	4.4719
AIC.T	9731.2426	9592.3965	9704.3048	9584.7765	9765.9894	9604.6301	9710.8264	9595.9987
$\lambda$	0.93971	0.96422	0.95423	0.96606	0.94079	0.96892	0.952474	0.98009
Mean $h_t$	9.2175	9.5328	8.6405	9.1126	8.6990	9.1372	8.3548	8.8230
Var $h_t$	421.0260	419.5530	208.5350	269.4070	250.3830	287.4148	168.5090	206.2640
Res: Mean	0.04986	0.10512	0.05566	0.09663	0.06923	0.11557	0.06345	0.10550
Res: SD	1.92773	1.92846	1.91894	1.91744	1.92752	1.92829	1.91867	1.91684
Res: Sk	1.22073	1.19914	0.49997	0.76995	1.23288	1.20364	0.51565	0.74150
Res: Exc.K.	14.01408	13.91947	5.58071	8.12427	14.06646	13.93932	5.65667	7.75685
Res: Min	-24.04774	-24.04458	-24.04075	-24.02325	-23.99830	-24.02342	-24.02858	-24.00446
Res: Max	36.76634	36.72549	19.47762	28.19959	36.84117	36.75570	19.98599	27.37541

Notes: (1) The estimation sample covers 2156 observations, from 13 May 1994 to 16 August 2002, (2)  $\varepsilon_t = y_t - \phi_0 - y_{t-1}$ ,  $y_t = 100 * [\ln(EMBI)_t - \ln(EMBI)_{t-1}]$ ,  $E[y_t] = 0.026579$ ,  $\text{Var}[y_t] = 9.196970$ , (3) Exogenous variable in mean: vector of dummy variables for the 909th, 910th and 911th observations corresponding to 23, 24 and 27 October 1997, (4) LL = Log-Likelihood,  $\lambda$  = persistence (TGARCH:  $\alpha_1 + \beta_1 + \kappa_2/2$ , AGARCH:  $\alpha_1 + \beta_1$ ),  $h_t$  = conditional variance, Res = residuals, SD = standard deviation, Sk = skewness, Exc.K. = excess kurtosis, Min = minimum, Max = maximum, (5)  $p$ -value of all  $\chi^2$  asymptotic and normality tests: 0.0000.

**Table 6** Diagnostic tests for estimated GARCH and EGARCH models for EMBI Global Brazil Sovereign Spread returns

	GARCH (1,1)	GARCH (1,1)	GARCH (1,1)	GARCH (1,1)	EGARCH (1,1)	EGARCH (1,1)	EGARCH (1,1)	EGARCH (1,1)
Distribution	Gaussian	Student t	Gaussian	Student t	Gaussian	GED	Gaussian	GED
Exog. Var.	-	-	dummy	dummy	-	-	dummy	dummy
Q (6)	0.0173 *	0.0318 *	0.2273	0.3505	0.0761	0.0396 *	0.3727	0.2059
Q (12)	0.0073 **	0.0126 *	0.2436	0.2643	0.0094 **	0.0072 **	0.3433	0.2320
Q (24)	0.0132 *	0.0212 *	0.1732	0.2908	0.0101 *	0.0090 **	0.2221	0.1794
Q (36)	0.0190 *	0.0339 *	0.2292	0.3991	0.0085 **	0.0100 *	0.2368	0.2203
Q (48)	0.0491 *	0.0800	0.1807	0.4225	0.0296 *	0.0331 *	0.2051	0.1938
Q <sup>2</sup> (6)	0.0000 **	0.0000 **	0.3255	0.9005	0.0000 **	0.0000 **	0.0002 **	0.0000 **
Q <sup>2</sup> (12)	0.0000 **	0.0000 **	0.5690	0.9883	0.0004 **	0.0001 **	0.0073 **	0.0002 **
Q <sup>2</sup> (24)	0.0000 **	0.0000 **	0.7021	1.0000	0.0136 *	0.0071 **	0.0519	0.0060 **
Q <sup>2</sup> (36)	0.0000 **	0.0000 **	0.9615	1.0000	0.1494	0.1042	0.3713	0.1136
Q <sup>2</sup> (48)	0.0000 **	0.0000 **	0.9419	1.0000	0.4402	0.3756	0.6510	0.3617
LM (2)	0.0000 **	0.0000 **	0.2578	0.6106	0.0000 **	0.0000 **	0.0001 **	0.0000 **
LM (3)	0.0000 **	0.0000 **	0.2913	0.8045	0.0000 **	0.0000 **	0.0003 **	0.0000 **
LM (4)	0.0000 **	0.0000 **	0.3658	0.9120	0.0000 **	0.0000 **	0.0006 **	0.0000 **
LM (5)	0.0000 **	0.0000 **	0.4993	0.9605	0.0000 **	0.0000 **	0.0015 **	0.0000 **
LM (6)	0.0000 **	0.0000 **	0.6049	0.9830	0.0000 **	0.0000 **	0.0030 **	0.0001 **

Notes: (1) The estimation sample covers 2156 observations, from 13 May 1994 to 16 August 2002, (2)  $\varepsilon_t = y_t - \phi_0 - y_{t-1}$ ,  $y_t = 100 * [\ln(EMBI)_t - \ln(EMBI)_{t-1}]$ ,  $E[y_t] = 0.026579$ ,  $\text{Var}[y_t] = 9.196970$ , (3) Exogenous variable in mean: vector of dummy variables for the 909th, 910th and 911th observations corresponding to 23, 24 and 27 October 1997, (4)  $Q(x) = p$ -value of residuals autocorrelation test with  $x$  lags,  $Q^2(x) = p$ -value of squared residuals autocorrelation test with  $x$  lags,  $LM(x) = p$ -value of Lagrange multiplier test with  $x$  lags, with \* 1% and \*\* 5% significance.

**Table 7** Diagnostic tests for estimated TGARCH and AGARCH models for EMBI Global Brazil Sovereign Spread returns

	TGARCH (1,1)	TGARCH (1,1)	TGARCH (1,1)	TGARCH (1,1)	AGARCH (1,1)	AGARCH (1,1)	AGARCH (1,1)	AGARCH (1,1)
Distribution	Gaussian	Student t						
Exog. Var.	-	-	dummy	dummy	-	-	dummy	dummy
Q (6)	0.0559	0.1055	0.2825	0.2966	0.0293 *	0.0780	0.3087	0.3913
Q (12)	0.0168 *	0.0393 *	0.3188	0.2490	0.0082 **	0.0257 *	0.3153	0.3077
Q (24)	0.0243 *	0.0468 *	0.2214	0.2950	0.0131 *	0.0327 *	0.2091	0.3356
Q (36)	0.0184 *	0.0440 *	0.2246	0.2976	0.0136 *	0.0384 *	0.2170	0.3823
Q (48)	0.0385 *	0.0835	0.1600	0.3450	0.0322 *	0.0785	0.1575	0.4242
Q <sup>2</sup> (6)	0.0000 **	0.0000 **	0.1276	0.9984	0.0000 **	0.0000 **	0.2148	0.9992
Q <sup>2</sup> (12)	0.0000 **	0.0000 **	0.4132	0.9999	0.0000 **	0.0000 **	0.5674	0.9998
Q <sup>2</sup> (24)	0.0031 **	0.0000 **	0.6615	1.0000	0.0000 **	0.0000 **	0.7164	1.0000
Q <sup>2</sup> (36)	0.0520	0.0026 **	0.9536	1.0000	0.0000 **	0.0000 **	0.9694	1.0000
Q <sup>2</sup> (48)	0.1640	0.0238 *	0.9515	1.0000	0.0002 **	0.0000 **	0.9624	1.0000
LM (2)	0.0000 **	0.0000 **	0.0450 *	0.9811	0.0000 **	0.0000 **	0.1135	0.9849
LM (3)	0.0000 **	0.0000 **	0.0934	0.9973	0.0000 **	0.0000 **	0.1656	0.9983
LM (4)	0.0000 **	0.0000 **	0.1640	0.9997	0.0000 **	0.0000 **	0.2616	0.9998
LM (5)	0.0000 **	0.0000 **	0.2530	0.9999	0.0000 **	0.0000 **	0.3693	1.0000
LM (6)	0.0000 **	0.0000 **	0.3181	1.0000	0.0000 **	0.0000 **	0.4660	1.0000

Notes: (1) The estimation sample covers 2156 observations, from 13 May 1994 to 16 August 2002, (2)  $\varepsilon_t = y_t - \phi_0 - y_{t-1}$ ,  $y_t = 100 * [\ln(EMBI)_t - \ln(EMBI)_{t-1}]$ ,  $E[y_t] = 0.026579$ ,  $\text{Var}[y_t] = 9.196970$ , (3) Exogenous variable in mean: vector of dummy variables for the 909th, 910th and 911th observations corresponding to 23, 24 and 27 October 1997, (4)  $Q(x) = p$ -value of residuals autocorrelation test with  $x$  lags,  $Q^2(x) = p$ -value of squared residuals autocorrelation test with  $x$  lags,  $LM(x) = p$ -value of Lagrange multiplier test with  $x$  lags, with \* 1% and \*\* 5% significance.

**Table 8** *MS-TGARCH and MS-GARCH models for EMBI Global Brazil Sovereign Spread returns*

	MS-TGARCH (2,1,1)	MS-GARCH (3,1,1)	MS-GARCH (2,1,1)
Distribution	Student t	Student t	Student t
Exog. Var.	-	-	dummy
Parameters	10	13	10
LL	-4,798.9	-4,798.3	-4,793.6
AIC	4.4630	4.4653	4.4581
BIC	4.4894	4.4995	4.4844
AIC.T	9622.3343	9627.1128	9611.6599
$\lambda$	0.9800825	0.93931178	0.89956726
Mean $h_t$	12.4881	11.1732	9.0997
Var $h_t$	12282.7997	1671.2473	240.8415
Res: Mean	0.05342	0.05265	0.05631
Res: SD	0.99608	0.96799	1.02541
Res: Sk	0.59194	0.60818	1.44130
Res: Exc.K.	3.24344	3.12269	15.53617
Res: Min	-23.95844	-23.94776	-23.94100
Res: Max	36.84365	36.86448	26.41661
Q (6)	0.2647	0.1716	0.5647
Q (12)	0.0880	0.0825	0.5079
Q (24)	0.0971	0.1071	0.5440
Q (36)	0.1044	0.1237	0.6050
Q (48)	0.1364	0.1530	0.5311
Q <sup>2</sup> (6)	0.8776	0.0936	0.9916
Q <sup>2</sup> (12)	0.8882	0.2480	0.9919
Q <sup>2</sup> (24)	0.9738	0.6816	1.0000
Q <sup>2</sup> (36)	0.9988	0.9401	1.0000
Q <sup>2</sup> (48)	0.9978	0.9472	1.0000

Notes: (1) The estimation sample covers 2156 observations, from 13 May 1994 to 16 August 2002, (2)  $\varepsilon_t = y_t - \phi_0 - y_{t-1}$ ,  $y_t = 100 * [\ln(EMBI)_t - \ln(EMBI)_{t-1}]$ ,  $E[y_t] = 0.026579$ ,  $\text{Var}[y_t] = 9.196970$ , (3) Exogenous variable in mean: vector of dummy variables for the 909th, 910th and 911th observations corresponding to 23, 24 and 27 October 1997, (4) LL = Log-likelihood,  $\lambda$  = persistence (MS-TGARCH:  $\alpha_1 + \beta_1 + \kappa_1/2$ , MS-GARCH:  $\alpha_1 + \beta_1$ ),  $h_t$  = conditional variance, Res = residuals, SD = standard deviation, Sk = skewness, Exc.K. = excess kurtosis, Min = minimum, Max = maximum, (5)  $p$ -value of all  $\chi^2$  asymptotic and normality tests ( $\chi^2$ ): 0.0000. (6)  $Q(x)$  =  $p$ -value of autocorrelation test for residuals with  $x$  lags,  $Q^2(x)$  =  $p$ -value of autocorrelation test for squared residuals with  $x$  lags, LM ( $x$ ) =  $p$ -value of Lagrange multiplier test with  $x$  lags, with \* 1% and \*\* 5% significance.

According to the best MS-TGARCH (2,1,1) estimated, as the threshold parameter *à-la*-GJR  $\kappa_2$  is not significant, there really is no leverage effect as already indicated by the previously described stylised facts<sup>13</sup>. Besides, the estimated parameter  $g_2$  indicates that the second volatility regime is almost 33 times greater than the first. The estimated transition probability of remaining in the second regime,  $\hat{p}_{22}$ , is 0.6123, which is relatively very low in comparison to other studies that make use of this algorithm (Hamilton, 1989, Hamilton and Susmel, 1994, Susmel, 1999, Valls

Pereira et al, 1999, among others). This means that this second regime is very temporary and short-lived. However, since  $g_2$ , and the estimated transition probability coefficients  $\hat{\theta}_{11}$  and  $\hat{\theta}_{22}$  are not significant, such conclusions cannot be drawn, implying that the model does not seem to be well specified.

By analysing the smoothed probabilities estimated by such model (Figure 2)<sup>14</sup>, one can conclude that the source of the bad specification seems to be the domain exercised by some extreme observations over the second regime. Such observations refer primarily to the daily percent log-returns on the EMBI Global Brazil Sovereign Spread that occurred on 23, 24 and 27

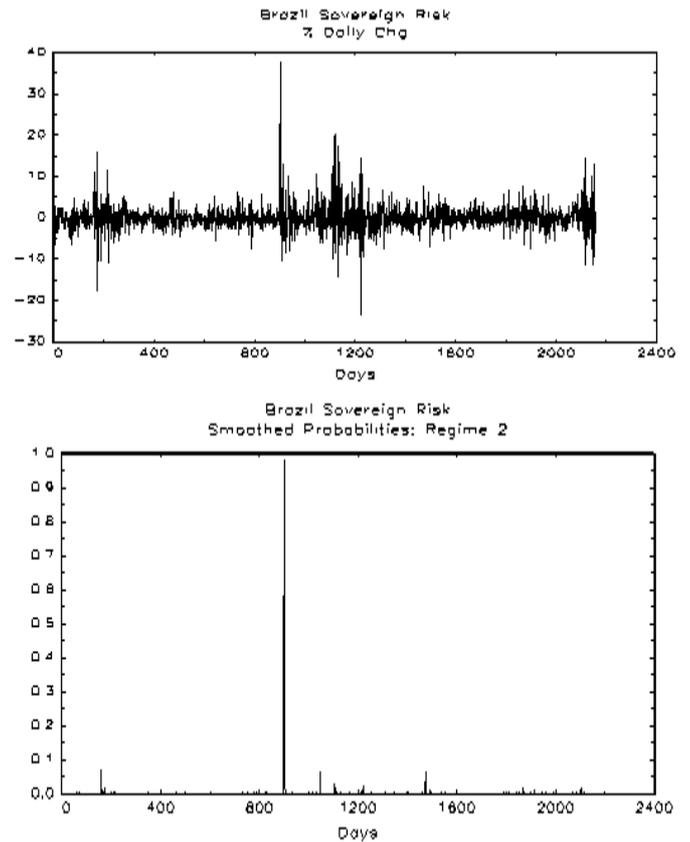
<sup>13</sup> The code used by the authors only detects the asymmetry that is due to negative shocks, replicating the leverage effect by Glosten et al. (1993). Minor modifications that account for positive shocks might also be introduced but were not attempted by the authors.

<sup>14</sup> The smoothed probability of the first regime is simply the “mirror” image of the lower panel.

October 1997, when the index not only suddenly reversed a long-term downward trend but also immediately skyrocketed with the outbreak of the Asian crisis<sup>15</sup>.

The occurrence of these extreme values produces so strong a bias in the volatility patterns that it compromised the power of fitting tests of the MS-TGARCH models. The intuition behind the impact of the dominance of these extreme values is that the iteration algorithm filter infers a high volatility regime only when extreme values occur. And this took place only three times over more than 2100 observations in the sample. That is, all other medium volatility clusters observed before were condensed back into the low volatility regime, mixing tranquil and turbulent periods altogether. Therefore, it would be necessary to estimate another MS-TGARCH model with more regimes that could better separate different volatility patterns, obtaining a more consistent inference and a better specification.

Next, some more specifications of an MS-TGARCH with three regimes were estimated in order to infer low, medium and high volatility regimes with a better fit. Among the models that converged and eliminated the autocorrelation present among the residuals and the squared residuals, the MS-TGARCH (3,1,1) was the chosen model according to the log-likelihood, the AIC and the BIC. Once again, the coefficient  $\kappa_2$  kept on being not significant and, as a matter of fact, the re-estimation of such model without it practically did not alter much the other estimated parameters. Hence, by the parsimony criteria, in this paper, we opted to report the model MS-GARCH (3,1,1) instead of the MS-TGARCH (3,1,1). Moreover, as the estimated transition probability that state 2 is preceded by state 3,  $\hat{p}_{32}$ , is very low, it would also be recommended to impose a restriction on this parameter, i.e.,  $\hat{p}_{32} = 0$ , (Hamilton and Susmel, 1994). The quality of the fit and the estimated parameters of this model can be found in tables 8 and 13.



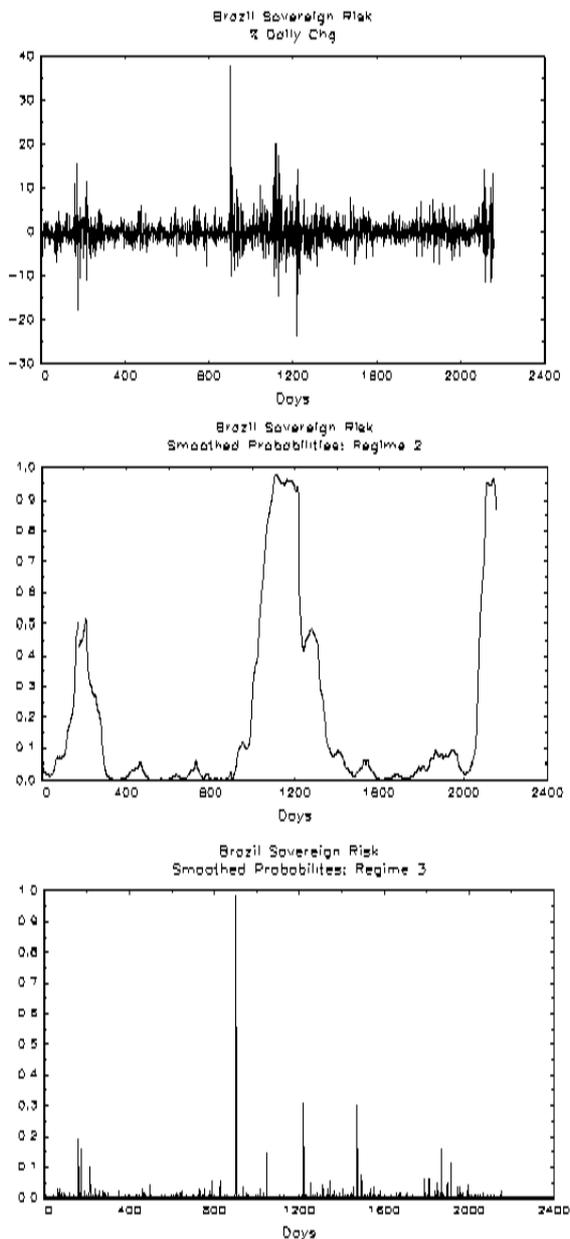
**Figure 2:** *Upper panel:* EMBI Global Brazil Sovereign Spread returns from 13 May 1994 until 16 August 2002, totalling 2156 observations. *Lower panel:* Student  $t$  distributed MS-TGARCH (2,1,1) smoothed probabilities [ $\text{Prob}(s_t=2|y_T, y_{T-1}, \dots, y_r)$ ] of being in regime 2.

The estimation of the MS-GARCH (3,1,1) model with one transition probability restriction managed the correct inference of the low and medium volatility regimes that were condensed in the previous estimated specifications, isolating the effects of the October 1997 observations in the third regime, as figure 3<sup>16</sup> shows. The parameter  $g_2$  is significant as well as the estimated probabilities of remaining in the first and in the second regime. In addition, the smoothed probabilities managed to mimic the visual volatility clusters described by the stylised facts.

However, despite these good partial results, such estimated specification surprisingly also indicates that neither some parameters  $\theta_{ij}$  of the transition probabilities nor the parameter  $g_3$  – the multiplying factor of the third volatility regime – are significant. Also, the estimated probability of remaining in the third state,  $\hat{p}_{33}$ , is very low indicating once more a poor specification probably due to overparameterisation.

<sup>15</sup> In order to deter the speculative attack against the Hong Kong dollar, on 23 October 1997, the monetary authority of the former British colony raises the annual interbank rates from 7% to 150%, causing geographically close stock markets to plummet and dragging down the whole world with them. The black Thursday could still be felt on the following Monday, 27 October 1997, when the international financial market witnesses another generalized day of panic, reminding the black Monday of the October 1987 crash of the New York stock exchange.

<sup>16</sup> The smoothed probability of being in a low volatility regime, not displayed, is only the difference between 1 and [ $P(s_t=2)+P(s_t=3)$ ].



**Figure 3:** *Upper panel:* EMBI Global Brazil Sovereign Spread returns from 13 May 1994 until 16 August 2002, totalling 2156 observations. *Central panel:* Student  $t$  distribution MS-GARCH (3,1,1) smoothed probability of being in state 2 for each given day [ $\text{Prob}(s_t=2|y_T, y_{T-1}, \dots, y_{t-1})$ ]. *Lower panel:* Smoothed probability for the third regime.

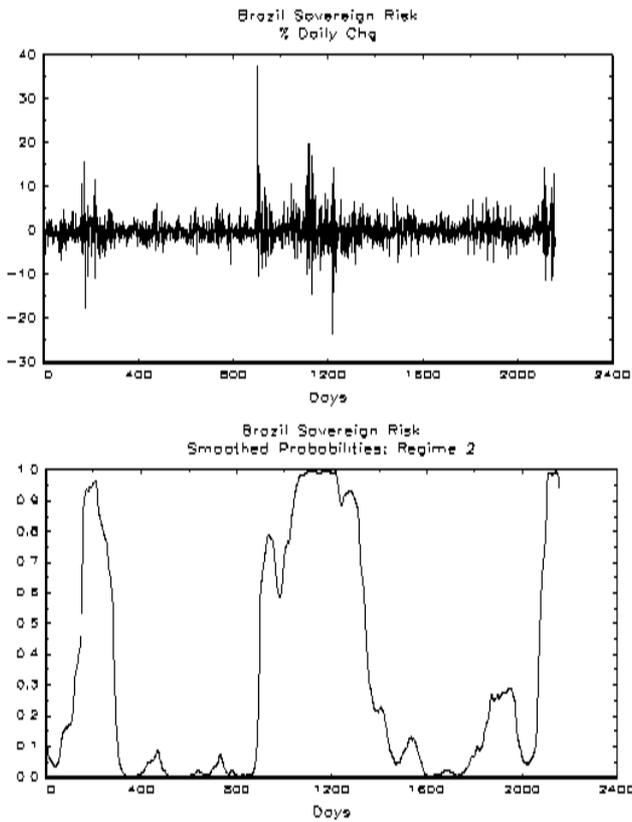
One way to identify the improvement produced by the addition of the third regime would be by performing a likelihood ratio test. Notwithstanding, although an MS-TGARCH (2,1,1) is a special case of an MS-TGARCH (3,1,1) under the null hypothesis of an MS-TGARCH (2,1,1), the parameters of an MS-TGARCH (3,1,1) are not identified. This implies that in this case the  $\chi^2$  asymptotic distribution is not valid for the likelihood ratio test (Hamilton and Susmel, 1994). But yet, for

illustrative purposes, the comparison between the likelihood values of the MS-TGARCH (2,1,1) and MS-GARCH (3,1,1) models denote only a slight improvement, therefore not justifying then the inclusion of a third regime. Also, the information criteria became too sensitive to the inclusion of so many parameters, rejecting the estimation of a third regime. Moreover, when Hamilton and Susmel (1994) estimated a fourth regime, they evaluated that only two observations were detected by the latter and concluded that the fourth regime would not be necessary, classifying the model with three regimes as the one that optimises the number of regimes. Since the third regime of the Brazilian country risk conditional volatility is strongly dominated by the October 1997 observations, one would conclude then that the third regime is unnecessary.

Nevertheless, the elimination of the third regime would bring us back to the estimation of the MS-TGARCH (2,1,1) model and such solution has already proved unreasonable for being poorly specified. The conclusion is that the MS-GARCH (3,1,1) formulation denotes a third volatility regime that is nothing but a special state for aberrant observations, implying that the third regime acts like an intervention variable. In these cases, as the third state seems to play the role of an intervention variable, or an additive dummy variable in the conditional mean, it is recommended that the third regime should be replaced with a vector of intervention constants in the mean that take on value one in the corresponding observations to the three impact ones: 23, 24 and 27 October 1997. According to Susmel (1999), this is a pulse variable that affects only the observations of these specific dates. This implies the estimation of an MS-GARCH (2,1,1)<sup>17</sup> with interventions.

Finally, the estimation of the MS-GARCH (2,1,1) model with interventions also obtains residuals with white noise properties and eliminates conditional heteroscedasticity, apart from considerably improving the likelihood function and the information criteria, even compared to the MS-TGARCH (2,1,1) without interventions. All parameters are significant and very much similar to the relevant parameters of the low and medium volatility regimes of the MS-GARCH (3,1,1) model. The model seems to be finally well fitted and the smoothed probabilities manage to detect the three volatility clusters described by the stylised facts, as figure 4 shows.

<sup>17</sup> Susmel (1999) makes use of the same resource in the case of the United Kingdom and Australia stock exchanges where the third regime of an MS-ARCH(3,  $q$ ) model is completely dominated by the third week of October 1997.



**Figure 4:** *Upper panel:* EMBI Global Brazil Sovereign Spread returns from 13 May 1994 until 16 August 2002, totalling 2156 observations. *Lower panel:* Student t distributed MS-GARCH (2,1,1) smoothed probability [ $\text{Prob}(s_T=2|y_T, y_{T-1}, \dots, y_1)$ ] of being in regime 2.

Hence, according to the MS-GARCH (2,1,1) estimated with interventions<sup>18</sup>, the best Markov switching conditional volatility model regarding the information criteria, there really is no leverage effect and the second regime conditional volatility is 30.6% greater than the volatility of the first regime, as  $\sqrt{g_2} = 1.7060077 = 1.306142297$ . The transition probabilities of each state are highly persistent but are not absorbent. The low volatility state lasts an average period of  $(1 - \hat{p}_{11})^{-1} \cong 311$  days, hence much longer than the  $(1 - \hat{p}_{22})^{-1} \cong 161$  day period which is the expected average of days for remaining in the second regime. The estimated persistence, computed following Almeida and Valls Pereira (1999), is 0.89956726, which means that, after 30 days, the volatility effect practically disappears since  $\hat{\lambda}^{30} \cong 0.04$ .

It is also interesting to note that the higher volatility regime periods coincide with the visual clusters described by the stylised facts. According to the

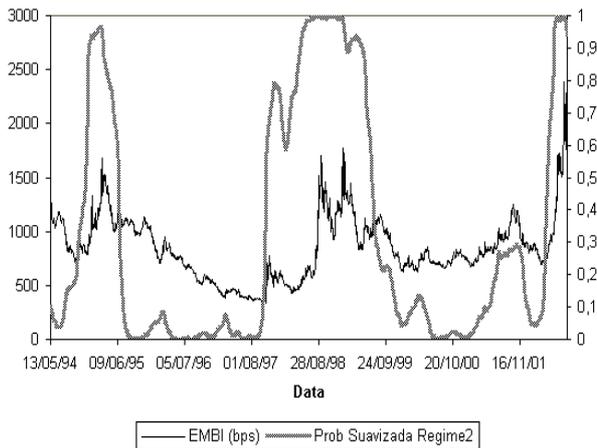
estimated model, the periods with greater probability of being in a higher volatility regime (considering the days when smoothed probability of being in the second regime was arbitrarily assumed to be equal to or greater than 0.5) are: between 7 December 1994 and 14 June 1995 (136 days), between 24 October 1997 and 2 July 1999 (441 days) and between 24 April 2002 and 16 August 2002 (83 days), when the series used in the estimation is interrupted. Yet, although the probability of being in the higher volatility regime augments progressively since 8 May 2001, it is interrupted on 29 October 2001. All these higher volatility periods mimic the long periods of crisis in the Brazilian sovereign risk perception, producing local maxima in the country risk (see figure 5): Tequila effect in the first period, Asia, Russia, LTCM and Brazil crises in the second and the Brazilian electoral process in the third. The estimated results also suggest that the Turkish and the Argentinean crises in 2001 did not really provoke a country risk volatility regime switch, since the Brazilian country risk volatility did not increase substantially in the period, although Brazilian country risk levels remained considerably high.

The introduction of interventions in the MS-GARCH models also stimulated the introduction of the same interventions in the GARCH models with observable components previously estimated. As a matter of fact, the intervention made in the three aberrant observations (corresponding to 23, 24 and 27 October 1997) finally allows one to obtain residuals with white noise properties and the elimination of heteroscedasticity of the residuals<sup>19</sup>.

Generally speaking, the presence of shock asymmetry could also be proved since asymmetry parameters in the AGARCH, TGARCH and EGARCH models with interventions were significant. But, the leverage effect was again rejected, since such parameters had negative sign. In other words, results suggest that positive shocks really provoke the subsequent volatility regime switch from a lower volatility regime to a higher one.

<sup>18</sup> An MS-TGARCH (2,1,1) with interventions was also estimated. However, the asymmetry parameter was not significant and the other parameters were almost identical to the ones of the MS-GARCH (2,1,1) model with interventions.

<sup>19</sup> Both the Gaussian distributed and the GED distributed specifications of the EGARCH model have the homoscedastic hypothesis rejected by the LM test but not by the LB test for the squared residuals.



**Figure 5** EMBI Global Brazil Sovereign Spread observations in basis points, or “EMBI (*bps*)”, (darker thin line) covering the period comprised between 13 May 1994 and 16 August 2002, totalling 2156 observations and smoothed probability of being in regime 2, or “Prob Suavizada Regime2”, (lighter thick line) [ $\text{Prob}(s_t=2|y_T, y_{T-1}, \dots, y_r)$ ] as computed in the Student t distributed MS-GARCH (2,1,1) specification with interventions.

The persistence of GARCH models with observable components and interventions is far higher than in the MS-GARCH models, corroborating the empirical results obtained by Hamilton and Susmel (1994), Susmel (1999), Issler (1999a) and Valls Pereira et al. (1999), using MS-ARCH models, and Almeida and Valls Pereira (1999) using MS-GARCH models. The persistence value of GARCH models with observable components and interventions varies from 0.938242, as in the case of the GED distributed EGARCH (1,1) with interventions<sup>20</sup>, up to 0.985935, as in the Student t distributed GARCH (1,1) with interventions. This implies that, after 30 days, the persistence value of volatility effects in a Student t distributed GARCH (1,1) with interventions is 0.147722314 and in a GED distributed EGARCH (1,1) with interventions is 0.354963503. This means that the persistence of GARCH models with observable components and interventions is 3.5 to 8.5 times greater than the persistence of the MS-GARCH (2,1,1) with interventions.

### **Comparing the results obtained through conditional volatility switching regime models**

The comparison of the results obtained through the different models specified was done by means of loss functions computations of in-sample and out-of-sample forecast. The in-sample forecast exercise is simply the estimation of the model with the 2156 observations comprised between 13 May 1994 and 16 August 2002. The out-of-sample forecast consists of

<sup>20</sup> The coefficient  $\beta_1$  is generally used as a persistence proxy of the EGARCH models.

keeping the parameters fixed and computing the 30 steps following the last observation used in the estimation process. This means that the forecast observations are comprised between 19 August 2002 through 27 September 2002, with the conditional volatility parameters observed until 16 August 2002. The loss function computation with the out-of-sample forecast is a good exercise since it allows testing the different models during situations of stress, the months prior to the Brazilian presidential elections in 2002, where the Brazilian country risk oscillated considerably. The results of the computed loss functions of in-sample and out-of-sample forecast can be found in tables 9 and 10 respectively.

In general, the interventions were vital to obtain better in-sample and out-of-sample forecasts. Almost all models that suffered interventions substantially improved the computed statistics for in and out of sample loss function, testifying the effects of the Asian crisis on the Brazilian country risk.

Among the in-sample forecast exercises, the best models were the Gaussian EGARCH (1,1) with interventions (lowest MSE and MAE) and the Student t distributed AGARCH (1,1) with interventions (lowest  $[\text{LE}]^2$  and  $|\text{LE}|$ ). The greater success these models present is due to the fact that they capture better the positive shock asymmetry effects over the conditional subsequent conditional volatility. The Gaussian TGARCH with interventions also had a good performance according to the HMSE.

Among the out-of-sample forecast exercises, the best models were the Student t distributed MS-GARCH (2,1,1) with interventions (lowest MAE and HMSE) and the Gaussian AGARCH (1,1) (lowest MSE and  $|\text{LE}|$ ). The Gaussian EGARCH (1,1) also performs very well, due to the lowest  $[\text{LE}]^2$  presented.

It should be noted that this last 30 day period is of great importance because the approaching Brazilian presidential elections at the end of October 2002 brings a sudden increase in the Brazilian country risk levels and volatility. As the MS-GARCH (2,1,1) persistence is lower than the other estimated models, it manages to adapt faster in times of great regime switching. In general, the forecast benefits of Markov switching conditional volatility models, compared to Student t distributed GARCH models, are at best marginal (Susmel, 1999). However, such conclusion might depend on the regime that is in force at the time of the estimation, whether in the middle of a turmoil period, just like the EMBI Global Brazil Sovereign Spread, or in the middle of a bonanza.

Finally, the estimated model parameters are displayed in tables 11, 12 and 13.

**Table 9** Comparison of in-sample forecast loss functions for different GARCH models for EMBI Global Brazil Sovereign Spread

Model	Loss Function (percentage improvement over constant variance)				
	MSE	MAE	[LE] <sup>2</sup>	LE	HMSE
Constant Variance	1,656.2545	11.8231	12.4132	2.5914	1.2849
Gaussian GARCH (1,1)	1,476.9732 (12.14)	9.8809 (19.66)	8.6582 (43.37)	2.0809 (24.54)	5.5260 (-76.75)
Student t GARCH (1,1)	1,470.6510 (12.62)	10.0337 (17.83)	8.4507 (46.89)	2.0576 (25.95)	6.2086 (-79.30)
Gaussian GARCH (1,1) dummy	667.0328 (148.30)	9.1197 (29.64)	8.1336 (52.62)	2.0378 (27.17)	3.9462 (-67.44)
Student t GARCH (1,1) dummy	829.4671 (99.68)	9.4701 (24.85)	8.3817 (48.10)	2.0547 (26.12)	20.5422 (-93.74)
Gaussian EGARCH (1,1)	1,380.2870 (19.99)	9.1985 (28.53)	8.9600 (38.54)	2.0945 (23.73)	5.4806 (-76.55)
GED EGARCH (1,1)	1,400.7268 (18.24)	9.3198 (26.86)	8.7292 (42.20)	2.0778 (24.72)	6.1053 (-78.95)
Gaussian EGARCH (1,1) dummy	* 635.2675 (160.72)	* 8.7145 (35.67)	8.3082 (49.61)	2.0563 (26.02)	4.2821 (-69.99)
GED EGARCH (1,1) dummy	646.9076 (156.03)	8.8052 (34.27)	8.5953 (44.42)	2.0720 (25.07)	4.5989 (-72.06)
Gaussian TGARCH (1,1)	1,496.3716 (10.68)	9.8248 (20.34)	8.9113 (39.30)	2.0833 (24.39)	4.8262 (-73.38)
Student t TGARCH (1,1)	1,489.5558 (11.19)	9.9991 (18.24)	8.2542 (50.39)	2.0364 (27.25)	5.4649 (-76.49)
Gaussian TGARCH (1,1) dummy	643.9219 (157.21)	9.0056 (31.29)	8.3108 (49.36)	2.0458 (26.67)	* 3.8938 (-67.00)
Student t TGARCH (1,1) dummy	886.3547 (86.86)	9.3953 (25.84)	8.1835 (51.69)	2.0336 (27.43)	38.9570 (-96.70)
Gaussian AGARCH (1,1)	1,439.4019 (15.07)	9.5513 (23.78)	8.9675 (38.42)	2.0949 (23.70)	5.6652 (-77.32)
Student t AGARCH (1,1)	1,443.0223 (14.78)	9.7861 (20.82)	8.0117 (54.94)	2.0331 (27.46)	6.4574 (-80.10)
Gaussian AGARCH (1,1) dummy	652.4955 (153.83)	8.9228 (32.50)	8.5042 (45.97)	2.0564 (26.02)	3.9141 (-67.17)
Student t AGARCH (1,1) dummy	847.8262 (95.35)	9.2865 (27.31)	* 7.9934 (55.29)	* 2.0316 (27.56)	41.3410 (-96.89)
Student t MS-TGARCH (2,1,1)	12,837.0990 (-87.10)	12.9163 (-8.46)	8.4669 (46.61)	2.0587 (25.88)	5.1787 (-75.19)
Student t MS-GARCH (3,1,1)	2,613.0388 (-36.62)	11.2399 (5.19)	8.4915 (46.18)	2.0778 (24.72)	4.6234 (72.21)
Student t MS-GARCH (2,1,1) dummy	831.7665 (99.12)	9.4552 (25.04)	8.2668 (49.79)	2.0467 (26.62)	19.7353 (-93.49)

Notes: (1) Estimation sample: 2156 observations, since 13 May 1994 through 16 August 2002, (2) Vector of dummy variables in mean for the 909th, 910th and 911th observations, corresponding to 23, 24 and 27 October 1997, (3) MSE = mean squared error, MAE = mean absolute error, [LE]<sup>2</sup> = mean squared log-error, |LE| = mean absolute log-error, HMSE = MSE with adjusted heteroscedasticity, (4) For the constant variance model, it was assumed that  $\hat{\epsilon}_t = (y_t - E[y_t])$  and  $\hat{h}_t = s^2 = T^{-1} \sum_t (\hat{\epsilon}_t)^2$  in the loss functions. (5) \* Best model according to the loss function.

**Table 10** Comparison of out-of-sample forecast loss functions for different GARCH models for EMBI Global Brazil Sovereign Spread

Model	Loss Functions (percentage improvement over constant variance)				
	MSE	MAE	[LE] <sup>2</sup>	LE	HMSE
Constant Variance	470.5621	14.1743	7.0057	1.8942	0.9489
Gaussian GARCH (1,1)	698.5185 (-32.63)	20.8273 (-31.94)	11.7859 (-40.56)	2.2520 (-15.89)	1.4371 (-33.97)
Student t GARCH (1,1)	992.3404 (-52.58)	27.5996 (-48.64)	10.0497 (-30.29)	2.3463 (-19.27)	0.8368 (-13.39)
Gaussian GARCH (1,1) dummy	765.6045 (-38.54)	22.6576 (-37.44)	10.5750 (-33.75)	2.2568 (-16.07)	1.2061 (-21.32)
Student t GARCH (1,1) dummy	1,008.0938 (-53.32)	27.8767 (-49.15)	10.0242 (-30.11)	2.3506 (-19.41)	0.8308 (14.22)
Gaussian EGARCH (1,1)	632.6625 (-25.62)	17.1684 (-17.44)	* 5.8596 (19.56)	1.8873 (0.37)	5.3027 (-82.10)
GED EGARCH (1,1)	692.5103 (-32.05)	18.3885 (-22.92)	5.9507 (17.93)	1.9252 (-1.61)	5.7520 (-83.50)
Gaussian EGARCH (1,1) dummy	874.8868 (-46.21)	21.4344 (-33.87)	7.4322 (-5.74)	2.0639 (-8.22)	3.6617 (-74.09)
GED EGARCH (1,1) dummy	887.4249 (-46.97)	21.4069 (-33.79)	7.7314 (-9.39)	2.0836 (-9.09)	4.8255 (-80.34)
Gaussian TGARCH (1,1)	7,256.6179 (-93.52)	74.1833 (-80.89)	13.0713 (-46.40)	2.9982 (-36.82)	0.7415 (27.98)
Student t TGARCH (1,1)	8,585.6597 (-94.52)	81.4171 (-82.59)	13.6142 (-48.54)	3.0776 (-38.45)	0.7526 (26.09)
Gaussian TGARCH (1,1) dummy	2,671.5314 (-82.39)	48.1602 (-70.57)	11.0684 (-36.71)	2.7018 (-29.89)	0.6957 (36.39)
Student t TGARCH (1,1) dummy	7,749.2480 (-93.93)	77.8579 (-81.79)	13.4383 (-47.87)	3.0508 (-37.91)	0.7483 (26.80)
Gaussian AGARCH (1,1)	* 587.0092 (-19.84)	16.7397 (-15.33)	6.0230 (16.32)	* 1.8868 (0.39)	3.7283 (-74.55)
Student t AGARCH (1,1)	730.5850 (-35.59)	21.6113 (-34.41)	10.0826 (-30.52)	2.2097 (-14.28)	1.3726 (-30.87)
Gaussian AGARCH (1,1) dummy	637.1452 (-26.15)	18.4092 (-23.00)	7.3807 (-5.08)	1.9949 (-5.05)	2.5595 (-62.93)
Student t AGARCH (1,1) dummy	746.4062 (-36.96)	22.0813 (-35.81)	10.6234 (-34.05)	2.2449 (-15.62)	1.3006 (-27.04)
Student t MS-TGARCH (2,1,1)	612.6383 (-23.19)	14.8582 (-4.60)	15.8852 (-55.90)	3.0934 (-38.77)	0.6695 (41.73)
Student t MS-GARCH (3,1,1)	639.7693 (-26.45)	13.7188 (3.32)	16.2188 (-56.81)	3.1617 (-40.09)	0.5614 (69.02)
Student t MS-GARCH (2,1,1) dummy	658.6661 (-28.56)	* 13.1665 (7.65)	16.6463 (-57.91)	3.2078 (-40.95)	* 0.5023 (88.90)

Notes: (1) Estimation sample: 2156 observations, since 13 May 1994 through 16 August 2002, (2) Vector of dummy variables in mean for the 909th, 910th and 911th observations, corresponding to 23, 24 and 27 October 1997, (3) MSE = mean squared error, MAE = mean absolute error, [LE]<sup>2</sup> = mean squared log-error, |LE| = mean absolute log-error, HMSE = MSE with adjusted heteroscedasticity, (4) For the constant variance model, it was assumed that  $\hat{\epsilon}_t = (y_t - E[y_t])$  and  $h_t = s^2 = T^{-1} \sum_t (\hat{\epsilon}_t)^2$  in the loss functions. (5) \* Best model according to the loss function.

**Table 11** *Estimated parameters of the GARCH and EGARCH models for the EMBI Global Brazil Sovereign Spread returns*

	Distribution	$\phi_0$	$\phi_1$	I	$\alpha_0$	$\alpha_1$	$\beta_1$	$\alpha_1'$	$\gamma_1$	$\nu$	GED log ( $\nu/2$ )
GARCH (1,1)	Gaussian	-0.101659 (0.04212)	0.101808 (0.02462)		0.354820 (0.056670)	0.206356 (0.022370)	0.757613 (0.021910)				
GARCH (1,1)	Student t	-0.129397 (0.038810)	0.114561 (0.022240)		0.215855 (0.060860)	0.178176 (0.028350)	0.807267 (0.020800)			5.264710 (0.597500)	
GARCH (1,1)	Gaussian	-0.113044 (0.041610)	0.114329 (0.023970)	16.577200 (1.183000)	0.259713 (0.045210)	0.176890 (0.019190)	0.794202 (0.018960)				
GARCH (1,1)	Student t	-0.131216 (0.038830)	0.111317 (0.022310)	10.332100 (1.681000)	0.204265 (0.058960)	0.172470 (0.028050)	0.813465 (0.027910)			5.314380 (0.605900)	
EGARCH (1,1)	Gaussian	-0.007297 (0.043270)	0.116388 (0.023160)		0.141032 (0.018040)		0.927833 (0.009479)	0.129187 (0.016540)	0.307269 (0.026890)		
EGARCH (1,1)	GED	-0.060069 (0.038520)	0.103602 (0.022080)		0.094401 (0.020330)		0.942856 (0.011460)	0.111862 (0.020710)	0.305490 (0.035320)		-0.476780 (0.040790)
EGARCH (1,1)	Gaussian	-0.057019 (0.042060)	0.118413 (0.022800)	17.151900 (0.822100)	0.122605 (0.018310)		0.938242 (0.009856)	0.069485 (0.015920)	0.342527 (0.027200)		
EGARCH (1,1)	GED	-0.073517 (0.040220)	0.101924 (0.021590)	16.492000 (1.693000)	0.088972 (0.020760)		0.946352 (0.011770)	0.076988 (0.020160)	0.331312 (0.035760)		-0.468264 (0.041650)

Notes: (1) The estimation sample covers 2156 observations, from 13 May 1994 through 16 August 2002, (2)  $\varepsilon_t = y_t - \phi_0 - \phi_1 y_{t-1}$ , with  $y_t = 100 * [\ln(EMBI)_t - \ln(EMBI)_{t-1}]$ , (3) Exogenous variable I in mean: vector of dummy variables for the 909th, 910th and 911th observations, corresponding to 23, 24 and 27 October 1997, (4) Values in parenthesis denote the standard deviation of the estimates.

**Table 12** *Estimated parameters of the TGARCH and AGARCH models for the EMBI Global Brazil Sovereign Spread returns*

	Distribution	$\phi_0$	$\phi_1$	I	$\alpha_0$	$\alpha_1$	$\beta_1$	$\kappa_1$	$\kappa_2$	$\nu$
TGARCH (1,1)	Gaussian	-0.026360 (0.044560)	0.110958 (0.023460)		0.422088 (0.062260)	0.284931 (0.032930)	0.768540 (0.022500)		-0.227522 (0.026610)	
TGARCH (1,1)	Student t	-0.081963 (0.040380)	0.123070 (0.022260)		0.283762 (0.066120)	0.255615 (0.041140)	0.797527 (0.027510)		-0.177854 (0.033970)	5.719580 (0.697800)
TGARCH (1,1)	Gaussian	-0.056347 (0.044200)	0.116270 (0.022980)	17.276400 (0.857500)	0.341264 (0.060880)	0.257139 (0.033430)	0.770998 (0.023980)		-0.147818 (0.027830)	
TGARCH (1,1)	Student t	-0.085268 (0.040380)	0.118908 (0.022290)	8.562350 (1.42500)	0.268439 (0.064670)	0.245067 (0.040870)	0.805192 (0.027630)		-0.168389 (0.033780)	5.671670 (0.673000)
AGARCH (1,1)	Gaussian	-0.045535 (0.045350)	0.103968 (0.023560)		0.337849 (0.060340)	0.180217 (0.020680)	0.760576 (0.022460)	-0.775237 (0.078910)		
AGARCH (1,1)	Student t	-0.092336 (0.040480)	0.120578 (0.022390)		0.218985 (0.061630)	0.177113 (0.027490)	0.791804 (0.028210)	-0.632238 (0.094890)		5.561750 (0.66200)
AGARCH (1,1)	Gaussian	-0.063417 (0.044410)	0.115092 (0.022780)	16.784500 (0.786100)	0.283075 (0.054390)	0.177342 (0.020620)	0.775132 (0.023230)	-0.637739 (0.077740)		
AGARCH (1,1)	Student t	-0.095261 (0.040490)	0.116875 (0.022510)	9.412700 (1.959000)	0.211830 (0.060430)	0.173334 (0.027730)	0.797045 (0.028510)	-0.611164 (0.097950)		5.569290 (0.654200)

Notes: (1) The estimation sample covers 2156 observations, from 13 May 1994 through 16 August 2002, (2)  $\varepsilon_t = y_t - \phi_0 - \phi_1 y_{t-1}$ , with  $y_t = 100 * [\ln(EMBI)_t - \ln(EMBI)_{t-1}]$ , (3) Exogenous variable I in mean: vector of dummy variables for the 909th, 910th and 911th observations, corresponding to 23, 24 and 27 October 1997, (4) Values in parenthesis denote the standard deviation of the estimates.

**Table 13** Estimated parameters of the *MS-GARCH* and *MS-TGARCH* models for the *EMBI Global Brazil Sovereign Spread* returns

	$\phi_0$	$\phi_1$	I	$\alpha_0$	$\alpha_1$	$\beta_1$	$\kappa_2$	$\nu$
MS-TGARCH (2,1,1)	-0.129954 (0.039947)	0.114258 (0.022222)		0.187305 (0.054860)	0.159915 (0.025939)	0.820171 (0.027044)	-1.3e-009 (0.016534)	5.970620 (0.851402)
MS-GARCH (3,1,1)	-0.128536 (0.039644)	0.111464 (0.022149)		0.255904 (0.094290)	0.132478 (0.026194)	0.806833 (0.039374)		5.970326 (0.807501)
MS-GARCH (2,1,1)	-0.131762 (0.039607)	0.110649 (0.022447)	10.457589 (1.981781)	0.375913 (0.148576)	0.138320 (0.029646)	0.761247 (0.050290)		5.754655 (0.768651)
	$g_2$	$g_3$	$\theta_{11}$	$\theta_{22}$	$\theta_{21}$	$\theta_{31}$	$\theta_{12}$	
MS-TGARCH (2,1,1)	33.539816 (37.000358)		35.299256 (22.071849)	1.256829 (0.761828)				
MS-GARCH (3,1,1)	1.618357 (0.267838)	18.986519 (25.432410)	15.510429 (6.851111)	12.274573 (6.498998)	0.858379 (1.024710)	1.191368 (0.562420)	0.762589 (0.411704)	
MS-GARCH (2,1,1)	1.706008 (0.382152)		17.600355 (7.263957)	12.647268 (5.111503)				

Notes: (1) The estimation sample covers 2156 observations, from 13 May 1994 through 16 August 2002, (2)  $\varepsilon_t = y_t - \phi_0 - \phi_1 y_{t-1}$ , with  $y_t = 100 * [\ln(EMBI)_t - \ln(EMBI)_{t-1}]$ , (3) Exogenous variable I in mean: vector of dummy variables for the 909th, 910th and 911th observations, corresponding to 23, 24 and 27 October 1997, (4) Values in parenthesis denote the standard deviation of the estimates, (5) The transition probability  $p_{32}$  was restricted to zero because in the unrestricted model it presented a value very close to zero.

The transition probabilities are parameterised by  $\theta_{ij}$  as follows:

$$p_{ij} = d / \left( 1 + \sum_{j=1}^k \theta_{ij}^2 \right), \text{ where } d = \begin{cases} \theta_{ij}^2, \forall j = 1, 2, \dots, k-1 \\ 1, \forall j = k \end{cases}$$

Thus, the resulting transition probabilities of the estimated parameters  $\theta_{ij}$  are:

- MS-TGARCH (2,1,1):

$$P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.9992 & 0.3877 \\ 0.0008 & 0.6123 \end{bmatrix}$$

- MS-GARCH (3,1,1):

$$P = \begin{bmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.9935 & 0.0048 & 0.5867 \\ 0.0024 & 0.9886 & 0.0000 \\ 0.0041 & 0.0066 & 0.4133 \end{bmatrix}$$

- MS-GARCH (2,1,1) with interventions:

$$P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.9968 & 0.0062 \\ 0.0032 & 0.9938 \end{bmatrix}$$

## 5. Concluding Remarks

Traditional country risk literature attributes small role to the country risk conditional volatility in the determination of the macroeconomic equilibrium in a small developing open economy (DSOE). Two of the main results of such recent literature are: 1) Due to the high correlation between country and currency risk, the former, as measured by the spread between the DSOE's sovereign bonds and the equivalent U.S. Treasury bonds, is not only explained by macroeconomic fundamentals but also by market sentiments; and 2) In the presence of multiple equilibria and self-fulfilling prophecies, bad equilibria occur with high country risk premia, high debt service,

sudden stops – leading to liquidation of international reserves, reduction of current account deficit, lower aggregate demand and social spending – and capital reversals. Although such literature seems to describe exception cases pretty well, the same cannot be said of all DSOEs. High country risk premia do not necessarily exhaust the argument since country risk levels may not be the only source of bad equilibria.

The prime hypothesis of this paper is that the uncertainty about fundamentals is one of the sources of bad equilibria and such noise should be incorporated into the literature in order to model liquidity crises in DSOEs. In a currency crisis game adapted to the country risk literature, the rise in the investors' uncertainty alone is able to increase the distribution

variance of investors' expected beliefs concerning the DSOE's macroeconomic fundamentals. This leaves agents no choice but to speculate on the DSOE's bonds in the secondary market, raising country risk levels consequently leading to a bad equilibrium. One testable implication of this finding is to verify whether bad equilibria occur with higher regime country risk conditional volatility.

Various GARCH-based models with observable (AGARCH, TGARCH and EGARCH) and non-observable components (MS-TGARCH) were applied to the EMBI Global Brazil Sovereign Spread, which is the thermometer of how investors perceive Brazilian macroeconomic fundamentals, from May 1994 through September 2002. In all models, interventions had to be included as a dummy variable in the mean because of the strong and sudden reversal of a long term downward trend in the Brazilian country risk levels as of the occurrence of the Asian Crisis.

The results of all models indicate that after some specific events, great positive shocks to the series indeed promote a volatility switch towards a higher regime. Even though country risk levels might recover even in short time after such shocks, the higher country risk volatility regime still persists, giving rise to the periods that typify more severe liquidity crises suffered by Brazil from 1994 through September 2002. The higher volatility regimes identified by MS-GARCH models, e.g., coincided with all such episodes. During the episodes of the Tequila Crisis (136 days: 7 December 1994 through 14 June 1995), the Asian, Russian, LTCM and Brazilian Crisis (441 days: 24 October 1997 through 2 July 1999) and the Brazilian Electoral Crisis (83 days: 24 April 2002 through 16 August 2002, when the series used in the estimation was interrupted), the conditional volatility increased about 30.6%. As a matter of fact, such result is very intuitive at hindsight: even though agents are conscious that short-lived extreme shocks might denote only spot events that will not repeat themselves without cease, agents are very uncertain about the possibility of the occurrence of another shock in the near future and thus perpetuate the crisis behaviour for a longer time.

This means that higher country risk volatility regimes, motivated by sudden great positive shocks, are in fact one of the generating sources of bad equilibria. This fact engenders two policy implications. In the short run, while macroeconomic fundamentals have not deteriorated and reforms being implemented have not actually brought more efficiency to the DSOE yet, public policies that involve the announcement of sound fundamentals should be enforced to lessen the probability of an eventual uncertainty speculative attack. In the long run, the design of transparent stable

policy rules and strong institutions should diminish the costs associated with the fear of disruption.

Extensions of this empirical analysis should still include (X)ARCH-M models, with observable and non-observable components, as well as endogenous macroeconomic variables. This would help to empirically verify a theoretical by-product of such analysis: which parameter is more decisive in the determination of bad equilibria, conditional mean or conditional variance? In other words, *can hope can beat fear or can fear beat hope?*

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