

Fiscal and Monetary Interaction in Brazil

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Abstract

This paper investigates the interaction between fiscal and monetary authorities in Brazil in order to measure the degree of fiscal dominance in the Brazilian economy. To do that, a dynamic stochastic general equilibrium model is used. The model was developed for an economy with sticky prices and inflationary trend, whose parameters of interest are estimated by Bayesian inference. It is concluded that the degree of fiscal dominance in the Brazilian economy is low vis-à-vis the U.S. and Canadian economies. This result has a direct impact on the conduct of policies targeted at reducing inflation, and this probably means having to bring inflation targets down, which would directly influence the agents' expectation about future inflation.

Keywords: Fiscal dominance, DSGE, fiscal and monetary authorities, inflation.

JEL Classification: E43, E52, E63.

1 Introduction

Several research studies have looked into fiscal and monetary policies, but separately. This modus operandi implicitly assumes strong hypotheses about the behavior of unassessed policies that might not be empirically founded, resulting in equilibrium models that are a far cry from reality. Therefore, the joint analysis of monetary and fiscal policies plays a crucial role.

Recent facts have underscored the importance in evaluating the interaction between monetary and fiscal policies. Owing to the economic crisis in which the world was engulfed in 2007, governments mapped out strategies for the promotion of substantial fiscal incentives in an attempt to stave off an economic recession in their economies – including the Brazilian one – in the short run. Some of these countries are at a disadvantage because of their fast ageing

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population which, combined with fiscal incentives, warns against some strong fiscal pressure in the future. Fiscal pressure can cast some doubt upon the ability of the central bank to curb inflation and anchor inflation expectations, even if the bank follows an inflation target policy and is deeply committed towards these targets.

The lengths to which the monetary authority will go to control inflation depend on how monetary and fiscal policies are conducted, and thus, the concepts of fiscal dominance and monetary dominance take on added importance. The economy is under fiscal dominance when the fiscal authority independently determines the current and future budget, defining the share of revenues from bonds and seigniorage¹. This way, the monetary authority faces restrictions imposed by the demand for bonds issued by the government, having to finance the difference between the revenue demanded by the fiscal authority and the value of bonds sold to the public by means of the revenue obtained from the issuance of currency. Therefore, as the fiscal authority's deficits cannot be financed only by the issuance of new bonds, the monetary authority may be coerced to issue currency and to put up with some inflation.

As highlighted by Sargent and Wallace [29], even though the monetary authority gains some control over inflation, this control is less efficient than in an economy under monetary dominance. Analogously, an economy is under monetary dominance when the monetary authority defines its policy independently, determining the amount of revenue from the issuance of currency the fiscal authority will be entitled to if needed. In this case, the fiscal authority faces an additional restriction, given that any deficit in its budget must be financed by the combination of bonds sold to the public and the seigniorage determined by the monetary authority. An economy whose monetary policy predominates over the fiscal one could also be regarded as an economy in which fiscal policy is passive and monetary policy is active, as Leeper [23] puts it.

In the Brazilian economy, the conflict of interests between the Central Bank of Brazil and the National Treasury stresses the importance to assess the interaction between monetary and fiscal policies. The conflict arises from the distinct obligations of each of these organizations, which are directly bound to the same price in the economy: the interest rate.

The Central Bank of Brazil is in charge of price control in the economy. For this control, it uses the short-term interest rate as instrument, which constitutes a price that chiefly responds to changes in money supply. Thus, the bank utilizes money supply for domestic price stabilization. Therefore, its actions are somewhat free so that it can obtain the desired interest rate by applying the monetary policy.

The National Treasury is responsible for the management of both domestic and foreign public debt. This means that the National Treasury should seek to get the best financing

¹Some authors use the term seigniorage as the act of issuing currency while some use it as the revenue from the issuance of currency. In this paper, the term seigniorage refers to the latter definition.

deals for government's operation, and best financing deals here should necessarily translate into a smaller debt value and longer maturity dates. To a certain degree, that is to say that the National Treasury is in charge of applying the fiscal policy.

Because the benchmark interest rate is applied to the government's public debt, and the interest rate responds mainly to the central bank's actions, the necessity of the bank to meet its obligations may be a major hindrance to the fulfillment of obligations by the National Treasury. As an example, in times of high inflationary pressure, the Central Bank tries to reduce money supply, which eventually pushes the interest rate up. In this case, the Central Bank's action runs counter to the desires of the National Treasury.

On the other hand, the increase in public debt caused by the National Treasury may compel the Central Bank to hike the interest rate. This occurs on the aggregate demand side if the debt increase is closely related to a rise in public spending, boosting the demand for money supply and thus raising the interest rate if money supply remains constant, or it occurs because the growth of public debt leads to a crisis of confidence among the government's savers and creditors, which then want a larger premium by perceiving a greater probability of default by the government. In this case, the actions of the National Treasury have a direct influence on the Central Bank's actions.

Some studies have sought to identify the regime under which the Brazilian economy operates. Tanner and Ramos [35] assess the economic environment of the 1990s in order to make a distinction between the fiscal and monetary dominance regimes. They use a backward-looking approach to check whether the government reduces its primary deficits when its obligations increase and whether the primary deficit is consistent with the current interest rate movements. Conversely, they also use a forward-looking method to investigate whether the current reductions in primary deficit help lower the government's future obligations. The results of both methods indicate poor evidence of monetary dominance in the Brazilian economy in the analyzed period. However, some evidence in favor of monetary dominance is obtained for years 1995 to 1997, and the implementation of the Real Plan is implicated as a cause for the adoption of such regime².

Blanchard [5] also finds evidence that the Brazilian economy was under fiscal dominance in 2002 and 2003. The author points out possible counterproductive outcomes from inflation target policies due typically to the use of a monetary policy, as the economy had a high level of indebtedness and a large share of the public debt denominated in foreign currency. In these cases, the rise in interest rate would heighten the likelihood of default, causing depreciation of domestic currency, which could lead to new inflationary pressures. Therefore, inflation needed to be controlled by the fiscal authority.

²The 1990s were marked by bouts of high inflation and budget deficits in the Brazilian economy. The Real Plan was implemented in 1994 as a program for economic stabilization.

Likewise, Favero and Giavazzi [14] corroborate the idea that the default risk is a key variable to be considered by a central bank committed towards inflation targets, as the restrictions imposed by fiscal variables further limit the monetary authority's actions. Also, the authors indicate 2002 as the year in which the economy presented an unstable trend in the debt/GDP ratio, as a result of an international financial shock.

Based on the Fiscal Theory of the Price Level, Loyo [24] goes against the conventional wisdom and explores a fiscalist approach to explain hyperinflation. The author asserts that monetary policy may have an impact on fiscal variables, causing inflation to edge up, using the Brazilian economy of the 1970s and 1980s as example. Using the same theory, Rocha and Silva [27] build on the work conducted by Loyo [24], who considers the inclusion of the economy under fiscal dominance to be the underlying cause of inflationary behavior in part of the second half of the 20th century. Thus, using annual series from 1996 to 2000 and autoregressive vectors, the authors do not find evidence that the Brazilian economy was under fiscal dominance in the analyzed period.

Fialho and Portugal [15] analyze the post-Real period to verify the predominance of a fiscal dominance regime in the Brazilian economy. By using autoregressive vectors and assessing their impulse-response functions, the authors find a negative response of the government's debt to a surplus shock. This way, every time the fiscal surplus increases, the debt stock increases, characterizing a monetary dominance regime. In addition, Fialho and Portugal [15] extend the work of Muscatelli, Tirelli and Trecroci [25]³ in order to allow changes in the behavior of policies, depending on the regime under which the economy is operating. Thus, the authors use an autoregressive Markov regime shifting model and find the same result obtained with the autoregressive model.

This paper belongs with the same group of studies on the modeling of fiscal and monetary authorities' behavior available for the Brazilian economy. However, unlike other studies, it uses a dynamic stochastic general equilibrium model exclusively developed for an economy with an inflationary trend to measure the level of interaction between the two authorities. The set of tools used models the economic environment in a way that allows determining responsibilities and analyzing the actions of fiscal and monetary authorities, as well as their impact on the remaining objects of the economy.

Aside from this introduction, the paper is organized as follows. Section 2 introduces the model used for the analysis. The method for the solution and estimation of the model is described in Section 3. Section 4 presents the estimation results, including the values for the calibrated parameters and the posteriori distributions inferred by the Bayesian method. Section 5 concludes.

³Muscatelli, Tirelli and Trecroci [25] use autoregressive models to assess the response of fiscal and monetary policies to macroeconomic targets.

2 The Model

To assess the interdependence between fiscal and monetary policies, a dynamic stochastic general equilibrium (DSGE) model is used. This DSGE model was developed for an economy with sticky prices and a non-zero inflation trend, based on the work of De Resende and Rebei [12]. The model is applied to the Brazilian economy using a time series with data for years 1999 through 2009, with inference of parameters made by calibration and Bayesian techniques.

The model used herein consists of an economy comprised of a representative consumer, with an infinite horizon; a representative firm that produces a single final good (hereinafter referred to as final firm); a continuum of firms that produce intermediate goods (hereinafter referred to as intermediate firms), inserted in a market with monopolistic competition; a fiscal authority, which collects taxes from the consumer, purchases consumer goods and issues debts; and a monetary authority in charge of money supply. The latter two agents make up the government. All in all, there are three classes of agents (consumer, firms and government) as described in what follows.

2.1 Consumer

The representative consumer maximizes the current value of her current and future utility, according to her intertemporal budgetary constraint. At time t , the consumer derives revenue from the sale of her services, h_t , from the profits of her capital stock at $t - 1$, k_{t-1} , and from nominal dividends, D_t , to which she is entitled for owning the firms. After all taxes have been paid, labor, capital, dividends and government bonds are used by the consumer to consume, invest in physical capital and adjust her portfolio of financial assets, which consist of government bonds and monetary reserves.

Hence, let w_t be the consumer's real wage from the sale of h_t and r_t the capital profit rate on k_{t-1} . Then, it is possible to formalize the consumer's optimization problem as:

$$\max_{\{c_t, m_t, h_t, b_t, k_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \gamma \frac{\psi}{\psi - 1} \left(\frac{m_t}{p_t} \right)^{\frac{\psi-1}{\psi}} + \eta \log(1 - h_t) \right],$$

subject to

$$(1 + \tau_t^c) c_t + x_t + CAC_t + \frac{m_t}{p_t} + \frac{b_t}{p_t} \leq (1 - \tau_t) \left(w_t h_t + r_t k_{t-1} + \frac{D_t}{p_t} \right) + \tau_t \delta k_t + \frac{m_{t-1}}{p_{t-1} \pi_t} + i_{t-1} \frac{b_{t-1}}{p_{t-1} \pi_t}, \quad (1)$$

$$k_t = (1 - \delta)k_{t-1} + x_t, \quad (2)$$

where c_t is consumption, τ_t^c is the tax rate levied on consumption, x_t is the real investment, m_t is the nominal currency reserve, p_t is the aggregate price level and b_t is the nominal amount of government bonds at the end of time t . The inflation rate is defined by $\pi_t = \frac{p_t}{p_{t-1}}$, and i_{t-1} is the nominal interest rate of government bonds between $t-1$ and t . In addition, $\beta \in (0, 1)$ stands for the consumer's subjective discount factor, $\psi > 0$ is the elasticity of demand for currency relative to the interest rate and $\delta \in (0, 1)$ is the capital depreciation rate. The term CAC_t is consistent with the convex adjustment cost for the capital stock and is defined by $CAC_t = \left(\frac{\phi_k}{2}\right) \left(\frac{x_t}{k_{t-1}} - \delta\right)^2 k_{t-1}$, for $\phi_k > 0$.

Thus, we have the Lagrangian

$$\mathcal{H}(c_t, m_t, h_t, b_t, k_t) = \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{l} \beta^t \left[\log(c_t) + \gamma \frac{\psi}{\psi-1} \left(\frac{m_t}{p_t}\right)^{\frac{\psi-1}{\psi}} + \eta \log(1-h_t) \right] + \\ \lambda_t \left[(1-\tau_t) \left(w_t h_t + r_t k_{t-1} + \frac{D_t}{p_t} \right) + \tau_t \delta k_t + \frac{m_{t-1}}{p_{t-1} \pi_t} + \right. \\ \left. i_{t-1} \frac{b_{t-1}}{p_{t-1} \pi_t} - (1+\tau_t^c) c_t - x_t - CAC_t - \frac{m_t}{p_t} - \frac{b_t}{p_t} \right] \end{array} \right\},$$

where the first-order conditions are derived for the consumer's problem:

$$\lambda_t = \frac{1}{(1+\tau_t^c) c_t}, \quad (3)$$

$$\lambda_t = \gamma \left(\frac{m_t}{p_t}\right)^{-\frac{1}{\psi}} + \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}}\right), \quad (4)$$

$$\lambda_t = \frac{\eta}{(1-\tau_t)(1-h_t) w_t}, \quad (5)$$

$$\lambda_t = \beta \mathbb{E}_t \left(\lambda_{t+1} \frac{i_t}{\pi_{t+1}}\right), \quad (6)$$

$$\lambda_t \left[1 + \phi_k \left(\frac{x_t}{k_{t-1}} - \delta\right) \right] = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \left[1 + (1-\tau_{t+1})(r_{t+1} - \delta) + \phi_k \left(\frac{x_{t+1}}{k_t} - \delta\right) + \frac{\phi_k}{2} \left(\frac{x_{t+1}}{k_t} - \delta\right)^2 \right] \right\}, \quad (7)$$

where λ_t is the Lagrange multiplier of the consumer's budgetary constraint at t .

From the first-order conditions of the problem, we have

$$1 = \beta \mathbb{E}_t \left[\frac{(1 + \tau_t^c) c_t}{(1 + \tau_{t+1}^c) c_{t+1}} \cdot \frac{i_t}{\pi_{t+1}} \right], \quad (8)$$

$$\frac{m_t}{p_t} = \left[\gamma (1 + \tau_t^c) c_t \left(\frac{i_t}{i_t - 1} \right) \right]^\psi, \quad (9)$$

$$h_t = 1 - \frac{\eta (1 + \tau_t^c) c_t}{(1 - \tau_t) w_t}, \quad (10)$$

where equation (8) corresponds to the Euler equation, equation (9) is the consumer's demand for real reserves and equation (10) is the labor supply.

2.2 Firms

2.2.1 Final firm

As mentioned earlier, there is a single representative firm that produces a final good and a continuum of firms that produce intermediate goods. For the second set of firms, each component is indexed by $j \in [0, 1]$. Hence, the final firm uses $y_t(j)$ of units of intermediate goods produced by firm j to produce y_t units of final good following a production function with constant elasticity of substitution, as follows:

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad (11)$$

where $\theta > 1$ is the elasticity of substitution between different intermediate goods.

Therefore, the problem of the final firm is to maximize its profit:

$$\max_{y_t(j)} \left\{ p_t \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} - \int_0^1 p_t(j) y_t(j) dj \right\},$$

where p_t is the price of the final good and $p_t(j)$ is the price of the intermediate good produced by firm j .

By solving the unrestricted maximization problem, we have:

$$p_t^\theta y_t y_t(j)^{-1} = p_t(j)^\theta.$$

Then the final firm chooses the input based on the following first-order condition:

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{-\theta} y_t. \quad (12)$$

In equilibrium, the profit of the final firm is null, hence, the price of the final good is given by:

$$p_t = \left(\int_0^1 p_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (13)$$

2.2.2 Intermediate firms

The final firm uses only intermediate goods to produce the final good. However, intermediate firm j combines $k_{t-1}(j)$ units of capital, $h_t(j)$ units of labor and a technology a_t to produce $y_t(j)$ units of intermediate good j . The production functions of intermediate firms are assumed to be of the Cobb-Douglas type and are given by:

$$y_t(j) = a_t k_{t-1}(j)^\alpha h_t(j)^{1-\alpha}, \quad \forall j \in (0, 1), \quad (14)$$

where the logarithm of the level of technology is assumed to follow an AR(1) process and, in the long-term equilibrium, $a = 1$. Then:

$$\log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_{a,t}, \quad (15)$$

where $\rho_a \in (0, 1)$ and $\varepsilon_{a,t} \sim \mathcal{N}(0, \sigma_a^2)$.

To add rigidity to the price dynamics, the model proposed by Calvo [7] for price adjustment will be used. In this model, firms adjust their prices at irregular time intervals, and the opportunity for adjustment occurs as a result of an exogenous stochastic process, that is, firms do not decide when to adjust the price of their goods. Nevertheless, whenever the opportunity to adjust the price of its goods comes, the firm adjusts it in an optimized way. Thus, the j -th intermediate firm chooses $k_{t-1}(j)$, $h_t(j)$ and $p_t(j)$ in such a way that they maximize the sum discounted from the expected dividends, assuming w_t , r_t , p_t and (12) as given.

Therefore, the problem of firm j is as follows:

$$\max_{\{k_{t-1}(j), h_t(j), p_t(j)\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \mu^t \left(\beta^t \frac{\lambda_t}{\lambda_0} \right) \left(\frac{D_t(j)}{p_t} \right),$$

subject to (12), (14),

$$D_t(j) = p_t(j) y_t(j) - [p_t w_t h_t(j) + r_t k_{t-1}(j)], \quad (16)$$

$$p_{t+n}(j) = p_t(j), \quad \forall n \geq 0, \quad (17)$$

where $D_t(j)$ stands for the nominal dividends of firm j and μ^t is the probability that the price adjusted at 0 will be in force at time t . We have that $\left(\beta^t \frac{\lambda_t}{\lambda_0}\right)$ is the consumer's stochastic discount factor⁴ used to assess profits at time t .

The Lagrangian of the problem of intermediate firms is given by:

$$\mathcal{J}(k_{t-1}(j), h_t(j), p_t(j)) = \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \varphi_t(j) \left[\begin{array}{l} \mu^t \left(\beta^t \frac{\lambda_t}{\lambda_0} \right) \left(\frac{D_t(j)}{p_t} \right) + \\ p_t(j) a_t k_{t-1}(j)^\alpha h_t(j)^{1-\alpha} - \\ [p_t w_t h_t(j) + r_t k_{t-1}(j)] - D_t(j) \end{array} \right] \right\},$$

for $j \in (0, 1)$.

Therefore, the first-order conditions for the problem of the intermediate firm j are given by:

$$\varphi_t(j) = \frac{r_t k_{t-1}(j)}{(1-\alpha) y_t(j)} \quad (18)$$

$$\varphi_t(j) = \frac{w_t h_t(j)}{\alpha y_t(j)}, \quad (19)$$

$$\frac{p_t(j)}{p_t} = \left(\frac{\theta}{\theta-1} \right) \frac{\mathcal{X}_t(j)}{\mathcal{Z}_t(j)}, \quad (20)$$

where $\varphi_t(j)$ is the Lagrange multiplier for firm j which includes all the restrictions of the maximization problem, in addition to indicating its real marginal cost at time t , and $\mathcal{X}_t(j)$ and $\mathcal{Z}_t(j)$ are defined as follows:

$$\mathcal{X}_t(j) \equiv \mathbb{E}_t \sum_{n=0}^{\infty} (\mu\beta)^n \lambda_{t+n} \varphi_{t+n}(j) y_{t+n} \left(\frac{p_{t+n}}{p_t} \right)^\theta, \quad (21)$$

$$\mathcal{Z}_t(j) \equiv \mathbb{E}_t \sum_{n=0}^{\infty} (\mu\beta)^n \lambda_{t+n} y_{t+n} \left(\frac{p_{t+n}}{p_t} \right)^{\theta-1}. \quad (22)$$

Or in their recursive forms:

$$\mathcal{X}_t(j) \equiv \lambda_t \varphi_t(j) y_t + \mu\beta \mathbb{E}_t \left(\pi_{t+1}^\theta \mathcal{X}_{t+1} \right), \quad (23)$$

$$\mathcal{Z}_t(j) \equiv \lambda_t y_t + \mu\beta \mathbb{E}_t \left(\pi_{t+1}^\theta \mathcal{Z}_{t+1} \right), \quad (24)$$

⁴The stochastic discount factor is the marginal rate of intertemporal substitution of consumption in two different periods. In this case, the Lagrange multiplier refers to equation (3).

Equations (18) and (19) are conditions under which the marginal product of labor and the marginal product of capital, adjusted by the real marginal cost, equal their prices. Moreover, by combining these equations, we have:

$$\frac{k_{t-1}(j)}{h_t(j)} = \frac{1-\alpha}{\alpha} \cdot \frac{w_t}{r_t}. \quad (25)$$

This way, the ratio between capital and labor used optimally does not depend on the intermediate firm, being the same for all of them. This result implies a common marginal cost between firms, such that $\varphi_{t(j)} \equiv \varphi_t, \forall j \in [0, 1]$. Equation (20) shows the optimal relative price of the firm in a dynamic context, which implies marginal costs and benefits that are the same for all firms when they change the prices of their products.

2.3 Government

The government is an economic agent that demands resources, g_t , for its expenditures, which are determined exogenously, and for paying the interests of debts issued in the previous period. The government derives revenue by levying taxes on consumer's consumption, labor, net capital stock and dividends, and by issuing currency and bonds. Thus, we can express the government's intertemporal budgetary constraint as:

$$g_t + (i_{t-1} - 1) \frac{B_{t-1}}{p_t} = \tau_t^c c_t + \tau_t (w_t h_t + r_t k_{t-1} + d_t - \delta k_t) + \frac{(M_t - M_{t-1})}{p_t} + \frac{(B_t - B_{t-1})}{p_t}, \quad (26)$$

where $d_t = \frac{D_t}{p_t}$ is the real dividend.

We assume that the ratio between the logarithm of the values of fiscal variables and their steady-state follow an AR(1) stochastic process, so:

$$\log\left(\frac{g_t}{g}\right) = \rho_g \log\left(\frac{g_{t-1}}{g}\right) + \varepsilon_{g,t}, \quad (27)$$

$$\log\left(\frac{\tau_t^c}{\tau^c}\right) = \rho_{\tau^c} \log\left(\frac{\tau_{t-1}^c}{\tau^c}\right) + \varepsilon_{\tau^c,t}, \quad (28)$$

$$\log\left(\frac{\tau_t}{\tau}\right) = \rho_\tau \log\left(\frac{\tau_{t-1}}{\tau}\right) + \varepsilon_{\tau,t}, \quad (29)$$

where $\rho_g, \rho_{\tau^c}, \rho_\tau \in (0, 1)$ and $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$, with $\varepsilon_t = \begin{bmatrix} \varepsilon_{g,t} & \varepsilon_{\tau^c,t} & \varepsilon_{\tau,t} \end{bmatrix}'$ and $\sigma^2 = \begin{bmatrix} \sigma_{g,t}^2 & \sigma_{\tau^c,t}^2 & \sigma_{\tau,t}^2 \end{bmatrix}'$.

Subtracting the financial expenses $\left((i_{t-1} - 1) \frac{B_{t-1}}{p_t}\right)$ and the financial revenues $\left(\frac{(M_t - M_{t-1})}{p_t} + \frac{(B_t - B_{t-1})}{p_t}\right)$ from the government's dynamic budget, we can define the primary surplus and seigniorage, respectively, as:

$$s_t^\tau = \tau_t^c c_t + \tau_t (w_t h_t + r_t k_{t-1} + d_t - \delta k_t) - g_t, \quad (30)$$

$$s_t^M = \frac{(M_t - M_{t-1})}{p_t}. \quad (31)$$

Then, solving (26) and using the No-Ponzi Game condition, we obtain:

$$\begin{aligned} i_{t-1} \frac{B_{t-1}}{p_{t-1} \pi_t} &= \sum_{n=0}^{\infty} \frac{s_{t+n}^\tau}{R_t^{(n)}} + \sum_{n=0}^{\infty} \frac{s_{t+n}^M}{R_t^{(n)}}, \\ &= \mathcal{T}_t + \mathcal{S}_t, \end{aligned} \quad (32)$$

where $R_t^{(n)} \equiv \prod_{v=1}^n \left(\frac{i_{t+v-1}}{\pi_{t+v}}\right)$ is the real discount factor of market n periods ahead, then, \mathcal{T}_t is the current value of the primary surplus and \mathcal{S}_t is the current value of seigniorage. Thus, it is possible to split the government debt into a part paid with its primary surpluses and into another one paid with seigniorage.

Definition 1. Given a sequence of prices $\{i_{t-1}, w_t, r_t, p_t\}_{t=0}^{\infty}$ and an initial nominal debt stock B_{-1} , we define a Fiscal Policy in κ as a sequence $\{g_t, \tau_t^c, \tau_t, B_t\}_{t=0}^{\infty}$ such that:

$$\mathcal{T}_t = \kappa i_{t-1} \frac{B_{t-1}}{p_{t-1} \pi_t}, \quad \forall t \geq 0, \quad (33)$$

where $\kappa \in [0, 1]$.

Therefore, the government follows a policy whose constant value κ of its debt should be backed on the current value of the primary surplus. As equation (32) has to be satisfied, that is, the government's intertemporal budgetary constraint must be met, we have:

$$\mathcal{S}_t = (1 - \kappa) i_{t-1} \frac{B_{t-1}}{p_{t-1} \pi_t}, \quad \forall t \geq 0, \quad (34)$$

Therefore, if value κ of the debt is backed on the current value of the primary surplus, then, value $(1 - \kappa)$ of the debt should be backed by the current value of seigniorage.

From the definitions of \mathcal{T}_t and \mathcal{S}_t in (32), we can rewrite them as:

$$\mathcal{T}_t = s_t^\tau + \mathbb{E}_t \left(\frac{\pi_{t+1}}{i_t} \mathcal{T}_{t+1} \right), \quad (35)$$

$$\mathcal{S}_t = s_t^M + \mathbb{E}_t \left(\frac{\pi_{t+1}}{i_t} \mathcal{S}_{t+1} \right), \quad (36)$$

that is, defining them as the sum of the values of variables at t and the expectation of the current value at $t + 1$ corrected by the real discount factor of the market.

The possible set of fiscal regimes in force in an economy is made up of all regimes indexed by κ , the value of the debt backed on the primary surplus. As $\kappa \in [0, 1]$, the set is infinite and bounded, so it is interesting to analyze the regimes that match the bounds of this set.

By analyzing (33), we can notice that, when $\kappa = 1$, all of the government debt is backed on the current value of the primary surplus. In this regime, the fiscal authority is committed towards adjusting its flow of primary surplus so that it is compatible with the current value of the issued bonds. Therefore, when the monetary authority sells government bonds in the open market, the fiscal authority increases the tax burden to the consumers in order to be able to pay for the principal value of the newly acquired debt and for the interests. As a consequence, the monetary authority does not respond to a rise in public debt stock produced by a budget deficit in the government. As pointed out, this case refers to a Ricardian regime, as proposed by Sargent [28] and by Aiyagari and Gertler [1].

At the other bound, we obtain $\kappa = 0$, where, analyzing (34), we note that the whole debt is supported by the monetary authority. In this regime, the payment of the principal value and of the interests of a newly issued bond will be made by seigniorage. Thus, the fiscal authority does not worry about the monetary policy; so, taxes and expenditures do not react to changes in the government's debt stock. Sargent [28] and Aiyagari and Gertler [1] refer to this regime as non-Ricardian.

After the explanation about and analysis of the interdependence between fiscal and monetary authorities, it is possible to observe that parameter κ reflects the government's preferences concerning the support of the debt. Therefore, the Fiscal Policy in κ is a rule that can parameterize the government's behavior; hence, both an analytical and applied analysis of the relation between fiscal and monetary authorities is appropriate.

2.4 Equilibrium

Economic equilibrium is focused on a symmetric competitive equilibrium, which will be defined in what follows. In the economy, there are two different sets of intermediate firms: one made up of firms that can choose optimally, and one made up of firms that follow a non-optimal rule for price selection; however, both sets are identical in terms of the price they choose. Let

μ be the proportion of intermediate firms that are unable to adjust the price of their goods optimally at time t , then keeping the prices realized at $t-1$, and let $(1 - \mu)$ be the proportion of firms that readjust the price of their goods at time t according to equation (20). This way, we can express the relative prices of firms in the symmetric equilibrium as:

$$\frac{p_t(j)}{p_t} = \begin{cases} \frac{p_{t-1}}{p_t}, & \forall j \in [0, \mu] \\ p_t^*, & \forall j \in (\mu, 1] \end{cases}. \quad (37)$$

As the marginal cost is the same for intermediate firms, we have $\varphi_t(j) = \varphi_t \Rightarrow \begin{bmatrix} \mathcal{X}_t(j) & \mathcal{Z}_t(j) \end{bmatrix} = \begin{bmatrix} \mathcal{X}_t & \mathcal{Z}_t \end{bmatrix}$. So, we can rewrite equation (20) as

$$p_t^* = \left(\frac{\theta}{\theta - 1} \right) \frac{\mathcal{X}_t}{\mathcal{Z}_t}, \quad (38)$$

in such a way that, combining equation (38) with (13) and (37), we obtain:

$$p_t^* = \left(\frac{1 - \mu \pi_t^{\theta-1}}{1 - \mu} \right)^{\frac{1}{1-\theta}}. \quad (39)$$

Definition 2. *Given that structural shocks are realizations of known stochastic processes and considering that we know the initial stocks of currency, M_{-1} , of nominal debt, B_{-1} , and of aggregate capital, k_{-1} , a symmetric competitive equilibrium corresponds to a sequence of prices $\{i_{t-1}, w_t, r_t, p_t, p_t(j) \forall j\}_{t=0}^{\infty}$, a sequence of allocation $\{c_t, x_t, m_t, b_t, h_t, k_t\}_{t=0}^{\infty}$ and a sequence of government policies $\{g_t, \tau_t^c, \tau_t M_t, B_t\}_{t=0}^{\infty}$ such that, $\forall t > 0$, we have:*

- (i) $\frac{k_{t-1}(j)}{h_t(j)} = \frac{k_{t-1}}{h_t}$ and $D_t(j) = D_t, \forall j \in [0, 1]$;
- (ii) $\frac{p_t(j)}{p_t} = p_t^*, \forall j \in [0, \mu]$ and $\frac{p_t(j)}{p_t} = \frac{p_{t-1}}{p_t}, \forall j \in (\mu, 1]$;
- (iii) consumer and firms optimize, given the government policy and the price system;
- (iv) the government policy is consistent with its dynamic budgetary constraint and satisfies the Fiscal Policy in κ , given the price system and the choices of consumer and firms;
- (v) and the following conditions are fulfilled:

$$h_t = \int_0^1 h_t(j) \, dj, \quad (40)$$

$$k_t = \int_0^1 k_t(j) \, dj, \quad (41)$$

$$m_t = M_t > 0, \quad (42)$$

$$b_t = B_t, \quad (43)$$

$$y_t = c_t + x_t + g_t + \left(\frac{\phi_k}{2}\right) \left(\frac{x_t}{k_{t-1}} - \delta\right)^2 k_{t-1}. \quad (44)$$

Therefore, in equilibrium, economic variables are such that all intermediate firms use capital and labor in the same proportion, with the same dividend, where total capital is the sum of all capital used by intermediate firms. The same holds for total labor. In addition, in equilibrium, money supply equals the representative consumer's nominal reserve, and the nominal debt assumed by the government equals the government's bonds owned by the consumer. Finally, the output can be split into consumption, investment, public spending, and convex cost adjustment for the capital stock.

2.5 Welfare loss function

Usually, a set of equilibrium conditions includes a restriction on resources, typically $f(k_{t-1}, h_t) = F(k_{t-1}, h_t)$, where $f(\bullet)$ is the production function of intermediate firms⁵ and $F(\bullet)$ is the production function of the final firm. Nevertheless, in the model used in the present paper, this restriction is not valid due to price dispersion to the different products of the intermediate firms, as a result of the hypothesis of price stickiness.

Therefore, a welfare loss function can be extracted from the model to assess the variation in welfare by changing κ in the Fiscal Policy in κ . As pointed out by Wickens [37], the necessity to use intermediate products to produce the final product generates a loss of efficiency with the use of labor and capital in the economy, that is, there is a loss of efficiency in the use of capital and labor to produce the final good, but not to produce intermediate goods.

We can rewrite equation (12) as follows:

$$f(k_{t-1}(j), h_t(j)) = \left(\frac{p_t(j)}{p_t}\right)^{-\theta} F(k_{t-1}, h_t).$$

Integrating both sides, considering that $f(\bullet)$ is homogeneous of degree 1, $a = 1$ and conforming to (40) and (41), we have:

⁵Note that, according to (40) and (41), the production function of intermediate firms uses total capital and labor as arguments.

$$\begin{aligned} \int_0^1 f(k_{t-1}(j), h_t(j)) dj &= \int_0^1 \left(\frac{p_t(j)}{p_t}\right)^{-\theta} F(k_{t-1}, h_t) dj, \\ f\left(\int_0^1 k_{t-1}(j) dj, \int_0^1 h_t(j) dj\right) &= \int_0^1 \left(\frac{p_t(j)}{p_t}\right)^{-\theta} dj F(k_{t-1}, h_t), \\ f(k_{t-1}, h_t) &= \mathcal{L}_t F(k_{t-1}, h_t). \end{aligned}$$

where $\mathcal{L}_t \equiv \int_0^1 \left(\frac{p_t(j)}{p_t}\right)^{-\theta} dj \geq 1$.

Therefore, output loss from inefficiency is determined by:

$$\mathcal{L}_t y_t = \int_0^1 y_t(j) dj.$$

Using a result in Schmitt-Grohe and Uribe [31], it is possible to rewrite inefficiency as:

$$\mathcal{L}_t = (1 - \mu)(p_t^*)^{-\theta} + \mu \pi_t^\theta \mathcal{L}_{t-1} \quad (45)$$

Using equilibrium conditions (40) and (41), the production functions of intermediate firms and the optimal demands for capital and labor are added, such that:

$$\mathcal{L}_t y_t = a_t k_{t-1}^\alpha h_t^{1-\alpha}, \quad (46)$$

$$r_t = (1 - \alpha) \varphi_t \frac{\mathcal{L}_t y_t}{k_{t-1}}, \quad (47)$$

$$w_t = \alpha \varphi_t \frac{\mathcal{L}_t y_t}{h_t}. \quad (48)$$

Also, we have the equality condition between revenue and output, in real terms, which is given by:

$$y_t = w_t h_t + r_t k_{t-1} + d_t. \quad (49)$$

This way, the necessary conditions, including first-order conditions and restrictions on the problems of each economic agent, are the rules that govern the economic environment of interest. Thus, these conditions will constitute the dynamic system to be used in parameter estimation, yielding results of interest according to the object of analysis in this paper.

2.6 Conclusion

A dynamic stochastic general equilibrium model is used to describe the objects of the Brazilian economy in order to assess the interaction between fiscal and monetary authorities. The model is proposed for an environment with inflationary trend and price stickiness, in which the Brazilian economy is inserted. For that purpose, the economic environment is split into a representative consumer, a set of firms, and the government.

The representative consumer is an intertemporal optimizer, with a utility function that provides positive utility to consumption and real reserves and negative utility to hours worked. The consumer owns the firms, from which she derives wage revenues, dividends and interests on capital, which together with real interests on bonds and past real reserves, determine its budgetary constraint used for consumption, investment in firms, new real reserves, acquisition of new bonds and adjustment of capital stock.

The firms are classified as intermediate and final ones, but it should be recalled that there is only one final good in the economy. Intermediate firms combine capital and labor in the current technological state into a typical production function. Only some intermediate firms can readjust the price of their products optimally at a random time interval, whereas the remainder of the firms maintain their past prices in force. This feature determines price stickiness in the economic environment, which leads to a welfare loss in the economy.

The government is an agent that can be categorized into two authorities based on its activities. We define fiscal authority as the branch of the government that collects taxes from the consumer, while the monetary authority is defined as the branch of the government in charge of money supply. Using the government's intertemporal budgetary constraint, it was possible to categorize its debt into one that is paid by primary surplus revenues and one paid by seigniorage, allowing us to define the Fiscal Policy in κ , which plays a central role in the analysis of the interaction between fiscal and monetary authorities. Finally, there is aggregation of the economy and its equilibrium conditions, with the aim of complementing the model.

3 Empirical Method

The empirical analysis of DSGE models should obligatorily go through two stages that complement each other. The first stage is related to the preparation of the model to be analyzed, which includes its solution. The method used in this stage is described in Appendix A. The second stage involves the preparation of data and parameter estimation, which are shown in what follows.

The estimation of the log-linearized model⁶ is based on quarterly and real per capita data on the government's total debt, on output and on private consumption, in addition to quarterly data on inflation. The model is estimated for the Brazilian economy between 1999 and 2009, and the data are available from the database of the Institute of Applied Economic Research (IPEA, www.ipeadata.gov.br) and of the Central Bank of Brazil (BCB, www.bcb.gov.br). Parameters $\alpha, \delta, \eta, \gamma, \beta$ and θ will be calibrated and parameters $\rho_a, \rho_g, \rho_{\tau^c}, \rho_{\tau}, \sigma_a, \sigma_g, \sigma_{\tau^c}, \sigma_{\tau}, \psi, \phi_k$ and κ will be estimated using Bayesian techniques⁷.

To find the solution to the model to be estimated, a state-space representation is obtained from the procedure developed by Blanchard and Kahn [6]. This representation is possible due to the log-linearization process, which allows for the analysis of the model by the Kalman filter. So to speak, the solution to the model in state-space form consists of a transition equation for the vector of endogenous state variables and of exogenous shocks, and a vector of equations that maps the state variables in the observed variables that will be used to estimate the model.

Once the solution to the model in state-space form is found, the estimation procedure consists of three stages: calibration of some parameters and proportion of steady-state variables, determination of priori distributions of parameters to be estimated using Bayesian techniques and likelihood estimation and use of an MCMC algorithm to numerically calculate the moments of posteriori distributions of the remaining parameters.

Calibration in macroeconomics consists in choosing parameters that equate moments of the model with those observed in the data. Nonetheless, certain parameters can be replicated from other studies, essentially those which do not contain an observed series, as in Hansen [18], who defined the parameters in his model based on parameter values obtained by Kydland and Prescott [22].

In this regard, parameters α and δ , which represent capital and the capital depreciation rate, respectively, are calibrated following the values obtained by Christiano, Eichenbaum and Evans [8]. On the other hand, parameter θ , which denotes the elasticity of substitution of the final firm, is calibrated following the value found by Basu [4], whereas the preference parameter γ is calibrated based on the value obtained by De Resende and Rebei [12] for the Canadian economy.

⁶The log-linearization technique was proposed in the context of real business cycles by King, Plosser and Rebelo [21], and consists in using the Taylor approximation around the steady-state value of the variable. For further details about the technique, see Uhlig [36].

⁷Some econometric procedures are used to parameterize and to assess DSGE models, such as calibration (for instance, Kydland and Prescott [22]) and the generalized method of moments (for instance, Christiano and Eichenbaum [10]). However, the Bayesian analysis will be used, whose strengths, according to An and Schorfheide [3] include the fact that the analysis is based on a system of equations, adjusting the DSGE model to a vector of aggregate time series and that the priori distributions can be used to add further information on parameter estimation.

Parameter η , which denotes the elasticity of labor supply, is calibrated using equations (3) and (5), whereas parameter β , que which stands for the consumer’s intertemporal discount rate, is calibrated using equation (6), with the inflation rate being calibrated as the historical mean for the period, as well as the nominal interest rate.

Table 1 summarizes the calibration procedure and the estimated parameter values.

Parameter	Definition	Estimate	Based on
α	Capital share	0.360	Christiano at. al. [8]
δ	Capital depreciation rate	0.025	Christiano at. al. [8]
θ	Elasticity of substitution	8.00	Basu [4]
γ	Preference parameter	5.91×10^{-6}	De Resende and Rebei [12]
η	Elasticity of substitution of labor supply	2.046	Equations (3) and (5)
β	Rate of intertemporal substitution	0.973	Equation (6)

Table 1: Parameter calibration

In addition to these parameters, some variables should have their steady-state values calibrated after the log-linearization of the model. As a matter of fact, calibration is performed on the ratio between these variables and output, except for tax rates. Therefore, the tax rate on consumption, represented by τ^c , is calibrated using the mean of the ratio between taxes on production and gross domestic product (GDP) at basic prices, whereas the tax rate on revenue, represented by τ , is calibrated using the mean of the ratio between taxes on revenue and the property on capital on the GDP at basic prices.

The public consumption ratio is calibrated using equation (44), after calibration of investment and private consumption ratios. The former is calibrated using historical data for the ratio between the sum of the gross fixed capital formation and stock variation over the GDP, whereas the latter is calibrated with historical data for the household consumption to GDP ratio. The dividend payout ratio is calibrated using equation (49), after calibration of wage revenue ratio with historical data on the wage revenue to GDP ratio and the capital spending ratio using the interest revenue to GDP ratio. Finally, the public spending and currency ratios are calibrated using historical data on the M1 money supply to GDP ratio.

Table 2 summarizes the calibration procedure for tax rates and for the ratios and their estimated steady-state values.

Parameter	Definition	Estimate	Based on
τ	Income tax	0.192	Production taxes/GDP
τ_c	Consumption tax	0.177	Income and capital tax/GDP
g	Government consumption	0.203	Equation (44)
x	Investment	0.173	GFCF and stock variation/GDP
c	Private consumption	0.624	Household consumption/GDP
d	Dividends	0.066	Equation (49)
wh	Labor expenses	0.797	Wage revenue/GDP
rk	Capital expenses	0.137	Capital revenue/GDP
b	Government debt	0.482	Net public debt/GDP
m	Currency	0.121	M1/GDP

Table 2: Calibration of tax rates and of ratios in steady state

The priori distributions have an important role for the estimation of DSGE models, with some advantages. One of the advantages is that their determination makes it possible to reduce the weight of parameter space regions that are not consistent with the economic theory, in addition to increasing the curvature of the likelihood function which could be almost flat in some dimensions of the parameter space, which would hinder the identification of a maximum by numerical methods.

In principle, priori distributions can be chosen based on the researcher's beliefs that reflect the validity of economic theories. However, in practice, most priori distributions are chosen based on observations. Hence, the priori distributions used in this paper follow those employed by De Resende and Rebei [12]. The authors follow the work of Smets and Wouters [34] and use the beta distribution for parameters that may take on values on the interval $[0, 1]$, the gamma distribution for parameters that have strictly positive values and the inverse gamma distribution for the standard deviations of structural shocks.

According to previous studies⁸, the point estimates for the autoregressive coefficient of the stochastic process guiding technology are on the interval $[0.60, 0.98]$, whereas for government spending, this interval corresponds to the interval $[0.76, 0.96]$. For standard deviations, studies indicate the interval $[0.004, 0.060]$ as the most likely for the estimations. Thus, the priori means for the values of ρ_a , ρ_g , ρ_τ and ρ_{τ_c} are all defined as 0.80, with the standard deviations of the first two defined as 0.01 and of the last two defined as 0.02. The priori standard deviations of

⁸See Dib [13], Ambler, Dib and Rebei [2], Smets and Wouters [34] and Christiano, Eichenbaum and Evans [8].

the latter group are larger as there is no previous study on autoregressive coefficients of the movements of tax rates, causing them to vary substantially.

Given the calibrated value of the preference parameter γ , the elasticity of demand for currency, ψ , has priori mean of 0.25, which is consistent with that reported in the literature⁹. For the priori mean of the capital adjustment cost parameter, ϕ_k , a value of 10, is assumed, which is estimated value for this parameter in Ortega and Rebei [26]. The priori mean for the parameter that denotes the proportion of firms that are unable to adjust their prices, μ , is defined as 0.75, as in Smets and Wouters [34]. Finally, the priori mean for parameter κ is defined as 0.80, based on the posteriori means estimated by De Resende and Rebei [12].

Table 3 summarizes the characteristics of the priori distributions.

Parameter	Definition	Mean	Variance	Distribution	Interval
ρ_a	Technology autocorrelation	0.80	0.01	Beta	[0, 1]
ρ_g	Public spending autocorrelation	0.80	0.01	Beta	[0, 1]
ρ_{τ^c}	Tax on consumption autocorrelation	0.80	0.02	Beta	[0, 1]
ρ_{τ}	Tax on income and capital autocorrelation	0.80	0.02	Beta	[0, 1]
σ_a	Technology standard deviation	0.01	4.00	Inversa Gamma	$[0, \infty)$
σ_g	Government spending standard deviation	0.01	4.00	Inversa Gamma	$[0, \infty)$
σ_{τ^c}	Tax on consumption standard deviation	0.02	4.00	Inversa Gamma	$[0, \infty)$
σ_{τ}	Tax on income and capital standard deviation	0.02	4.00	Inversa Gamma	$[0, \infty)$
ψ	Elasticity of demand for currency	0.25	0.20	Gamma	$[0, \infty)$
ϕ	Capital adjustment cost	10.00	5.00	Gamma	$[0, \infty)$
μ	Level of price stickiness	0.80	0.10	Beta	[0, 1]
κ	Fiscal dominance	0.80	0.10	Beta	[0, 1]

Table 3: Priori distributions

The likelihood function is created as the last estimation step using the Kalman filter, and is weighted by the priori density of the parameters. Its maximization provides the mode of the parameters, which is used in the initial step of the Metropolis-Hastings algorithm, which numerically calculates the moments of the posteriori distribution of the parameters. Table 4 shows the results obtained in this stage.

⁹See works cited in footnote 8.

Variable	Definition	Estimate	95% confidence interval
ρ_a	Technology autocorrelation	0.9945	[0.9896, 0.9989]
ρ_g	Public spending autocorrelation	0.9205	[0.8716, 0.9479]
$\rho_{\tau c}$	Tax on consumption autocorrelation	0.7507	[0.7403, 0.7619]
ρ_{τ}	Tax on income and capital autocorrelation	0.7656	[0.7348, 0.7958]
σ_a	Technology standard deviation	0.0916	[0.0710, 0.1113]
σ_g	Government spending standard deviation	0.2968	[0.2479, 0.3435]
$\sigma_{\tau c}$	Tax on consumption standard deviation	0.2144	[0.1762, 0.2626]
σ_{τ}	Income tax and capital standard deviation	0.0584	[0.0486, 0.0680]
ψ	Elasticity of demand for currency	0.4447	[0.4382, 0.4507]
ϕ	Capital adjustment cost	6.5991	[5.5502, 7.4802]
μ	Price stickiness	0.9099	[0.8790, 0.9411]
κ	Fiscal dominance	0.9330	[0.9318, 0.9342]

Table 4: Results of the posteriori distributions of the parameters

The results of posteriori densities suggest that technology shocks are highly persistent. The persistence of technology shocks may be due to the fact that the Brazilian economy has come out of a lag in technological development in recent decades, and therefore, the series that denotes technology has an ascending path, which results in a nonstationary series.

With respect to government accounts, shocks to its spending also show persistence, having the highest volatility among the shocks analyzed in this paper. The shocks to the tax rates also have persistence, but at a lower level. The shocks to tax on consumption were nearly four times more volatile than the shocks to the tax on capital and income. This fact may be connected to fluctuations in the world economy in 2001 and 2007, as household consumption is one of the most volatile components of output and the model used in the paper is for a closed economy, that is, abrupt results for international variables that influence domestic variables are obtained by means of their shocks.

As to the level of price stickiness and the level of fiscal dominance, it is important to compare the results obtained for the Brazilian economy with those assessed by De Resende and Rebei [12]. Table 5 shows the values found for the economies, in addition to the annual inflation rate (consumer prices) for 2001 to 2009.

	Brazil	USA	Canada	Mexico	South Korea
Fiscal dominance	0.9333	0.9664	0.9860	0.6286	0.7820
Inflation	6.73	2.78	2.27	5.37	3.03

Tabela 5: Comparisons across economies

The estimation results suggest that Brazil is under low fiscal dominance, at a higher level of dominance, but similar to those found for the U.S. and Canada. According to Loyo [24], economies under low fiscal dominance, i.e., which have large κ , values, have less difficulty in attaining their goals regarding inflation target policies.

In fact, the Brazilian economy was successful in the management of inflation, keeping it within its target and within the lower and upper target levels, except for 2001, 2002, and 2003, when a loss of confidence in the economy by foreign investors swept over the nation. On the other hand, the Mexican economy, which has a 3% inflation target (with 2% and 4% target levels), ended 2010 with a 4.4% inflation rate. The South Korean economy has presently faced some difficulty keeping inflation around the target level.

4 Conclusion

The literature on fiscal and monetary theories is up-to-date and relevant and these theories have been tested and analyzed by the modern macroeconomic approach, which deals with dynamic stochastic general equilibrium models. Therefore, this paper assessed the Brazilian economy as a whole, especially the government's behavior, in order to investigate the relationship between fiscal and monetary authorities to determine to what extent the actions of an authority limits those of another authority.

To achieve the proposed goal, the paper addresses an economy characterized by a representative consumer, which supplies labor and capital for a continuum of firms that produce intermediate goods that will be used as inputs by a single firm that produces final goods. In turn, the government is comprised of a fiscal authority and a monetary authority, whose actions are interconnected to support the public debt.

The relevant issue for the objective of this paper is focused on what is defined as Fiscal Policy in κ , which determines how the government supports its debt, paid either by way of primary surpluses or seigniorage. Determination of the share of the debt paid by primary surpluses defines the level of fiscal dominance of the economy. Thus, instead of assuming that an economy is inserted in a fiscal dominance environment, we can just define to what extent the fiscal authority predominates over the monetary one.

The model proposed here has its linearized form estimated by Bayesian methods, in addition to having some parameters calibrated. The estimation results reveal that the Brazilian economy is under low fiscal dominance compared to the U.S. and Canadian economies. According to the existing literature, this environment could be the major reason for the equilibrium in the domestic economy, as well as for the success in the management of the inflation targets proposed by the monetary policy.

Therefore, the Brazilian monetary authority enjoys great freedom to fight inflation, often succeeding in its monetary policy goals. However, the Brazilian economy still has a high inflation rate and a high interest rate for international standards. Thus, the decrease in inflation should have to include commitment by the authority towards pursuing a lower inflation target than the current one, as the expectation of agents about inflation is directly influenced by the target.

Finally, a limitation of this study lies in the fact that the model used was developed for a closed economy. Even though the Brazilian economy is not largely open, external events have an important impact on domestic price levels, especially through the exchange rate. This way, the development of a model that comprises the foreign market is of great value for capturing the level of fiscal dominance of the Brazilian economy, as well as its impacts on the price level.

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Appendix A - Approximation and Solution to the Model

The system represented by the first-order equations is made of nonlinear rational expectations equations, whose solution includes functions of unknown forms with equilibrium conditions with integers that do not usually allow for explicit solutions. A way to circumvent this problem is to use numerical methods to approximate the true solution to the system. In this respect, there are several methods¹⁰ that approximate and solve DSGE models with the use of approximation techniques.

Among the most widespread methods is one that linearizes the system of necessary equations by Taylor series expansion and then applies the available algorithms to find the solutions, as with the models developed by Blanchard and Kahn [6], Uhlig [36], Christiano [9] and Sims [33]. These methods use a first-order approximation¹¹ for the equilibrium functions, providing adequate answers to questions about the existence of equilibrium and about the analysis of second moments of endogenous variables, in addition to not succumbing to the “curse of dimensionality”¹², allowing for problems with a large number of state variables to be analyzed without further computational costs.

This paper utilizes disturbance methods, as in Schmitt-Grohé and Uribe [30], to solve the model by first-order approximation. The method is summarized in what follows.

The set of equilibrium conditions of a DSGE model can be written as

$$\mathbb{E}_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0, \quad (50)$$

where \mathbb{E}_t denotes the expectation operator based on the informational set at t , x_t is a vector of predefined variables of order $n_x \times 1$ and y_t is a vector of endogenous variables of order $n_t \times 1$ to be determined at t .

The vector x_t can be partitioned as $x_t = \begin{bmatrix} x_{1,t} & x_{2,t} \end{bmatrix}'$, where $x_{1,t}$ consists of a vector of predefined endogenous state variables and vector $x_{2,t}$ consists of a vector of exogenous state variables. It is assumed that exogenous variables follow the stochastic process given by

$$x_{2,t+1} = \Lambda x_{2,t} + \tilde{\eta} \sigma \varepsilon_{t+1},$$

where vectors $x_{2,t}$ and ε_t are of order $n_\varepsilon \times 1$, ε_t is identically and independently distributed with zero mean and variance matrix I_{n_ε} , $\sigma \geq 0$ is a known scalar, $\tilde{\eta}$ is a known $n_\varepsilon \times n_\varepsilon$ matrix

¹⁰Disturbance, projection, parameterized expectations, and iteration of the value function are some of the methods used. An explanation of their implementation is found in Judd [20] and Heer and Maussner [19].

¹¹There are methods that use higher-order approximations that provide accurate approximations to the solutions of DSGE models by direct expansion of the political function of the problem. The studies conducted by Sims [32], Collard and Juillard [11] and Schmitt-Grohé and Uribe [30] refer to these methods.

¹²It is related to the exponential increase of the space dimension in which the problem should be analyzed due to the linear increase in the number of variables to be considered by the problem.

and Λ is a $n_\varepsilon \times n_\varepsilon$ matrix in which all eigenvalues have a modulus smaller than one.

The solution to the model given by (50) is

$$y_t = g(x_t, \sigma), \quad (51)$$

$$x_{t+1} = h(x_t, \sigma) + \eta \sigma \varepsilon_{t+1}, \quad (52)$$

where η is a $n_x \times n_\varepsilon$ matrix defined by $\eta = \begin{bmatrix} 0 & \tilde{\eta} \end{bmatrix}'$.

Therefore, the objective of this method is to find a second-order approximation of functions g and h around the steady state $\begin{bmatrix} x_t & \sigma \end{bmatrix} = \begin{bmatrix} \bar{x} & 0 \end{bmatrix}$. Consequently, with the help from the Taylor series expansion, we have

$$\begin{aligned} g(x_t, \sigma) &\approx g(\bar{x}, 0) + g_x(\bar{x}, 0)(x_t - \bar{x}) + g_\sigma(\bar{x}, 0)\sigma \\ &\quad + \frac{1}{2} \left[g_{xx}(\bar{x}, 0)(x_t - \bar{x})^2 + 2g_{x\sigma}(\bar{x}, 0)(x_t - \bar{x})\sigma + g_{yy}(\bar{x}, 0)\sigma^2 \right], \end{aligned} \quad (53)$$

$$\begin{aligned} h(x_t, \sigma) &\approx h(\bar{x}, 0) + h_x(\bar{x}, 0)(x_t - \bar{x}) + h_\sigma(\bar{x}, 0)\sigma \\ &\quad + \frac{1}{2} \left[h_{xx}(\bar{x}, 0)(x_t - \bar{x})^2 + 2h_{x\sigma}(\bar{x}, 0)(x_t - \bar{x})\sigma + h_{yy}(\bar{x}, 0)\sigma^2 \right]. \end{aligned} \quad (54)$$

Using equations (51) and (52) in equation (50), we have:

$$F(x, \sigma) \equiv \mathbb{E}_t f(g(h(x, \sigma) + \eta \sigma \varepsilon', \sigma), g(x, \sigma), h(x, \sigma) + \eta \sigma \varepsilon', x) = 0, \quad (55)$$

where the time subscript is omitted, and the apostrophe indicates the variable at $t + 1$.

Finally, with the Implicit Function Theorem applied to equation (55) and given that $F_{x^i \sigma^j}(x, \sigma) = 0$, for all x , σ , j and k , where $F_{x^i \sigma^j}(x, \sigma)$ denotes the F derivative in relation to x taken i vezes and related to σ taken j times, it is possible to obtain the unknown coefficients of approximations (53) and (54).

This way, Schmitt-Grohé and Uribe [30] derive a second-order approximation for the political function of a general class of DSGE models without following a specific formulation for the value function. The authors use disturbance¹³ methods that include a scalar parameter for the variance of exogenous shocks as argument for the political function, eliminating the certainty equivalence principle that arises in models with first-order approximation.

¹³The disturbance method is well explained in Judd's book [20, chapters. 13, 14 e 15].

Appendix B - Bayesian Inference

Bayesian inference is based on simple probability rules and differs from classic inference. While in classic inference the probability of an event is the limit of its relative frequency and the parameters are fixed, for Bayesian inference, the probability, in general, captures the beliefs of a researcher about an event and the parameters are random variables with probability distribution.

The interest in Bayesian inference is focused on parameter distribution after observation of the data. Therefore, using the Bayes' theorem, we have

$$p(\theta|y) = \frac{f(y|\theta)p(\theta)}{f(y)}, \quad (56)$$

where θ is the vector of parameters of interest, y are the observed data and $f(y) = \int f(y|\theta)p(\theta) d\theta$ is the marginal distribution of y .

Thus, there is interest in using the data to learn about the parameters of the model¹⁴. As $f(y)$ does not rely directly on θ , it is possible to rewrite (56) as

$$p(\theta|y) \propto f(y|\theta)p(\theta), \quad (57)$$

where $p(\theta|y)$ is the posteriori density, $f(y|\theta)$ is the likelihood function and $p(\theta)$ is the prior density. Hence, we can estimate the likelihood function with the Kalman filter and simulate the posteriori distribution by Monte Carlo methods, more precisely, by the use of the Metropolis-Hastings (MH) algorithm. Since the Kalman filter method is explained, to a certain extent, in most Economics courses,¹⁵ here we focus on the MH algorithm.

The MH algorithm¹⁶ is a general simulation procedure that allows sampling distributions that are analytically intractable. So, from the samples and using the Law of Large Numbers, it is possible to obtain the mean and the posteriori variance of the estimators of θ .

The MH algorithm is as follows:

- Step 1: start the counter for $j = 1$ and choose an initial value θ^0 (the posteriori mode¹⁷ is indicated);
- Step 2: take θ^* from $q(\cdot|\theta^{(j-1)})$, where $q(\cdot)$ is any distribution that has been proposed;

¹⁴The vector of parameters should be indexed to an index related to the model and the distributions in (56) should be conditioned. However, we omit the indexation so as not to complicate the notation.

¹⁵For further details on the Kalman filter, see Hamilton [17].

¹⁶For a detailed analysis of Markov chain simulations using Monte Carlo methods, see Gamerman and Lopes [16].

¹⁷As the Kalman filter provides the likelihood function, $f(y|\theta)$, and $p(\theta)$ is known, the mode of θ calculated by $\theta = \operatorname{argmax}[f(y|\theta)p(\theta)]$.

- Step 3: calculate the ratio $r(\theta^{(j-1)}, \theta^*) = \frac{p(\theta^*)q(\theta^{(j-1)}|\theta^*)}{p(\theta^{(j-1)})q(\theta^*|\theta^{(j-1)})}$;
- Step 4:
 - if $r \geq 1$, then $\theta^{(j)} = \theta^*$;
 - if $r < 1$, then $\theta^{(j)} = \theta^*$ with probability r ; and $\theta^{(j)} = \theta^{(j-1)}$ with probability $(1 - r)$;
- Step 5: change the counter from j to $j + 1$ and go back to step 2 until convergence is obtained.

Therefore, once the chain with stationary distribution is generated, it is possible to calculate the posteriori moments of the parameters used in the paper.

Appendix C - Distribution graphs

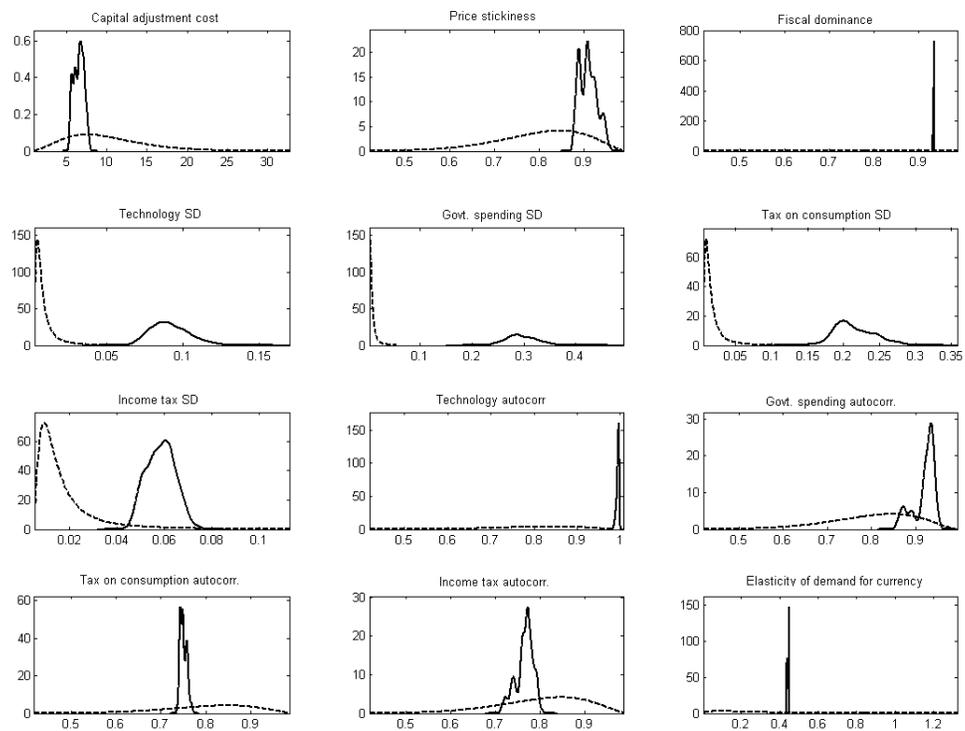


Figure 1: Priori density (dashed lines) and posteriori density (solid lines)