International Finance from Macroeconomics to Econophysics

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Abstract

This paper surveys the developments in the field of international finance, in particular the research of economists on foreign exchange rates. That might be of interest to physicists working on econophysics. We show how the econophysics agenda might follow naturally from the economists´ research as well as present our own work on the subject.

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1. Introduction

Economists working on the field of international finance traditionally felt uneasy with the ideas in modern finance theory, in particular with its notion of efficient markets. Instead, foreign exchange markets are widely believed to behave like the unstable and irrational asset markets described by Keynes [1].

Even the efficient markets assumption itself has been challenged recently by studies in behavioral finance. One argument, for instance, against the role of mass psychology in speculative markets is that since real returns are nearly unpredictable, the real price of stocks is close to the intrinsic value, i.e. the present value with constant discount rate of optimally forecasted future real dividends. Shiller [2], however, remarked that "this argument for the efficient markets hypothesis represents one of the most remarkable errors in the history of economic thought".

International finance has thus been in practice *open economy* macroeconomics. As it happens, macroeconomics seems to have failed as well to satisfactorily address exchange rate behavior, as this paper will show briefly. That circumstance makes international finance economists more prone to welcome the new ideas coming from physics. In so-

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called econophysics, the behavior of exchange rates and other financial assets are seen as *complex*. In complex systems with many interacting units, everything depends on everything else.

Section 2 discusses the role of expectations in macroeconomics. Section 3 focuses on the failure of modeling attempts in the framework of open economy macroeconomics. Section 4 shortly presents the econophysics agenda. Sections 5 and 6 introduce the Lévy distributions and show some algebra behind them. Section 7 displays our previous results on the econophysics of exchange rates. Section 8 shows our work on exchange rate multiscaling. And section 9 concludes.

2. Macroeconomics and expectations

Macroeconomics is the study of the economy as a whole by focusing on the forest and abstracting the trees. The discipline was practically single-handed launched by Keynes [3]. Contrary to the view of classical economists like Adam Smith, Keynes' basic insight was that a market economy is inherently unstable, and that the source of instability lies in the logic of financial markets. According to Keynes, market capitalism should be neither left alone nor abolished, but stabilized. After the developments that took place in macroeconomics after Keynes, what still arguably survives of Keynesian economics today is the above insight [4].

Keynes' book was greatly simplified in a paper by Hicks [5]. The original ideas collapsed to a graph which is standard in macroeconomics textbooks for undergraduates—the so-called IS-LM model. For tractability, the IS-LM model assumed stationary expectations, i.e. people forecast no change for future prices. Stationary expectations is a reasonable assumption in a stable zero-inflation environment, but it is not when inflation departures from nil.

Adaptive expectations came up to take the possibility of a non-zero inflation into account. Here people forecast by looking at previous inflation. Adaptive expectations is a fair assumption if prices are growing up at a constant rate. However, it is not if prices accelerate. Even if prices accelerate at a constant rate, people with adaptive expectations will make systematic forecast errors.

So rational expectations is the assumption that people also consider an accelerating inflation together with all past and current information, including what a government is doing. But rational expectations assumes, too, that people behave as if they have the "true model" of the economy in their minds, and that is too demanding.

As Shiller [6] observed, our (economists') economic models require their (the people in the economy) economic models, models that they use to generate their expectations. Rational expectations collapses the two models into one: to assume that people know (or behave as if they know) the true model that describes the economy.

Collapsing two models into one is quite convenient an insight that helped to solve complicated models analytically. After all, these can be studied without collecting any data about the models in people's minds. Rational expectations (which is a mere stochastic version of perfect foresight) is then a wonderful theory, but unfortunately for the wrong species. One must seriously accept that the models used by real world people ("popular models") are not the rational expectations one. Economic modeling has thus no choice but collecting data on the popular models themselves.

By using questionnaires to evaluate the popular models among US and Japanese (institutional and individual) investors during the stock market crash of 1987, Shiller [6] and colleagues concluded that "the suggestion we get of the causes of the crash is one of people reacting to each other with heightened attention and emotion, trying to fathom what other investors were likely to do, and falling back on intuitive models like models of price reversal or continuation. There appears to be no recognizable exogenous trigger for the crash. With such popular models, a feedback system is created with possibly complicated dynamics, and we do not need to refer to a trigger to explain a crash".

That rational expectations is a quite restrictive particular case can be illustrated with reference to the *El Farol* bar problem put forward by Arthur [7, 8]. Suppose that one hundred people must decide independently each week whether to show up at their favorite bar (*El Farol* in Santa Fe). If someone predicts, say, that more than 60 will attend, he will avoid the crowds and stay home. If he predicts fewer than 60 he will go.

Surely, predictions of how many attend depend on others' predictions of how many attend because that determines their attendance. But others' predictions depend in turn on their predictions of others' predictions. Deductively there is an infinite regress. No "correct" expectational model can be assumed to be common knowledge.

If all use an expectational model (say, rational expectations) that predicts few will go, all will go, invalidating that model. If all believe most will go, no one will go, invalidating that belief. Expectations will be forced to differ, i.e. expectations should be necessarily heterogeneous.

The example above shows that people cannot assume or deduce expectations but must discover them. As people visit the bar, they act inductively, i.e. they act as statisticians, each starting with a variety of subjectively chosen expectational models (the popular models above).

Each week they act on their currently most accurate model. People's beliefs compete for use in an "ecology" these beliefs create. By employing a computer simulation, Arthur showed that the mean attendance quickly converges to 60. The predictors "self-organize" such that 40 per cent on average are forecasting above 60, and 60 per cent below 60. So predictors split into a 60/40 average ratio that keeps changing in membership forever.

At the end of the day, one finds himself too far from the uniform forecasting models with rational expectations that are on average validated by the prices these forecast. In a sense, expectations were thus "endogenized".

Arthur and colleagues [9] extended the *El Farol* economy with endogenous expectations for an artificial financial world and generated out-of-equilibrium outcomes, of which the equilibrium with rational expectations is just a possible particular outcome.

Since mainstream economists feel somehow uneasy with "out-of-equilibrium" stories, it is worthwhile to illustrate their plausibility with reference to an example from behavioral finance.

Out-of-equilibrium decisions are likely to be the norm in the initial public offering (IPO) market, for instance. Stockbrokers in such markets seem to behave like impresarios (who manage musicians and entertainers) [6]. The public interprets empty seats as a signal of low quality. The impresarios then price tickets below equilibrium to create excess demand and long queues, and that will lead to greater demand the next days. People thus manifest "shortage illusion".

Likewise, underpricing IPOs creates the high initial returns that leave the impression that a stockbroker is giving good investment advice. Underwriters then let the high initial returns run for a while to generate publicity for an IPO.

Arthur and colleagues have designed artificially intelligent computer programs to generate and discard expectational hypotheses, and to make bids or offers based on their currently most accurate hypothesis. Endogenous expectations compete in the ecology the expectations create. The stock price forms from bids and offers, and thus ultimately from expectations.

They have found two possible regimes: (1) if parameters are set so that the artificial agents update their hypothesis slowly, the diversity of expectations collapses quickly into the homogeneous rational expectations one; (2) if the rate of updating of hypotheses is turned up, the artificial market displays several of the “anomalies” observed in real markets, such as unexpected price bubbles and crashes, random periods of high and low price variation, and the presence of technical trading.

These anomalies are thought of as out-of-equilibrium phenomena. It has to be said that mainstream macroeconomics and finance have also developed some equilibrium stories for such anomalies [10, 11]. But although “rational expectation bubbles” might still be useful as a limiting case, from the above discussion it would not be sensible to assume rational expectations from the start.

From the discussion above one can too infer that if expectations are endogenous they also have to be heterogeneous. One can also infer that international finance economists are likely to welcome the notion of complexity (the issue of complexity is exhaustively discussed elsewhere [12]). Indeed, preliminary attempts have already been made [13]. A reason good enough for that to happen is the failure of modeling attempts in the framework of open economy macroeconomics. In what follows, that will be discussed in some detail.

3. Open economy macroeconomics

In the open economy macroeconomics field, nominal and real exchange rate volatility of the floating period following Bretton Woods was explained by "overshooting" in the Dornbusch [14] model. In such a benchmark model of the field, output is exogenous and goods price stickiness is the critical reason for the exchange rate to overshoot its long run value in response to monetary shocks.

However, overshooting is possible in other models, and empirical evidence for it is thin [15-17]. In particular, one empirical regularity inconsistent with overshooting is the well documented [15, 18] tendency for spot and forward exchange rates to move in tandem.

Despite the fact that its empirical performance is not very successful [19], the Dornbusch model played a dominant role in shaping the literature on exchange rate dynamics through the early nineties [20]. That demonstrates "undeniable time-tested appeal of the traditional sticky price Keynesian model" [21]. Its prominence might also be related to the analytic simplicity of the model.

For macroeconomists, however, the Dornbusch model presents limitations related to its lack of microfoundations. The model provides a specification of the price determination process that they call "ad hoc", and ignores the current account in the exchange rate determination [22].

The quest for microeconomic foundations for macroeconomics is an almost consensus among macroeconomists, and is reminiscent of the so-called reductionism in physics. Wolfram [12] observed that "in the existing sciences much of the emphasis over the past century or so has been on breaking systems down to find their underlying parts, them trying to analyze these parts in as much detail as possible. And particularly in physics this approach has been sufficiently successful that the basic components of everyday systems are by now completely known. But just how these components act together to produce even some of the most obvious features of the overall behaviour we see has in past remained an almost complete mystery".

The research on microfoundations has begun when rational expectations stepped in. But reductionism is not useful when complexity is involved, as seen in the paragraph above. So microfoundations are not the issue if macroeconomic phenomena are complex.

Other criticisms on the model of Dornbusch by macroeconomists are as follows. The model disregards the intertemporal budget constraints needed to describe the current account and fiscal policy consistently, provides no clear description of how monetary policy affects production decisions, and has no meaningful welfare criteria, which may mislead policy prescriptions [15].

There are a number of reasons why open economy macroeconomists still continue to research using the Dornbusch model, though. First, most of the stories appearing in media reports on exchange rates are consistent with the Dornbusch model [21]. Also, despite its general empirical collapse, evidence is also emerging that data provide support for some of the long run relationships suggested by the model, an example being long run purchasing power parity [22].

The empirical evidence supportive of other macroeconomic models that are traditional alternatives to the Dornbusch model, namely the monetarist model and the portfolio balance model, is thin as well.

Despite some initial success, the monetarist model has failed empirically (reference [19] presents a discussion). In particular, estimates of equations for the US dollar-Deustche mark rate in the late seventies and beyond often produce coefficients implying that increases in Germany's money supply during this period caused its currency to appreciate. That has been called "the mystery of the multiplying marks" [19].

Although less empirical work had been carried out on the portfolio model, the supportive evidence has notwithstanding been weak (reference [19] shows details).

The traditional flexible price models of the exchange rate have been developed theoretically by the so-called intertemporal approach to the current account [23-25].

A widely accepted standpoint by international macroeconomists is, however, that most important problems—such as the effects of macro policies on output and exchange rates—cannot be satisfactorily addressed in the framework of perfect price flexibility. That is another reason why empirical macroeconomists and policymakers have continued to use the traditional aggregative Dornbusch model [15].

Overall, it can thus be said that modeling with the standard macroeconomic models has failed empirically. Such a poor performance was highlighted when studies demonstrated that a random walk predicts exchange rate behavior better than models based on the fundamentals of the economy.

The studies of Meese and Rogoff [26-28] showed that the traditional open economy macroeconomics models could not outperform a simple random walk in out-of-sample regressions. Since then, the inability to beat the random walk has been regarded as the standard by which to judge the empirical failure of the models of the open macroeconomy.

Usually the random walk hypothesis is tested by considering the exchange rate dependent on its past value plus white noise. In such a first order autoregressive process, a coefficient equaling one means that there is a unit root, i.e. the exchange rate series is non-stationary.

Several empirical studies found that exchange rate data do exhibit unit roots, but the error term does not have a constant variance [29-32]. The series is thus non-stationary and the error term shows time dependent heteroskedasticity.

To revive the standard macroeconomic models, Koedijk and Schotman [33] estimated an "error-correction" *real* exchange rate equation and showed that it is superior, in-sample, to a random walk.

The bilateral real exchange rates between the US, the UK, Germany, and Japan for the period February 1977 to June 1987 were considered in an econometric model (based on the Dornbusch model) and a significant mean reversion component was found.

However, despite the fact that the major trends of the non-dollar exchange rates could be explained by macroeconomic fundamentals, Koedijk and Schotman also discovered that the dollar bubble between March 1984 and February 1985 cannot be understood by appealing to fundamentals.

Also using dynamic error-correction techniques, Mark [34] considered an equation (derived from the Dornbusch model) to investigate the performance of the monetary exchange rate models concerning long run predictability. In forecasting tests over long horizons for several quarterly dollar exchange rates, evidence was found that macro fundamentals help to predict the nominal exchange rate, particularly at the four-year horizon.

The study by Chinn and Meese [35] also suggested that over long enough periods there is indeed a stationary relationship between the exchange rate and the fundamentals of the open macroeconomy models.

The revival of fundamentals is sometimes also associated with the rebirth of long run purchasing power parity (reference [22] presents a comprehensive discussion on that).

Despite the revival of the standard fundamentals of macro models, the hypothesis that the exchange rate follows a random walk is still to be taken seriously. An interesting development that makes it possible to conciliate the apparent divergence between random walk and fundamentals is the model of De Grauwe and Dewachter [36], and De Grauwe, Dewachter, and Embrechts [37].

De Grauwe and colleagues' model gives supplementary speculative dynamics to the Dornbusch model by considering chart rules concerning forecasting, and explains exchange rate movements by chaos. An advantage of chaotic models is to mimic the random walk pattern of the exchange rate with the "stochastic" behavior produced by deterministic solutions. The models above have been extended to show that massive foreign exchange intervention can remove the chaos [38].

In the mid 1980s the general sentiment among open economy macroeconomists was that the research into exchange rate economics appeared to have grown tired of searching for new macro models [39]. As a result, attention shifted from examination of macro models toward work related to the foreign exchange market as a financial market *per se*.

That trend was reinforced by the studies pointing out that the nominal exchange rate shows much greater variability than the important fundamental variables of the structural models [40-44].

Accounts of short run exchange rate movements based solely on fundamentals were then believed not to prove successful, owing exactly to the presence of speculative forces at work in the foreign exchange market. Speculation was not reflected in the usual set of fundamentals of the macro models [19].

Thus the literature on foreign exchange market microstructure focused on the behavior of agents and market characteristics rather than on the influence of macro fundamentals. One motivation for such work has been to understand the mechanisms generating deviations from fundamentals. A survey on that is provided by Flood [45], and another useful reference is Frankel, Galli, and Giovannini [46].

Other studies adopted the modeling strategy of reducing all structure of a model to just one single variable. This was intended to focus analysis on the effect of "news", i.e. unexpected changes in the exchange rate resulting from changes in the fundamentals that come as a surprise.

The news approach thus relied on the existence of unexpected shocks to explain every exchange rate movement. It was shown however that only a small proportion of movements of the spot exchange rate is caused by news [47]. A survey of the papers dealing with news is provided by Frankel and Rose [48].

If compared with the news approach, an advantage of chaotic models is that they do not rely on random shocks to explain shifts in the exchange rate. Indeed currency crashes can happen in these models with no change in their exogenous variables.

As an offshoot of the closed economy macro literature on real business cycles, the equilibrium exchange rate model [21, 49-53] gives a full account of the supply side.

At this stage it is not possible to draw any firm conclusions concerning the empirical validity of the equilibrium model [19]. The emerging challenger of the equilibrium model is that of Obstfeld and Rogoff [15, 16]. The studies collected together in Van Der Ploeg [54] allow a general appreciation of other new developments.

The model of the exchange rate developed by Obstfeld and Rogoff—the "redux" model—is a dynamic intertemporal two country model that assumes monopolistic competition and sticky nominal prices in the short run.

While preserving the sticky price feature of the Dornbusch model, it provides a more rigorous framework than the latter model by incorporating the intertemporal approach to the current account. That allows for evaluating the welfare effects of macro policies on output and the exchange rate, a possibility not contemplated by the flexible price intertemporal approach.

The results of Obstfeld and Rogoff sometimes differ sharply from those of either the Dornbusch model or the flexible price intertemporal approach to the current account. In particular, the model gives a different view of the international welfare spillovers due to monetary and fiscal policies.

The testable results of the redux model—such as whether the distortions affecting the welfare effects of international monetary policy are empirically significant [16]—still have to stand up to empirical scrutiny. If they succeed, macroeconomists might claim that lack of microfoundations partly explains the bad empirical performance of the Dornbusch model.

The wave of research initiated by the redux model is sometimes labelled "new open economy macroeconomics". Lane [55] and Sarno [56] present surveys.

Lane questions the relevance of this literature for policymaking because many welfare results are highly sensitive to the precise denomination of price stickiness and the specification of preferences. But the widespread commitment with microfoundations and the many unanswered questions that remain should ensure that the literature is likely to grow yet further in the coming years among macroeconomists.

It is still worthwhile to mention that the speculative dynamics side of the model of De Grauwe and colleagues has also been blended with the model of Obstfeld and Rogoff to produce a chaotic nominal exchange rate [57].

Where do we then stand in the open economy macroeconomics literature? The Dornbusch model demonstrates undeniable time-tested appeal. But the redux model comes up to update the Dornbusch model as regards microfoundations and a supposed breakthrough is to allow an explicit welfare analysis as far as policy is concerned.

The welfare results of the new open economy macroeconomics literature are highly sensitive to the precise denomination of price stickiness and the specification of preferences, though. For that reason, the literature is of only limited interest in policy circles.

Notwithstanding, the lack of welfare criteria of the Dornbusch model is claimed to yield misleading policy prescriptions; and that will encourage macroeconomists to further research on the new open economy macroeconomics.

4. The econophysics agenda

Like economists, physicists also use models—which they call "artificial worlds". Unlike economists, however, physicists are fundamentally empirical in their approach to science. Indeed, when doing their everyday research some physicists never make reference to models at all. Unlike mainstream economists, physicists usually think of the macroeconomy as a complex system with many interacting subunits. They also have decided to examine empirical economic and financial facts prior to the building up of models.

For the physicist's eye, the economy is a collection of interacting units where everything depends on everything else. The problem is then *how* does everything depend on everything else? Here physicists are looking for empirical laws that will describe this complex interaction. In particular, they are mostly interested in fluctuations [58].

For a start, take the biased random walk model of Bachelier [59], which is somewhat like a drunk with a coin and a metronome. Here a biased coin is one that has a tendency to go up. One might think of heads meaning one step to the right, and of tails as one step to the left.

By adapting the biased random walk for the S&P500 data which encompass the crash of 19 October 1987, Stanley and colleagues [58] showed that the huge drop of Black Monday is virtually impossible in the model. They observed that the probability that a walk will move two steps in the same direction is p^2, three steps is p^3, and the probability of many steps in the same direction is exponentially rare.

The biased random walk has a probability density function that is Gaussian. With returns (fluctuations) normalized by one standard deviation, the probability of having more than 5 standard deviations is essentially zero. However, there are many shocks in the S&P500 returns that exceed 5 standard deviations (30 or 40 hits on the positive side). And

Black Monday is more than 34 standard deviations [58]. Research on econophysics that take empirical data into account aims at showing that catastrophic, rare events like Black Monday must be considered as part of the overall picture; they are not (in a sense) "anomalies". Even the great stock market crashes would be simply ordinary (although infrequent) events.

Black Monday was the largest single-day free fall in market history. The crash was nearly twice as severe as the stock market collapse of 1929, although that time it did not trigger a depression. What made the Dow Jones industrial average sink? It is difficult to believe that there could be a sudden change in the fundamentals which would lead people simultaneously within half a day to the view that returns in the future had gone down by over 20 per cent. A dubious explanation is that the crash was caused by portfolio insurance computer programs which sold stocks as the market went lower.

In an efficient market, when supply matches demand, prices have their proper values, i.e. values that correspond to the underlying fundamentals. Market prices could still bounce up and down erratically, but huge fluctuations could not be accounted for. Price changes would behave like the bell curve. Greater than some typical size, price changes ought to be extremely rare. Prices would somewhat follow a gentle random walk.

As observed [58], almost everything in nature, including disordered things, has scale. We can find the scale of even raindrops on a road by zooming in or out. Most functions in physics have a characteristic scale and almost all physics comes down to solving a differential equation. Once the scale is determined, a function can be expressed in an exponential form whose derivative is also an exponential. Solutions to this look like tractable Gaussian functions.

But some systems in nature lack a scale. In particular, systems with many interacting units (like the macroeconomy) generally do not have scales, i.e. they exhibit scale invariance that can be expressed by power laws. Indeed price fluctuations in the S&P500 were found to become about sixteen times less likely each time the size is doubled [60]. A scale-invariant power law has also been found for financial market volatility: the market does not have a typical wildness in its fluctuations [61].

Mandelbrot [62] looked at how random changes in cotton prices were distributed by size and did not find a bell curve. Instead, he discovered that price changes do not have a typical size, thereby being governed by a non-Gaussian power law. That allows one to see large fluctuations in market prices as a result of the natural, internal workings of markets; they can strike from time to time even if there are no sudden alterations of the fundamentals. Mandelbrot suggested a stable Lévy distribution [63] to model the cotton prices.

Financial asset prices are also unlikely to follow Gaussian distributions [64]. Sky-high peaks and fat tails are pervasive in financial data. Although leptokurtosis could be accounted for by stable Lévy distributions, these have never been established in mainstream finance. One reason is related to their property of infinite variance. Since volatility is a central concept to finance, it is useful for the variance to be finite. (The debate in the early days of modern finance can be appreciated in reference [65]).

To remedy such a deficiency, a truncated Lévy distribution has been put forward [66, 67]. A truncated Lévy flight aims at modeling financial series through a non-stable distribution which features non-normal scaling power laws and finite variance. The truncated Lévy flight is then a candidate to satisfactorily model financial data. Indeed, that has been shown for the S&P500 [67] and other stock markets [68-70], as well as foreign exchange rates [71]. An earlier study that found power laws in foreign exchange markets is that of Müller et al. [72].

Non-Gaussian power laws are expected to coexist uneasily with mainstream finance theory, which is built on the efficient market hypothesis. However, econophysics does not clash with mainstream finance. Overall physicists see the efficient market as an idealized system and real markets as only approximately efficient. They think the concept of efficient markets is still useful to model financial markets. But rather than simply assuming normality from the start, they try to fully characterize the statistical properties of the random processes observed in financial markets [73].

5. Lévy distributions

As observed, early attempts to collapse the Gaussian distribution as a special case include the suggestion of a stable Lévy distribution to model cotton prices. Ordinary stable Lévy distributions have fat power-law tails that decay more slowly than an exponential decay. Such a property can track extreme events, and that is plausible for financial data. But it also generates an infinite variance, which is implausible and undesirable.

As also observed, a sharply truncated Lévy flight has been put forward. But it is still possible to define a TLF with a smooth cutoff that yields an infinitely divisible characteristic function [74]. In a smoothly truncated Lévy flight (STLF), the cutoff is done by asymptotic approximation of a stable distribution valid for large values [75].

Yet the STLF breaks down in the presence of positive feedbacks [76, 77]. But the cutoff can still be alternatively combined with a statistical distribution factor to generate a gradually truncated Lévy flight (GTLF) [76, 77]. Nevertheless that procedure also brings fatter tails. The GTLF itself also breaks down if the positive feedbacks are strong enough. That apparently happens because the truncation function decreases exponentially.

We have ourselves put forward what we call an exponentially damped Lévy flight (EDLF) [78], in which the gradually truncated Lévy is modified and then combined with the smoothly truncated one. In the presence of increasing and positive feedbacks, our distribution smoothly and gradually deviates from the Lévy. The truncation parameters are estimated by nonlinear least squares to provide an optimized fit for the tails. Our EDLF seems to fairly fit data on daily exchange rates. Section 7 will discuss that in some detail.

Whether scaling is single or multiple depends on how a Lévy flight is broken. While the abruptly truncated Lévy flight (the TLF itself) exhibits mere single scaling, the STLF shows multiscaling [75, 79]. When employing the abruptly TLF [71] to fit the exchange rate data we have realized that the such data set might be fitted by an EDLF as well [78]. That is interesting because we can focus on the exchange-rate multiscaling properties stemming from the EDLF. Not surprisingly, and in accordance with the previous literature [80-85], we find multiscaling to be pervasive among exchange rates. This will be shown in section 8.

6. Some algebra behind the Lévy distributions

Let S_n be the sum of n independent and identically distributed random variables X_t,

$S_n = X_1 + X_2 + X_3 \ldots + X_n$ (1)

with $E(X_t) = 0$. It is usual to work with returns in finance, i.e.

$Z_{\Delta t}(t) = S_t - S_{t-\Delta t} = X_t + X_{t-1} + \ldots + X_{t-\Delta t+1}$ (2)

where Δt is a time lag. Now consider the symmetric Lévy distribution

$$L(Z_{\Delta t}) \equiv \frac{1}{\pi}\int_0^\infty \exp(-\gamma\Delta t q^\alpha)\cos(qZ_{\Delta t})\mathrm{d}q \quad (3)$$

where $0 < \alpha < 2$, and $\gamma > 0$ is a scale factor.

The characteristic function of (3), $\varphi(K)$, is such that

$\ln[\varphi(K)] = -\gamma\Delta t|K|^\alpha$ (4)

which satisfies $\Delta t \ln[\varphi(K)] = \ln[\varphi(\Delta t^{1/\alpha} K)]$. That means that the corresponding probability distribution is

$L(Z_{\Delta t}) = \Delta t^{-1/\alpha} L(\Delta t^{-1/\alpha} Z_{\Delta t}) = \Delta t^{-1/\alpha} L(Z_s)$ (5)

where $Z_s = \Delta t^{-1/\alpha} Z_{\Delta t}$ is a scaled variable at Δt.

Let us define a modified Lévy flight (MLF) through

$\mathrm{P}(Z_{\Delta t}) = \eta L(Z_{\Delta t}) f(Z_{\Delta t})$ (6)

where η is a normalizing constant, and $f(Z_{\Delta t})$ is the change carried out on the distribution.

The abruptly truncated Lévy flight (TLF) is an extension to which

$$f(Z_{\Delta t}) = f_{\text{abrupt}}(Z_{\Delta t}) = \begin{cases} 0, & (|Z_{\Delta t}| > l_{\max}) \\ 1, & (|Z_{\Delta t}| \le l_{\max}) \end{cases} \quad (7)$$

where $l_{\max}$ is the step size at which the distribution begins to departure from the ordinary Lévy. The TLF is not stable and has finite variance, thereby converging to a Gaussian equilibrium according the central limit theorem. The characteristic function of the TLF is no longer infinitely divisible. Nevertheless approximate scaling can still occur for a finite time interval [73]. But scaling must break down for longer time intervals.

Now consider the STLF [74, 75]. The cutoff parameter $\lambda_0 > 0$ is introduced into Eq. (6) as

$$f(Z_{\Delta t}) = f_{smooth}(Z_{\Delta t}) = \begin{cases} Ca\,|Z_{\Delta t}|^{-1-\alpha}\, e^{-\lambda_0 |Z_{\Delta t}|}, & (Z_{\Delta t} < 0) \\ CbZ_{\Delta t}{}^{-1-\alpha}\, e^{-\lambda_0 Z_{\Delta t}}, & (Z_{\Delta t} > 0) \end{cases} \quad (8)$$

Function $f_{smooth}(Z_{\Delta t})$ is based on the asymptotic approximation of a stable distribution of index α valid for large values of $|Z_{\Delta t}|$ when $\gamma = 1$. It exhibits a power law behavior. For $0 < \alpha < 1$, the first term of the expansion of $L(Z_{\Delta t})$ can be approximated by

$$L(Z_{\Delta t}) \approx \frac{\gamma\Delta t\Gamma(1+\alpha)sin(\pi\alpha/2)\,|Z_{\Delta t}|^{-(1+\alpha)}}{\pi} \quad (9)$$

By taking into account the particular case of the Lévy where $a = b$, we get

$$\ln[\varphi_{STLF}(K)] = \begin{cases} \gamma\{(\lambda_0^2 + K^2)^{\alpha/2}\cos(\alpha\theta) - \lambda_0^\alpha\}, & (0 < \alpha < 1) \\ \gamma\lambda_0^\alpha\{(1 + iK/\lambda_0)^\alpha - 1\}, & (1 < \alpha < 2) \end{cases} \quad (10)$$

where $\theta = \arctan(K/\lambda_0)$, and $\gamma = C\Gamma(-\alpha)$. Now the characteristic function ends up infinitely divisible.

The GTLF [76, 77] is defined as

$$
f(Z_{\Delta t}) = f_{gradual}(Z_{\Delta t}) = \begin{cases} 1, & (|Z_{\Delta t}| \leq l_c) \\ \exp\left\{-\left(\dfrac{|Z_{\Delta t}| - l_c}{\beta_1}\right)^{\beta_0}\right\}, & (|Z_{\Delta t}| > l_c) \end{cases} \tag{11}
$$

where l_c is the step size at which the distribution starts to deviate from the Lévy. Here β_0 and β_1 are the constants related to the truncation.

When using the currency data to be presented in section 7, we have realized that their distributions deviate from the Lévy in a smooth and gradual fashion after $|Z_{\Delta t}| > l_c$. Sometimes the deviations were also caught increasing. Such a class of deviations was already found to be positive [76, 77], which means even fatter tails. It has been argued [76, 77] that, since the physical capacity of a system is limited, the feedback begins to decrease exponentially (and not abruptly) after a certain critical step size. In contrast, in the presence of our previously found increasing deviations, we think that an abrupt truncation is necessary still. In such cases, using the truncation approaches as in Eqs. (7), (8), and (11) might prove not to be appropriate.

For that very reason we have suggested [78] the broader formulation for $f(Z_{\Delta t})$ dubbed EDLF. The EDLF encompasses the previous TLF, STLF, and GTLF. Our EDLF is defined as

$$
f(Z_{\Delta t}) = f_{damped}(Z_{\Delta t}) = \begin{cases} 1, & |Z_{\Delta t}| < l_c \\ (\Delta t^{-1/\alpha} |Z_{\Delta t}| + \vartheta)^{\tau_1} \exp\{H(Z_{\Delta t})\}, & (l_c < |Z_{\Delta t}| < l_{\max}) \\ 0, & |Z_{\Delta t}| > l_{\max} \end{cases} \tag{12}
$$

where

$$
H(Z_{\Delta t}) = \lambda_1 + \lambda_2 [1 - |Z_{\Delta t}| / l_{\max}]^{\tau_2} + \lambda_3 (|Z_{\Delta t}| - l_c)^{\tau_3} \tag{13}
$$

and ϑ, λ_1, $\lambda_2 \leq 0$, $\lambda_3 \leq 0$, τ_1, τ_2, and τ_3 are parameters describing the deviations from the Lévy, l_c is (as before) the step size at which the distribution begins to deviate from the Lévy, and l_{max} is the step size at which an abrupt truncation is carried out.

Note that when $l_{max} \to \infty$, we have

$$
H(Z_{\Delta t}) = \lambda_1 + \lambda_2 + \lambda_3 (|Z_{\Delta t}| - l_c)^{\tau_3} \tag{14}
$$

By setting $\vartheta = 0$, $\tau_1 = -1 - \alpha$, $l_c = 0$, and $\tau_3 = 1$ in Eqs. (12), (13), and (14), the resulting function is thus equivalent to the smooth case given by Eq. (8). When $l_{max} \to \infty$, the similar function for the gradual case can be found by setting $\vartheta = \lambda_1 = \lambda_2 = \tau_1 = 0$. The abrupt case is given by setting $l_c = 0$ and choosing the appropriate parameters such that $H(Z_{\Delta t}) \to -\infty$.

7. Illustration with exchange rate data

Now we turn to review our own work on the dollar prices of 30 currencies plus a fake euro. The data sets for the 30 currencies were taken from the Federal Reserve website. They are a currency value in US dollar terms. As standard, here we ignore "holes" from weekends and holidays; analysis thus concentrates on trading days. Since the series for the real euro is too short, we have decided to take a *false* euro instead in order to get a longer series. We build the fictitious series for the euro by following a methodology put forward by Ausloos and Ivanova [86]. Table 1 shows the 31 currencies, historical time period, and number of data points.

Fig. 1 displays the logarithm of the probability density functions (PDFs) of currency returns for selected countries in Table 1, namely Australia, Britain, Canada, Belgium, India, Brazil, China, and South Africa. Increases in time horizons range from Δt = 1, 2, and 5 trading days (a week) to 240 trading days (a year). A spreading of the PDFs characteristic of any random walk is observed. Fig. 2 shows a log-log plot of the "probability of return to the origin" $P(0)$ against Δt [67]. Roughly, scaling power laws emerge for the currencies within the time window of $1 \le \Delta t \le 100$; and that fact is at first consistent with the presence of a TLF.

Table 2 presents parameters α and γ for the currencies in Table 1. Parameter α is greater than two for six countries, namely Canada, China, Malaysia, South Africa, Thailand, and Venezuela; the currencies of these countries may (or may not) be outside the Lévy regime. For all the other currencies, a TLF might describe the data within a time window of (generally) 100 trading days (not shown).

Thus a Lévy PDF could model the modal region of such processes within a finite time interval. Thus we have made a case for the presence of such TLFs (with finite second moments) to be pervasive in daily time series of currency returns.

We have also moved up to assess how our EDLF adjusts to the same data. But here estimation of parameters α and γ departures from our previous approach, which is standard in this type of literature. Such parameters are usually estimated by plotting the probability of return to the origin against Δt. Our new hybrid estimation process takes a maximum likelihood approach for α and γ, and nonlinear least squares for the other parameters.

From Eq. (6), the log likelihood function is given by

$$\sum_z \ln(\mathrm{P}(z_{\Delta t})) = \sum_z \ln(\mathrm{L}(z_{\Delta t})) + \sum_z \ln(f(z_{\Delta t})) + \text{constant}\,. \tag{15}$$

The maximum likelihood estimates are obtained by minimizing Eq. (15) as a function of the distribution parameters. Note that (15) is composed of three parts. For Δt = 1, the first part depends only on α and γ, and the second one depends on the other parameters. As a result, the estimation process can be carried out separately for Δt = 1. The maximum likelihood estimates of α and γ are the same as those obtained if the process were a Lévy. Such a practice is discussed elsewhere [87, 88], and a computer program for implementing it (called Stable.exe) is available online at http://academic2.american.edu/~jpnolan/.

With $\hat{\alpha}$ and $\hat{\gamma}$ being the maximum likelihood estimates of α and γ, let $\hat{P}(z_s)$ be the sample probabilities of the scaled variable at Δt, i.e. $z_s = \Delta t^{-1/\hat{\alpha}} z_{\Delta t} = \Delta t^{-1/\alpha} Z_{\Delta t}$. Also let $\hat{L}(z_1)$ be the Lévy distribution using $\hat{\alpha}$, $\hat{\gamma}$ for Δt = 1. The difference $\ln \hat{P}(z_s) - \ln \hat{L}(z_1)$ shows how data deviate from the original Lévy process. Assuming that $\ln \hat{P}(z_s) - \ln \hat{L}(z_1) = \ln f_{damped}(z_{\Delta t})$, which equals

$$\beta_1 \ln(|z_s| + \vartheta) + H(z_s), \quad (\Delta t^{-1/\alpha} l_c < |z_s| < \Delta t^{-1/\alpha} l_{\max})\,, \tag{16}$$

then the parameters describing the deviations from the Lévy can be estimated by a nonlinear least squares procedure. Results are displayed in Table 3.

The sample probabilities of a scaled variable at Δt, $\hat{P}(z_s)$ together with a Lévy distribution using $\hat{\alpha}$ and $\hat{\gamma}$ were then calculated. The differences $\ln \hat{P}(z_s) - \ln \hat{L}(z_1)$ are shown in Fig. 4. The resulting curves are shown as the continuous lines in Fig. 4. Their parameter estimates obtained by the nonlinear least squares method are also presented in Table 3. In all cases, parameters λ_3 and β_3 were dropped from the model. Here zero estimates for λ_3 and β_3 mean that the scaled PDFs exhibit heavy tails with increasing and positive feedbacks.

The examples in Fig. 5 show that our EDLF fits the exchange rate data reasonable well. It is worth emphasizing that the data in Fig. 4 show log differences $\ln \hat{P}(z_s) - \ln \hat{L}(z_1)$ that are increasing. And that is why fatter tails with increasing (instead of decreasing) and positive feedback emerge.

8. Multiscaling

Our suggested EDLF has been employed to study the multiscaling properties of the exchange rates above [89]. Here we briefly discuss our contribution on this subject.

By scaling $Z_{\Delta t}$ together with the truncation parameters, a distribution can be collapsed onto $\Delta t = 1$. We thus have

$$\mathrm{P}(Z_{\Delta t}) = \eta L(Z_{\Delta t}) f(Z_{\Delta t}) = \Delta t^{-1/\alpha} \eta L(Z_s) f_s(Z_s) = \Delta t^{-1/\alpha} \mathrm{P_s}(Z_s) \qquad (17)$$

where $Z_s = \Delta t^{-1/\alpha} Z_{\Delta t}$, and $f_s(Z_s)$ is a truncation function defined by the scaled parameters $l_{ms} = \Delta t^{-1/\alpha} l_{\max}$, $l_{cs} = \Delta t^{-1/\alpha} l_c$, and $\lambda_{3s} = \Delta t^{\tau_3/\alpha} \lambda_3$.

Power laws for both the K^{th} absolute moment and norm of the characteristic function of the EDLF can be derived as follows. By scaling $Z_{\Delta t}$ and using $l_{ms} = \Delta t^{-1/\alpha} l_{\max}$, $l_{cs} = \Delta t^{-1/\alpha} l_c$, and $\lambda_{3s} = \Delta t^{\tau_3/\alpha} \lambda_3$, the K^{th} absolute moment $E[|Z_{\Delta t}|^K]$ can be reckoned as

$$E[|Z_{\Delta t}|^K] = \Delta t^{K/\alpha}\ E_s[|Z_s|^K] \qquad (18)$$

Note that such a power law depends on α and $l_{\max}$, l_c, and λ_3. If it were dependent only on α, multiscaling could not emerge because $\ln E[|Z_{\Delta t}|^K]$ would be given by a linear function K/α.

Now let

$$<|Z_{\Delta t}(t)|^K> = \tfrac{1}{n}\sum_{t=1}^{n} |S_t - S_{t-\Delta t}|^K \qquad (19)$$

and

$$<|\widetilde{Z}|^K> = \tfrac{1}{n}\sum_{t=1}^{n} |S_t - S_{t-1}|^K \qquad (20)$$

be the K^{th} sample mean of the lagged absolute values of S_t at the time interval Δt and 1 respectively. For moments which are low enough (such as $0 < K < \alpha$), $\mathrm{P}(Z_{\Delta t})$ is expected to be approximated by $L(Z_{\Delta t})$, which in turn does not depend on the truncation parameters [75]. The reason why that might occur is that tails differ, and thereby they do not contribute a great deal to the low moment case [75]. Thus we expect $<|Z_{\Delta t}|^K> \approx \Delta t^{K/\alpha} <|\widetilde{Z}|^K>$ to hold for lower moments. That means that the ratio $R(K, \Delta t) = <|S_{\Delta t}|^K>/<|\widetilde{S}|^K>$ scales with Δt as $R(K, \Delta t) = \Delta t^{K/\alpha}$.

By considering the case with $K > \alpha$, l_{max} finite, $\vartheta = 0$, and $\lambda_3 = 0$, it can be shown [89] that

$$E[|Z_{\Delta t}|^K] \approx \frac{\Delta t^{K/\alpha}2\eta\gamma\Gamma(1+\alpha)sin(\pi\alpha/2)}{\pi}O_{damped} \qquad (21)$$

where

$$O_{damped} = \frac{l_{cs}^{(K-\alpha)}}{K-\alpha} + l_{ms}^{K-\alpha+\tau_1}e^{\lambda_1}\sum_{j\geq 0}\frac{\lambda_2^j}{j!}B_u(1+j\tau_2, K-\alpha+\tau_1) \qquad (22)$$

and $l_{ms} = \Delta t^{-1/\alpha}l_{\max}$, $l_{cs} = \Delta t^{-1/\alpha}l_c$, and $B_u(1+j\tau_2, K-\alpha+\tau_1)$ is the incomplete Beta function with $1 + j\tau_2 > 0$, $K - \alpha + \tau_1 > 0$, and $u = 1 - l_c/l_{max}$. Thus the ratio $E[|Z_{\Delta t}|^K]/E[|Z_1|^K]$ is approximately given by

$$\Delta t^{K/\alpha}\frac{l_{cs}^{K-\alpha} + (K-\alpha)l_{ms}^{K-\alpha+\tau_1}e^{\lambda_1}\sum_{j\geq 0}\frac{\lambda_2^j}{j!}B_d(1+j\tau_2, K-\alpha+\tau_1)}{l_c^{(K-\alpha)} + (K-\alpha)l_{\max}^{K-\alpha+\tau_1}e^{\lambda_1}\sum_{j\geq 0}\frac{\lambda_2^j}{j!}B_d(1+j\tau_2, K-\alpha+\tau_1)} \qquad (23)$$

In such a situation, the ratio $R(K, \Delta t)$ scales as $R(K, \Delta t) = \Delta t^{K/\alpha}$ if the ratio of (23) equals one. Nakao [75] previously noted that the self-similarity of $E_s[|Z_s|^K]$ breaks down if $K > \alpha$. If $K < \alpha$ then $E_s[|Z_s|^K] \approx E[|Z_1|^K]$; otherwise, $E_s[|Z_s|^K] \neq E[|Z_1|^K]$. In practice the power law is of type $R(K, \Delta t) = \nu\Delta t^{K/\alpha}$ for some time interval Δt, where ν is a constant describing quasi-stable processes [71]. In such situations, ratio (23) gets approximately equal to ν for $\Delta t_1 \leq \Delta t \leq \Delta t_2$. By depending solely on how the truncation parameters are set, a number of distinct scaling patterns can be uncovered. For example, if $l_c = 0$ then $E[|Z_{\Delta t}|^K]/E[|Z_1|^K] \approx \Delta t^{1-\tau_1/\alpha}$.

The norm of the characteristic function can also be used to assess parameter γ by taking into account the same assumption that $P(Z_{\Delta t}) \approx L(Z_{\Delta t})$ for low values of $|K|$. Since

$\varphi(K) \equiv \mathrm{E}[e^{iKZ_{\Delta t}}] = \mathrm{E}[\cos(KZ_{\Delta t}) + i\sin(KZ_{\Delta t})] = \mathrm{E}[\cos(KZ_{\Delta t})] + i\mathrm{E}[\sin(KZ_{\Delta t})]$ (24)

then the squared norm of $\varphi(K)$ is $\|\varphi(K)\|^2 = \mathrm{E}^2[\cos(KZ_{\Delta t})] + \mathrm{E}^2[\sin(KZ_{\Delta t})]$. For some K and Δt, $\|\varphi(K)\|$ can be estimated by

$\|\hat{\varphi}(K)\|^2 = \langle\cos(KZ_{\Delta t})\rangle^2 + \langle\sin(KZ_{\Delta t})\rangle^2$ (25)

By assuming that $\ln[\varphi(K)] \approx -\gamma\Delta t|K|^\alpha$ for $0 < K < \alpha$, the "estimated" norm in logs of the characteristic function is $\ln\|\varphi(K)\|$, and then we can expect that $\ln\|\hat{\varphi}(K)\| = -\gamma\Delta t|K|^\alpha$.

Fig. 6 displays sample ratios $R(K, \Delta t)$ for several values of K in log-log plots of the exchange rates. Some ratios exhibit power law dependence on Δt. Pictures with lines which are dependent on K emerge in some of the plots.

By fitting $\log R(K, \Delta t) = \xi\log\Delta t$ for every K, we get the corresponding scaling exponents shown in Fig. 7. Some curves show linear dependence on K, for $0 < K < \alpha$. However scaling breaks down after $K > \alpha$, and a nonlinear behavior steps in.

Fig. 8 displays the sample logarithm of the absolute characteristic function versus Δt for several values of K. A power law dependence on Δt seems again to emerge from the pictures.

By fitting $\ln\|\hat{\varphi}(K)\| = \zeta\Delta t$ for every K, the estimated values of ζ versus $|K|^{\hat{\alpha}}$ are plotted in Fig. 9. Britain, Brazil and Canada show a linear dependence for all $K < 3$. For all the other cases, the linear dependence on the initial values of K are followed by nonlinear patterns.

Table 4 shows results for all the currencies, where either single scaling or multiscaling is displayed in connection with both (ξ and ζ) exponents. As can be seen, multiscaling is pervasive among foreign exchange rates.

9. Conclusion

This paper is a survey of the work of economists in the field of exchange rates called international finance. It is also a presentation of our own previous work on the econophysics of exchange rates. That might be of interest to physicists working on the general subject of econophysics.

Overall the paper is intended to show how the econophysics agenda might follow naturally from the economists´ research as far as international finance is concerned. Our own work on the subject focuses mainly on the Lévy distribution and its applications to exchange rate data. Among other things, we present our suggested method to break the Lévy tails and show the multiscaling properties of actual exchange rates in connection with our exponentially damped Lévy flight.

Acknowledgements

We are grateful to Aline De Almeida, Aline Gandon, and Martha Scherer for research assistance.

Table 1
Description of data sets.

Country	Currency	Time Period	Data Points
Australia	Australian Dollar	4Jan71 – 10Jan03	8025
Austria	Shilling	4Jan71 – 31Dec98	6999
Belgium	Belgian Franc	4Jan71 – 31Dec98	7013
Brazil	Real	2Jan95 – 10Jan03	2014
Britain	Pound	4Jan71 – 10Jan03	8032
Canada	Canadian Dollar	4Jan71 – 10Jan03	8038
China	Yuan	2Jan81 – 10Jan03	5471
Denmark	Krone	4Jan71 – 10Jan03	8031
Euro Area	False Euro	4Jan93 – 10Jan03	2521
Finland	Markka	4Jan71 – 31Dec98	6976
France	Franc	4Jan71 – 31Dec98	7021
Germany	Deutsche Mark	4Jan71 – 31Dec98	7021
Ireland	Pound	4Jan71 – 31Dec98	7021
India	Rupee	2Jan73 – 10Jan03	7525
Italy	Lira	4Jan71 – 31Dec98	7020
Japan	Yen	4Jan71 – 10Jan03	8026
Malaysia	Ringgit	4Jan71 – 10Jan03	8010
Mexico	Peso	8Nov93 – 10Jan03	2300
Netherlands	Guilder	4Jan71 – 31Dec98	7021
New Zealand	New Zealand Dollar	4Jan71 – 10Jan03	8016
Portugal	Escudo	2Jan73 – 31Dec98	6518
Singapore	Singapore Dollar	2Jan81 – 10Jan03	5531
South Africa	Rand	4Jan71 – 10Jan03	8005
South Korea	Won	13Apr81 – 10Jan03	5416
Spain	Peseta	2Jan73 – 31Dec98	6521
Sri Lanka	Rupee	2Jan73 – 10Jan03	7172
Sweden	Krona	4Jan71 – 10Jan03	8031
Switzerland	Swiss Franc	4Jan71 – 10Jan03	8032
Taiwan	Taiwan Dollar	30Oct83 – 10Jan03	4548
Thailand	Baht	2Jan81 – 10Jan03	5428
Venezuela	Bolivar	2Jan95 – 10Jan03	2013

Table 2
Parameters α and γ for the currencies in Table 1.

Country	α	γ
Austrália	1.41487	.004656830
Áustria	1.90185	.000010368
Belgium	1.56042	.009849513
Brazil	.89059	.003707604
Britain	1.76454	.000078676
Canada	*2.04822*	.000005979
China	*4.19286*	2.5306E-11
Denmark	1.39021	.002288440
Finland	1.75114	.00040350
France	1.48668	.001021003
Germany	1.54737	.000330146
Ireland	1.61516	.000150068
India	1.87979	.000018721
Italy	1.27801	.014513000
Japan	1.43542	.011937000
Malaysia	*2.78363*	.000000169
Mexico	1.60305	.000923330
Netherlands	1.55999	.00034082
New Zealand	1.87623	.000011118
Portugal	1.33192	.00514115
Singapore	1.81272	.000027149
South Africa	*3.46313*	1.7417E-8
South Korea	.93298	.015343000
Spain	1.28282	.01962400
Sri Lanka	1.22370	.000898138
Sweden	1.53611	.000880928
Switzerland	1.68564	.000209469
Taiwan	1.19228	.003356487
Thailand	*2.03006*	.000033537
Venezuela	*4.13507*	6.6404E-9

Table 3
Parameter estimates for selected currencies.

Country	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\vartheta}$	l_c	l_{max}
Australia	1.4753	0.00180229	8.3372	0.0000	0.00000	2.2366	0.0000	0.00000	0.00428	0.0000	0.27
Brazil	0.5960	0.00157227	3.3317	0.0000	0.00000	1.0683	0.0000	0.00000	0.00277	0.0000	0.27
Britain	1.5513	0.00520366	1.0086	-3.7788	0.00000	-0.0276	2.4653	0.00000	0.00000	0.0050	0.09
Canada	1.5599	0.00181284	2.8675	-2.4219	0.00000	0.8203	3.3156	0.00000	0.00000	0.0035	0.04
Denmark	1.5117	0.02321810	4.7664	-3.4188	0.00000	0.8453	2.4294	0.00000	0.00000	0.0350	1.10
Hong Kong	0.7600	0.00085047	4.0242	-1.5663	0.00000	0.8316	4.6757	0.00000	0.00000	0.0000	0.70
Japan	1.4341	0.49351300	1.3605	0.0000	0.00000	1.9274	0.0000	0.00000	0.33850	0.7500	30.00
Mexico	1.4017	0.02356360	5.0413	-1.5195	0.00000	1.4748	2.7723	0.00000	0.00000	0.0350	1.30
New Zealand	1.4061	0.00154540	5.0986	-3.3148	0.00000	0.7782	4.4101	0.00000	0.00000	0.0000	0.22
Norway	1.5180	0.02001750	4.1884	-2.0023	0.00000	1.0801	1.3332	0.00000	0.00000	0.0350	0.50
Singapore	1.4696	0.00270447	5.4180	-0.7819	0.00000	1.4317	1.4658	0.00000	0.00000	0.0055	0.08
South Korea	0.6394	0.33273200	2.6583	0.0000	0.00000	1.1408	0.0000	0.00000	0.44520	0.0000	400.00
Sweden	1.4062	0.01853350	4.7843	-2.9571	0.00000	1.0297	2.3466	0.00000	0.00000	0.0250	1.10
Switzerland	1.5984	0.00717332	4.3446	-1.8323	0.00000	1.1556	3.2261	0.00000	0.00000	0.0125	0.17
Taiwan	1.0052	0.01627530	3.6706	0.0000	0.00000	1.6571	0.00000	0.00000	0.05930	0.0000	1.70

Estimates $\hat{\alpha}$ and $\hat{\gamma}$ are obtained by the maximum likelihood method for $\Delta t = 1$. Estimates $\hat{\lambda}_1$, $\hat{\lambda}_2$, $\hat{\lambda}_3$ $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\vartheta}$ are nonlinear least square estimates for the truncation parameters of the function using the SAS system (http://www.sas.com); and l_c and l_{max} are empirically found from our data.

Table 4
Single scaling and multiscaling.

Country	Currency	Exponent ξ	Exponent ζ
Australia	Australian Dollar	Multiscaling (2)	Multiscaling (2)
Austria	Shilling	Multiscaling (1.8)	Single Scaling
Belgium	Belgian Franc	Single Scaling	Multiscaling (0.3)
Brazil	Real	Single Scaling	Single Scaling
Britain	Pound	Single Scaling	Single Scaling
Canada	Canadian Dollar	Single Scaling	Single Scaling
China	Yuan	Multiscaling (1)	Multiscaling (1)
Denmark	Krone	Multiscaling (2)	Single Scaling
Euro Area	False Euro	Multiscaling (2.2)	Single Scaling
Finland	Markka	Multiscaling (2.1)	Single Scaling
France	Franc	Multiscaling (2)	Single Scaling
Germany	Deutsche Mark	Multiscaling (2)	Single Scaling
Ireland	Pound	Multiscaling (2)	Single Scaling
India	Rupee	Multiscaling (2)	Multiscaling (1.4)
Italy	Lira	Multiscaling (2.5)	Multiscaling (0.1)
Japan	Yen	Multiscaling (2)	Multiscaling (0.25)
Malaysia	Ringgit	Multiscaling (1.8)	Multiscaling (2)
Mexico	Peso	Multiscaling (2)	Multiscaling (1.5)
Netherlands	Guilder	Multiscaling (2.5)	Single Scaling
New Zealand	New Zealand Dollar	Multiscaling (1.5)	Single Scaling
Portugal	Escudo	Multiscaling (2.5)	Multiscaling (0.2)
Singapore	Singapore Dollar	Multiscaling (2)	Single Scaling
South Africa	Rand	Multiscaling (1)	Multiscaling (2)
South Korea	Won	Multiscaling (1.5)	Multiscaling (2)
Spain	Peseta	Multiscaling (2)	Multiscaling (~0)
Sri Lanka	Rupee	Multiscaling (~0)	Multiscaling (~0)
Sweden	Krona	Multiscaling (2)	Multiscaling (3)
Switzerland	Swiss Franc	Multiscaling (2.5)	Single Scaling
Taiwan	Taiwan Dollar	Multiscaling (1.5)	Multiscaling (0.5)
Thailand	Baht	Multiscaling (~0)	Multiscaling (0.5)
Venezuela	Bolivar	Multiscaling (1)	Multiscaling (~0)

An approximate linear behavior for all K (all $\kappa(\alpha) = |K|^{\alpha}$) gives evidence of mere single scaling. In turn, a linear behavior for initial values of $K < \alpha_o$ ($\kappa(\alpha) < \alpha_o$) followed by a nonlinear pattern after $K > \alpha_o$ ($\kappa(\alpha) > \alpha_o$) tracks the presence of multiscaling.

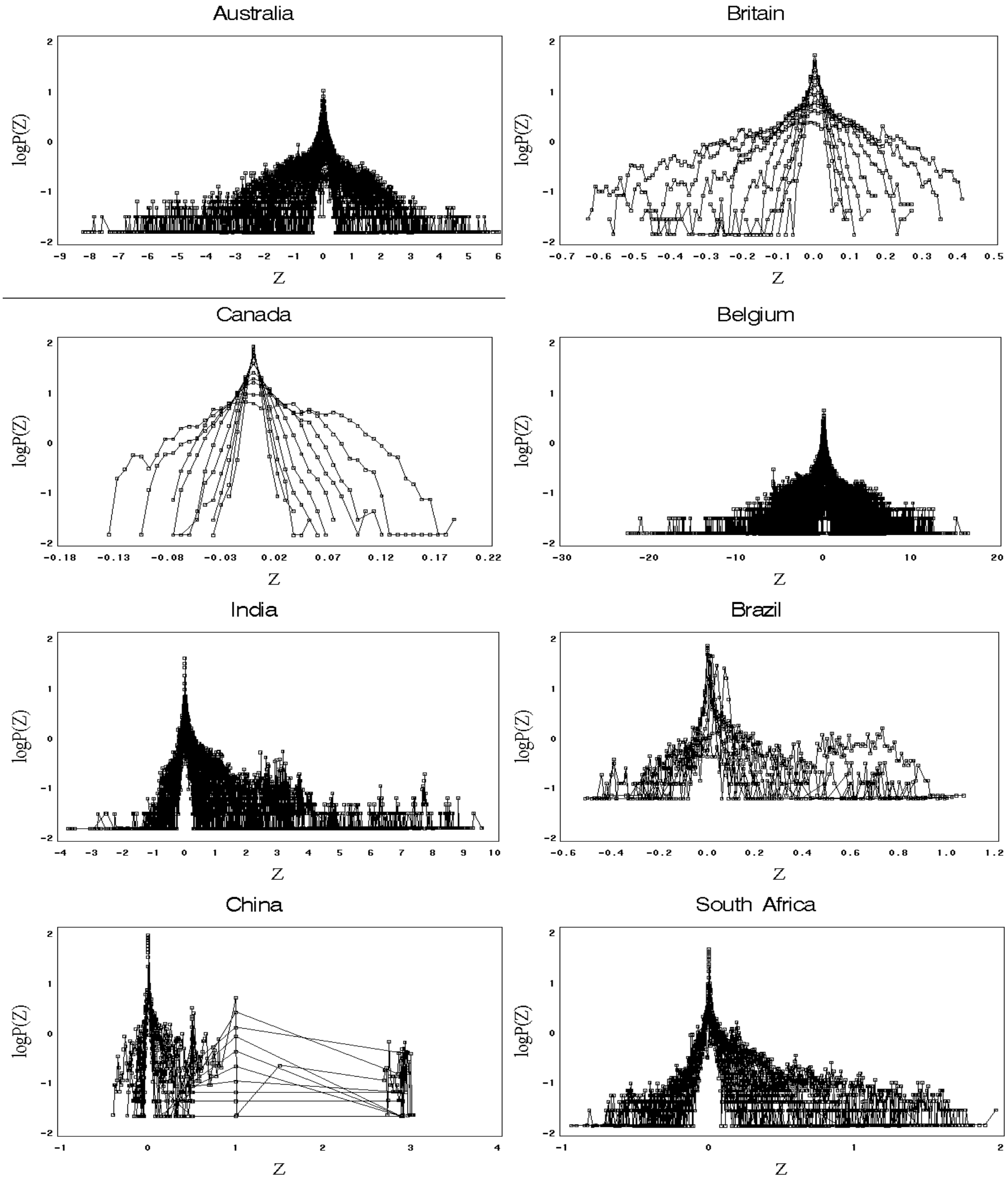

Fig. 1. Probability density functions of the currency returns of Australia, Britain, Canada, Belgium, India, Brazil, China, and South Africa observed at time intervals Δt, which range from 1 to 240 trading days. As Δt is increased, a spreading of the probability distribution characteristic of any random walk is observed.

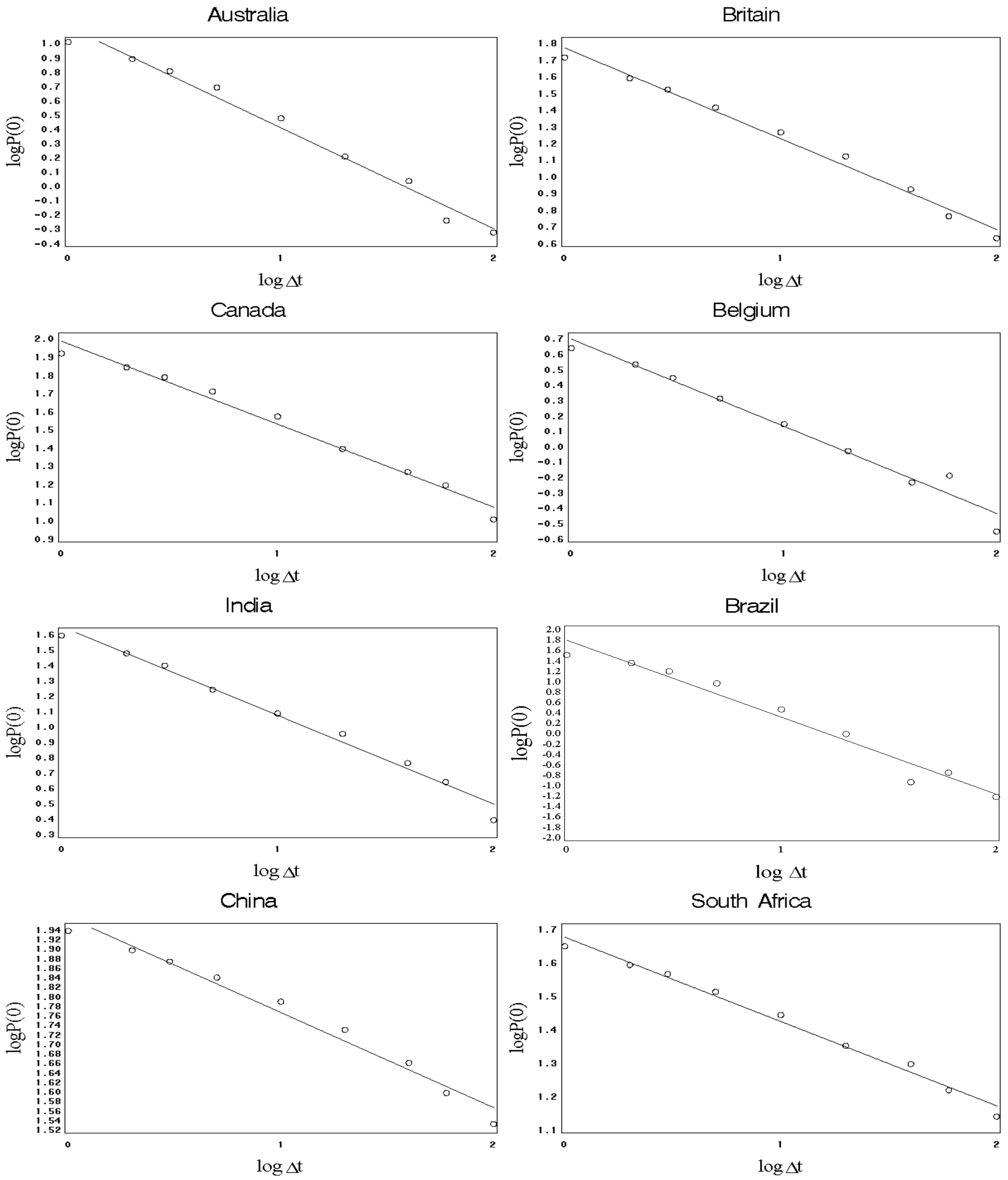

Fig 2. Log-log plot of the probability of return to the origin $P(0)$ against the time lag Δt for the currency returns of Australia, Britain, Canada, Belgium, India, Brazil, China, and South Africa. Power laws emerge within the time window of $1 \le \Delta t \le 100$. This non-Gaussian scaling is consistent with the presence of a TLF.

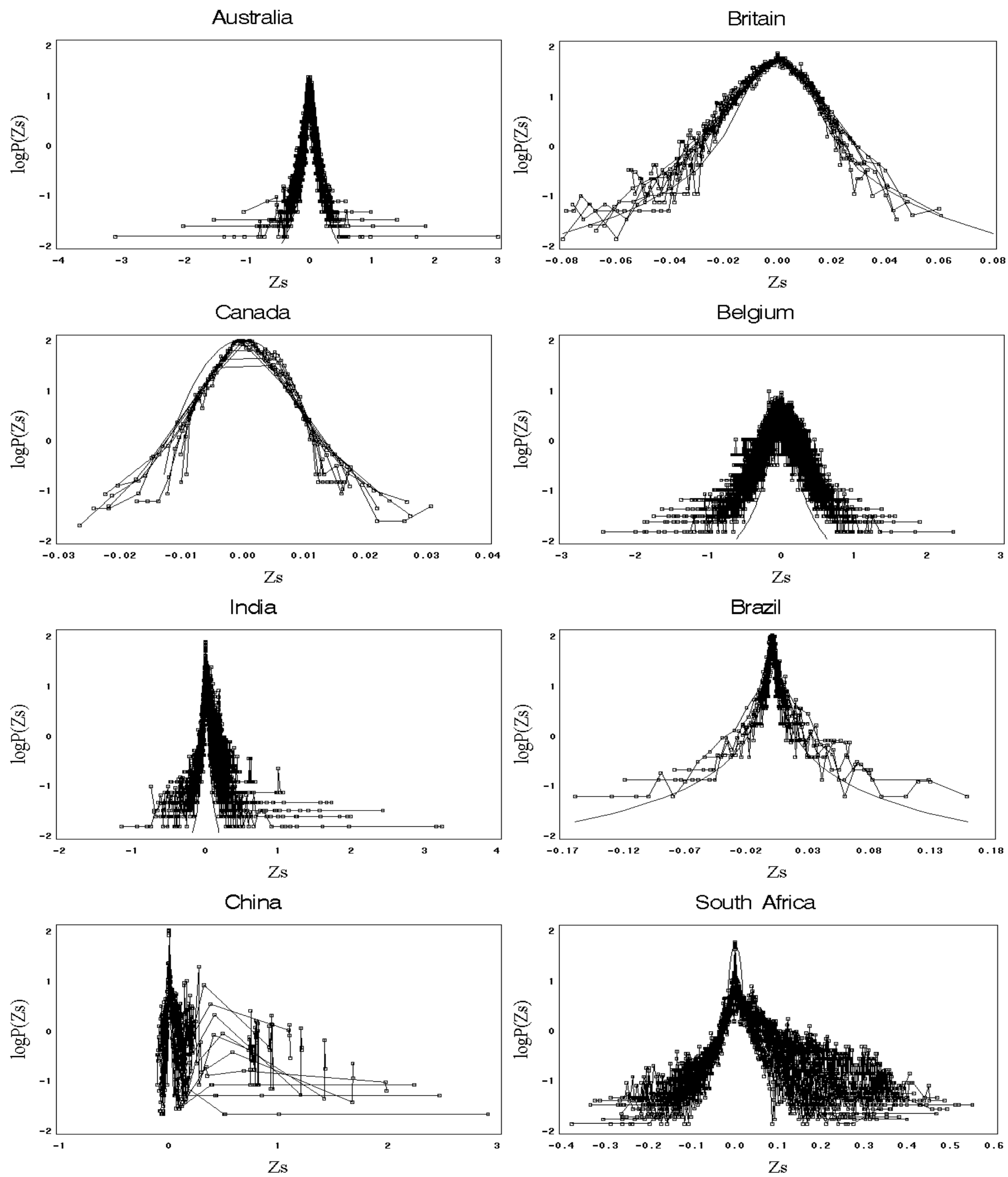

Fig. 3. The same PDFs as in Fig. 1, but now plotted in scaled units $P(Z)$. Given the scaling index α for a given currency, all the data is made to collapse onto a $\Delta t = 1$ distribution.

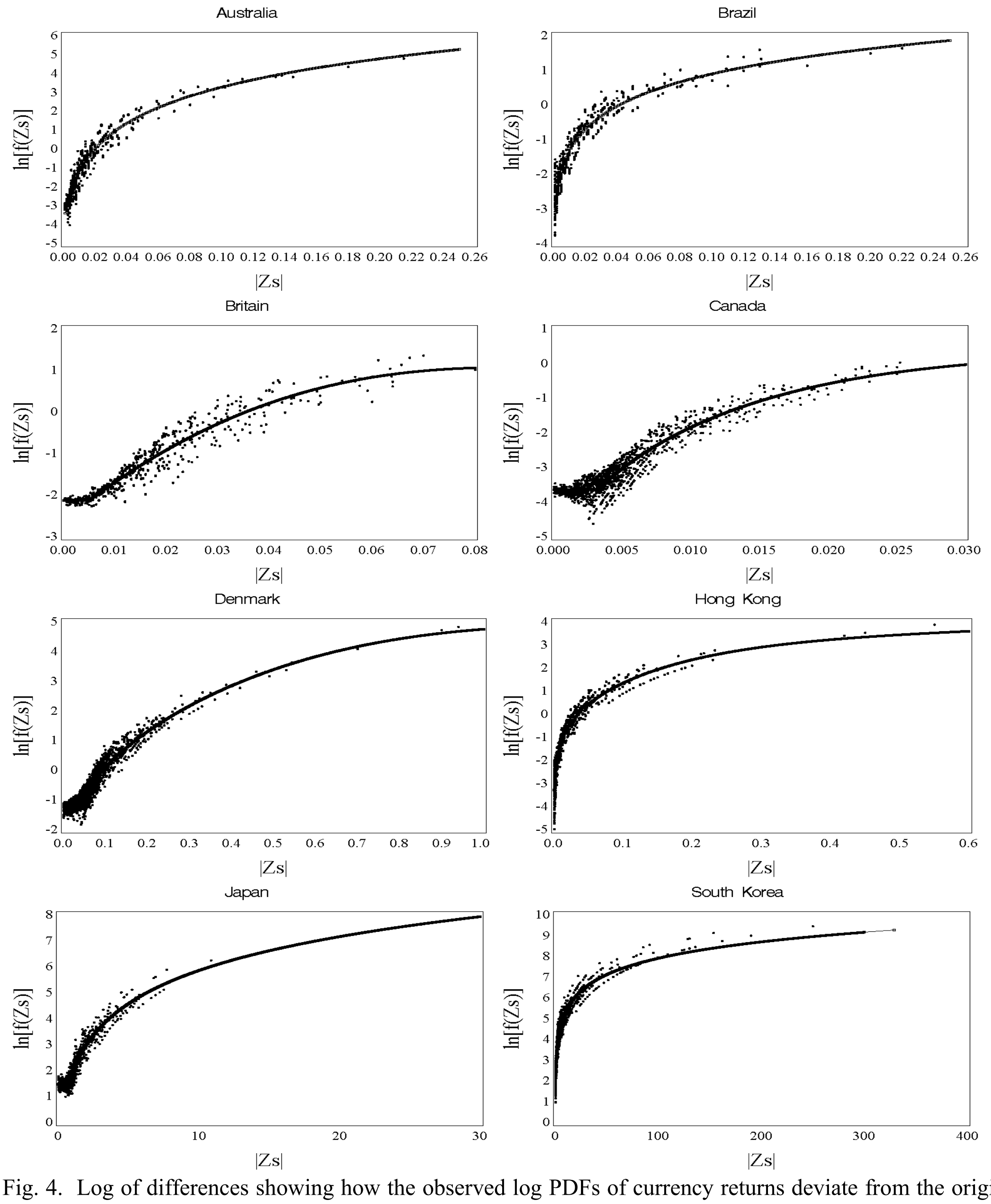

Fig. 4. Log of differences showing how the observed log PDFs of currency returns deviate from the original log Lévy process. The continuous lines are the fittings using the variance and $z_s = \Delta t^{-1/\hat{\alpha}} z_{\Delta t}$.

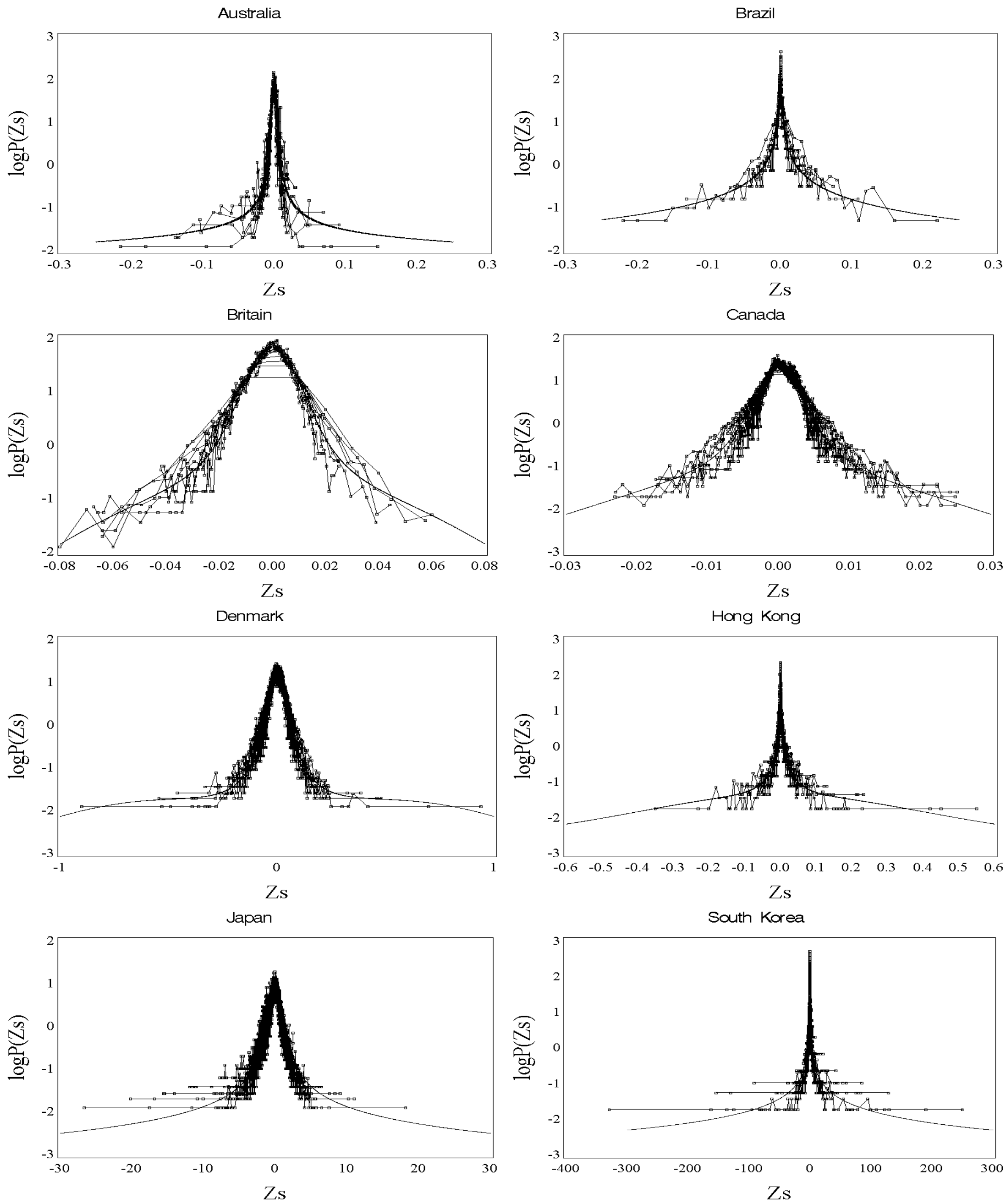

Fig. 5. The same PDFs as in Fig. 4 but now plotted in scaled units $P(Z_s)$, where $z_s = \Delta t^{-1/\hat{\alpha}} z_{\Delta t}$. Given the scaling index α for a currency, the data are made to collapse onto a $\Delta t = 1$ distribution. The curves are our suggested exponentially damped Lévy flights estimated from the data.

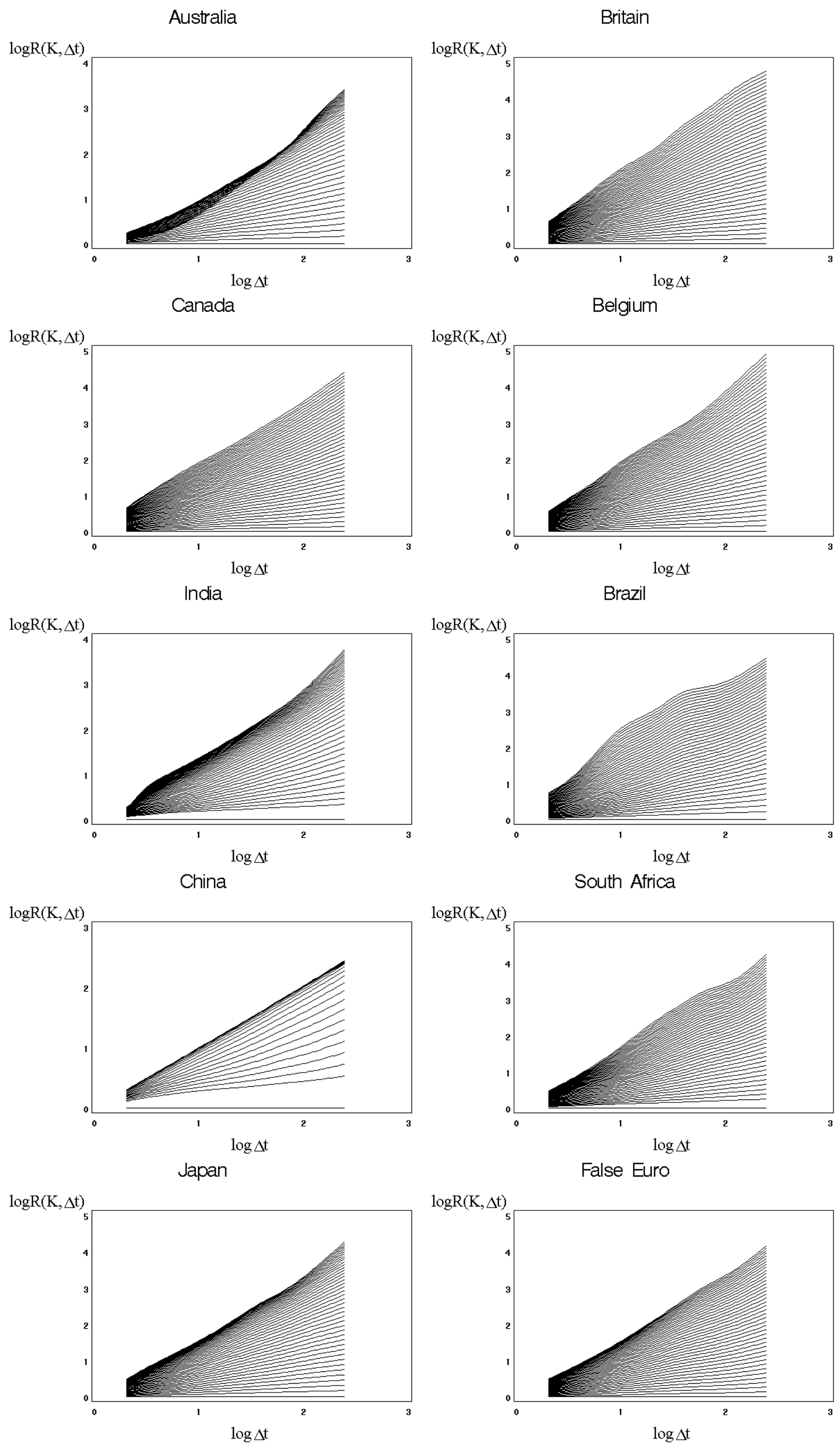

Fig.6. Estimated ratios $R(K,\Delta t)$ of selected exchange rates for K = 0.0–3.0 at intervals of 0.2. For each plot, the bottom line corresponds to K = 0.0, and the top one to K = 3.0.

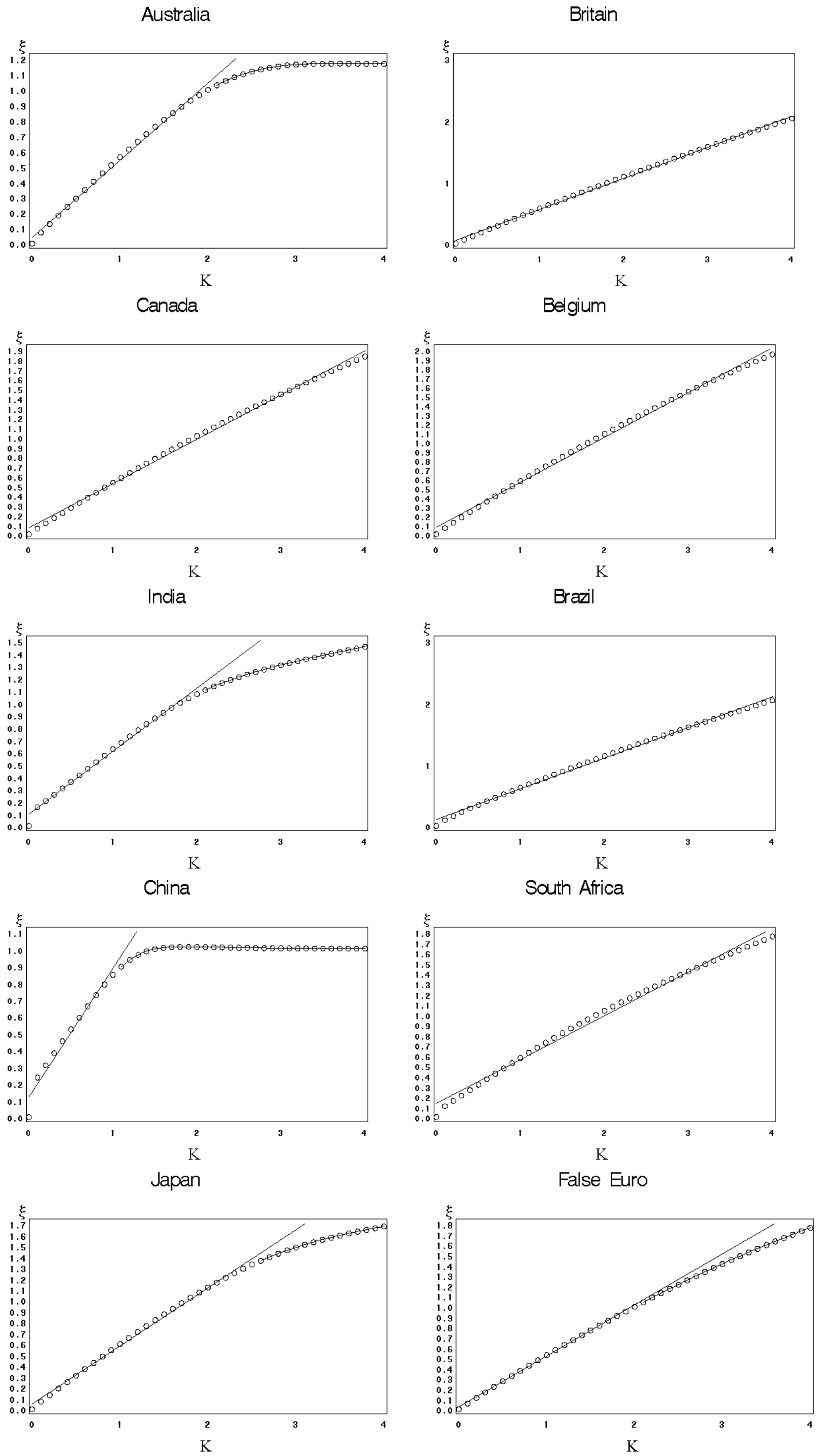

Fig. 7. Estimated multiscaling exponents ξ for selected exchange rates. An approximate linear behavior for all K gives a piece of evidence of mere single scaling. A linear behavior for initial values of $K < \alpha$ followed by a nonlinear pattern after $K > \alpha$ tracks the presence of multiscaling.

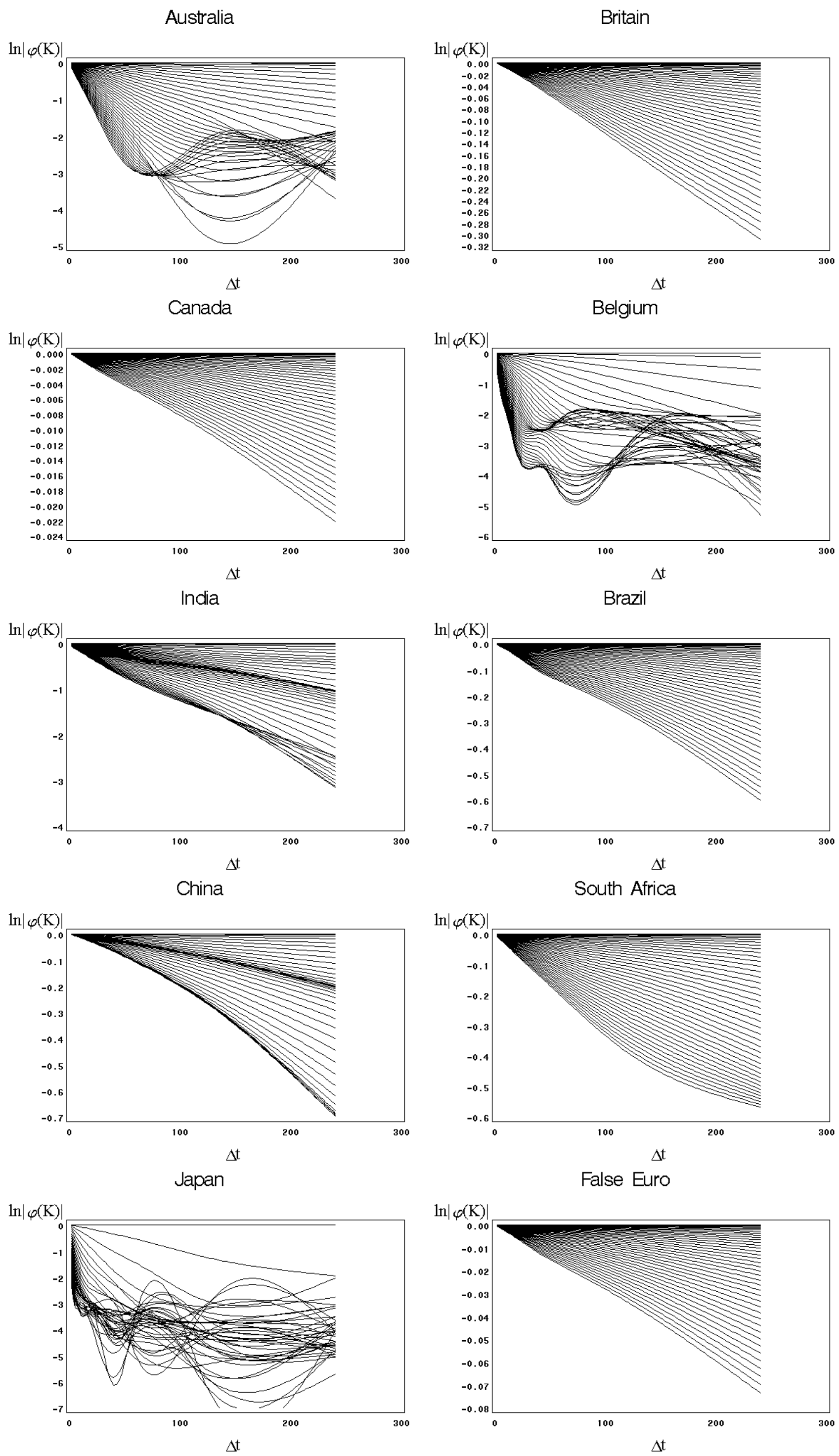

Fig. 8. Estimated ratios $\ln\|\varphi_L(K)\|$ for selected exchange rates for K = 0.0–3.0 at intervals of 0.2. For each plot, the upper line corresponds to K = 0.0, and the bottom one to K = 3.0.

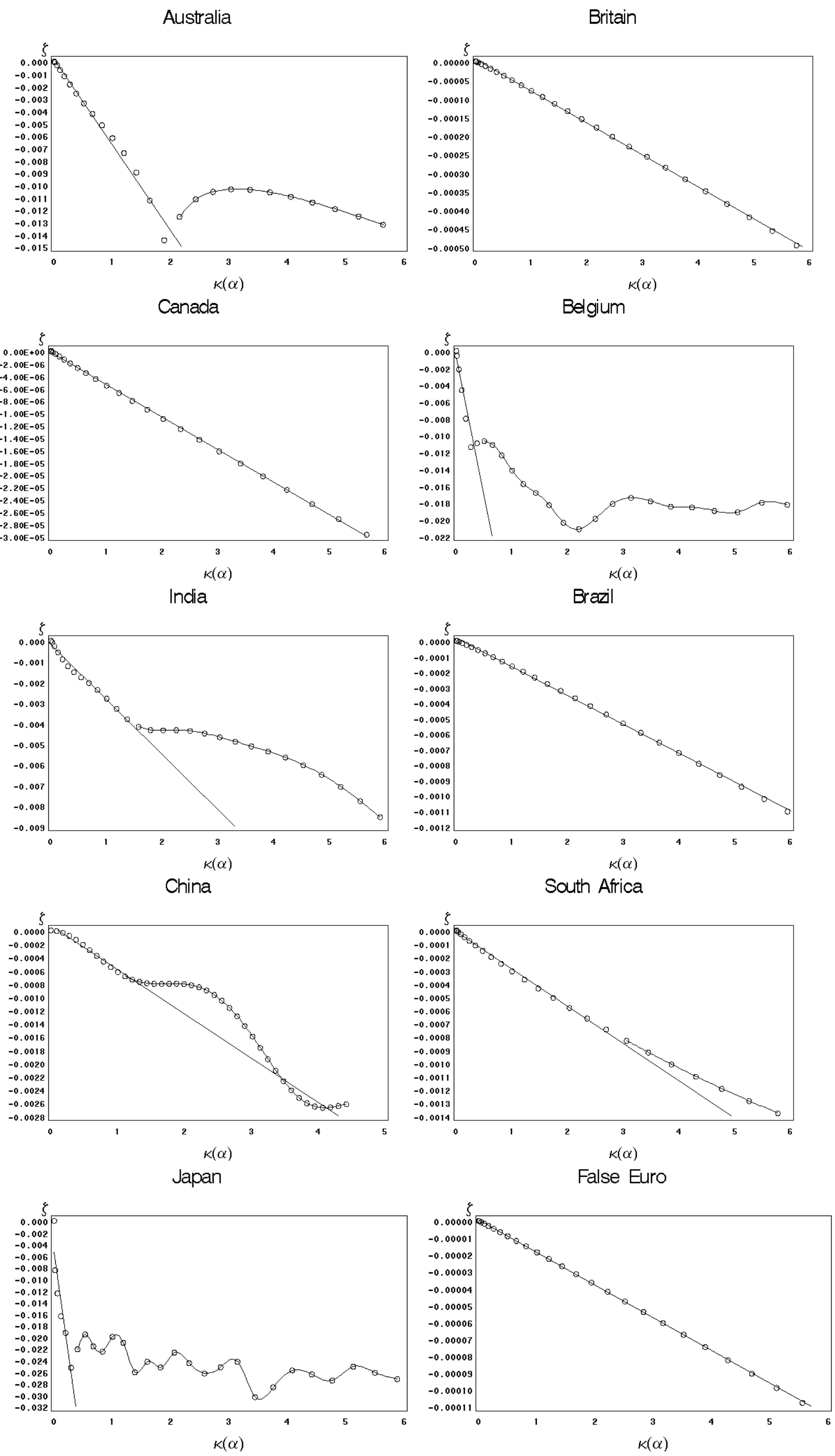

Fig. 9. Estimated multiscaling exponents ζ for selected exchange rates. An approximate linear behavior for all $\kappa(\alpha) = |K|^{\alpha}$ indicates mere single scaling. A linear behavior for initial values of $\kappa(\alpha) < \alpha_o$ followed by a nonlinear pattern after $\kappa(\alpha) > \alpha_o$ captures the presence of multiscaling.

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