Volatility and Value-at-Risk Forecasting with Realized Volatility and HAR: A comparative Approach

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Resumo

O presente trabalho buscou aplicar as novas técnicas de volatilidade realizada (RV) com modelos HAR para quatro ações pertencentes ao índice IBOVESPA, testando se os mesmos geram melhores previsões da volatilidade e se podem aumentar a eficiência do *value-at-risk* (VaR). Constata-se que o HAR gera melhores previsões que o GARCH e EWMA, mas na aplicação do VaR ele surge como um técnica complementar, pois nenhuma das técnicas apresentou-se superior às outras. Complementarmente, verificamos que o problema de microestrutura da RV parece ser pequeno ou não existir, pois os modelos que melhor se ajustaram foram aqueles com freqüência de 1 minuto.

Abstract

The present study tested the realized volatility (RV) with the HAR models for the four principal BOVESPA's stocks, trying to evaluate whether these models generate better forecast and could provide improvements in Value-at-Risk technique. We found that the HAR generates better forecast than the GARCH and EWMA; however, in the VaR's implementation with HAR models, it arises as complementary technique, because none of the techniques were completely superior to the others. In addition, our study found that the microstructure noise problem is very small or does not exist, because models which got the best fit were those with 1-minute frequency.

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1. Introduction

The development of a new technique for measuring and estimating asset volatility is a field of great interest in finance, especially concerning the asset pricing and risk management theory which is based on that statistical measure. To market makers and Banking Institutions in general, these new techniques should be more comprehensible and easier to measure, estimate and forecast. The current techniques, like conditional variance or stochastic volatility, are not simple to understand and estimation processes such as almost-maximum likelihood are complex and difficult to converge. To academics, it is necessary to develop a direct way of estimating volatility, which is a non-observed variable, by implementing models to capture all stylized facts of financial series.

In that sense, a new great wave in finance has been growing - the Realized Volatility (RV), which seeks to meet those needs. This technique is usually accompanied by econometric models denominated Heterogeneous Autoregressive (HAR), developed by Corsi (2004, 2009), aiming to model and forecast volatility. All this literature originates from the seminal article of Merton (1980), according to which it is possible to estimate an asset's latent or non-observed volatility over a given period using the sum of n intradaily squared returns, when n tends to infinity. However, this technique took many years to be employed due to a problem with the availability of ultra-high frequency data, i.e., 1-minute or more frequent samples. Further, these models could only be tested in reality by using supercomputers. Even nowadays, handling databases with many assets at this frequency requires a large data processing. Finally, and more importantly, Black (1976) developed a theory about the problem caused by high-frequency sampling, which was denominated as microstructure noise.

This problem partly prevented the theme to be further explored because the theory sustained that an asset's observable price consisted of the efficient, or real, price plus a random error. However, when the asset's squared return is calculated, the error no longer disappears when it is summed, systematically generating a bias in this estimate. Moreover, the higher the intraday return frequency, the larger the microstructure noise. Hence, the utilization of models for measuring the realized volatility depended on a way of solving those problems.

Therefore, in an attempt to solve such problems, some authors like Andersen and Bollerslev (1998) started to employ a sampling frequency that could statistically converge to the continuous function of latent volatility, but was not high enough to cause relevant bias due to the microstructure noise. However, some authors like Harris (1990), Zhou (1996) and Andersen and Bollserslev (1998), and later, employing simulation techniques, Zhang, Mykland and Aït-Sahalia (2005), demonstrate that ignoring this problem could lead to serious mensuration errors. Hence, it was necessary to develop techniques for estimating latent volatility without or with a minimum microstructure error.

In order to solve this problem, several techniques were found that successfully dealt with the microstructure noise - through the optimal choice of sampling frequency (Bandi and Russel (2006a) and Zhang, Mykland and Aït-Sahalia (2005); through filters based on the estimation of an AR(p) or MA(q) of the intraday data (Ebens (1999), Andersen, Bollerslev, Diebold and Ebens (2001) and Hansen, Large and Lunde (2006)); through the so-called Realized Kernel, which develops a HAC-type technique with Bartlett Kernel and covariance matrix of Newey and West (1987) to correct the microstructure noise (Barndorff-Nielsen, Hansen, Lunde, Shephard (2006a, 2006b, 2008a, 2008b)); and finally, through a technique that seeks to combine two different frequencies, with an aim to take advantage of each of them (Zhang, Mykland and Aït-Sahalia (2005) and Aït-Sahalia, Mykland, and Zhang (2006, 2009)).

As the microstructure noise started to be solved, a literature emerged with an aim at modeling and forecasting the Realized Volatility without the use of conditional variance, as well as capturing stylized facts of financial series that were not modeled before. This literature started with the work of Andersen, Bollerslev, Diebold and Labys (2003) and Corsi (2004, 2009), who presented a way of modeling and forecasting realized volatility from models estimated by Ordinary Least Squares (OLS). Moreover, these techniques demonstrated some properties that were not always present in models like GARCH *(*Generalized Autoregressive Conditional Heterocedastik), ARFIMA (Autoregressive Fractal Integrated Moving Avarage), FIGARCH *(*Fractal Integrated Generalized Autoregressive Conditional Heteroscedasticity*)* and Stochastic Volatility (SV), such as: long memory, low computational cost, multivariated extensions, quick responses to shocks in the short term and economic explanation for model designing.

Andersen, Bollerslev, Diebold and Labys (2003) and Bollerslev, Chou and Kroner (1992) find that GARCH and Stochastic Volatility models do not satisfy multivariate models, because the estimation is made by Almost-Maximum Likelihood or Kalman Filter, which is complex and of difficult convergence with many assets.

Moreover, Corsi (2004, 2009) points out that conditional variance models do not capture all the characteristics of financial series, such as quick responses to short-term shocks and long memory. He argues that since GARCH models (p,q) present high persistence, as identified by Bollerslev, Chou and Kroner (1992) and are often modeled by Integred GARCH (IGARCH), they have a p close to zero and a q close to 1, which causes the model to give a slow response to sudden changes. This happens because the parameter p, which models the conditional variance of the shock over the t-1 period, is small, and the parameter q, which models ARCH (p) of infinite order and has a longer dependence, is much more important in the determination of volatility. Thus, the short-term impacts take long to be assimilated by the models. If it were the opposite, i.e., a high p and a low q, the long-term effect would be once more neglected, as this technique imposes a strong trade-off on the short and long-term relationship and fails to adequately model these relations.

In an attempt to model the thusly neglected long memory relationship, Corsi (2004, 2009) utilizes traditional long memory models, such as the ARFIMA (p,d,q) and the FIGARCH (p,d,q); however those are also unable to fully perform their task. According to the author, the mathematical operation of the fractional difference operator may result in a loss of information and may not be able to capture fast changes in the long-term dynamics, which is often observed in financial data. Furthermore, he points out that operator precedence of the fractional difference operator (parameter d) together with the other parameters (p and q) is not trivial to estimate, making the estimation of these models often impossible. Additionally, this difficulty once more causes trouble in computational terms to multivariate extensions. Finally, estimating the parameter d separately in order to facilitate the convergence process may incur in bias and inefficiency of the estimators.

Thus, Corsi (2004, 2009) demonstrated that the Heterogeneous Autoregressive (HAR) model could correct those problems. Further, it could provide superior fitting and forecasting performance in relation to traditional models, because it would be easily implemented, would capture long memory and its parameters would adequately respond to short-term shocks. Later, a number of works found the same result for several assets - Chang and McAller (2010) for exchange rate; Scharth and Medeiros (2009) for stocks; Allen, McAleer, and Scharth (2009) and Jou, Wang, and Chiu (2010) for derivatives.

Others extended the model of Corsi (2004, 2009) - Markovian regime switching in Bordignon and Raggi (2010); jumps modeling and leverage effects in Corsi, Pirino and Reno (2009), and in Chung, Huang and Tseng (2008); Multiple-Regime Smooth Transition HAR model, in Medeiros and McAleer (2008); and multivariate extensions in Audrino and Corsi (2008), confirming the best performance of these models.

With respect to Brazilian data, there are no works to properly test these techniques and no use of the HAR model. Some of the works are Andrade and Tabak (2001) for exchange rate; Carvalho, Freire, Medeiros and Souza (2005) for IBOVESPA and Sá Mota and Fernandes (2004) for IBOVESPA stocks. However, they either apply the realized volatility without correcting the microstructure noise or utilize old econometric techniques, such as EWMA (exponentially weighted moving average) and GARCH. No one utilizes the HAR models, failing to take advantage of what is best in using realized volatility in practical terms.

Given the circumstances above, the paper seeks to meet two goals. First, to analyze whether the HAR models are superior to traditional models in forecasting ability, at the same time observing which is the best method and sampling frequency to minimize the microstructure noise in the Brazilian data, as we have a more volatile market, with less liquidity and more restrict data availability than the North-American market. Further, our second goal is to analyze whether the HAR models succeed in the empirical application for Value-at-Risk (VaR) and whether they are superior to the GARCH and EWMA models.

Results indicate that the HAR is superior to the GARCH and EWMA models in forecasting ability, especially at 2, 5 and 10 steps ahead. Besides, the correction method of Hansen, Large and Lunde (2008) did not fit the Brazilian data. Curiously, the 1-minute frequency which was the highest employed, produced the best models of forecasting and fitting to VaR.

In the Value-at-Risk application, the HAR models did not demonstrate superiority to the GARCH model. The realized volatility-based model performed well in three of the four stocks, as did the GARCH, especially at short forecasting horizons (1 and 2 steps ahead), being fit at all maximum loss levels for the next day. Furthermore, the model had an excellent performance for GGBR4, passing all tests and at all forecasting horizons. However, it failed to model PETR4 and was outperformed by the GARCH model for VALE5. We emphasize that the GARCH model did not fit the GGBR4 series and had a draw in performance when compared to the USIM5 series;

also, it was barely approved in VaR configurations with a maximum loss of 10% and 5%. Hence, we believe that the models are complementary to each other, with none demonstrating a significant superiority. The EWMA showed problems with the criterion de independence of violations, being rejected in a large part of the models estimated.

For our purpose, this paper is organized in four more parts, as follows. In the methodology, we introduce the microstructure noise and the options for correction, as well as the HAR model and the traditional estimated models. The third section is dedicated to the treatment of the database and the fourth provides the findings, where we detail the results of forecasting and application to VaR. Finally, we proceed to the conclusion of the paper.

2. Methodology

In this section, we will formally introduce the methodology employed in the paper, dividing it into five parts. In the first subsection, we will present the theoretical construction of realized volatility and its correction methods. In the second, we will introduce the econometric models HAR, GARCH and EWMA. In the third, we will analyze the criteria developed to evaluate the models' forecasting performance. In the fourth, we will show the use of Value-at-Risk and the empirical validation tests of the technique. Finally, in the last subsection we will show the empirical method employed by us that makes the HAR model's performance much superior to traditional methods.

2.1 Realized Volatility

Merton (1980) showed that it would be possible to create a proxy of latent volatility using the sum of N intraday squared returns over a given time period t. It would be possible, as when N tends to infinite there is a convergence in probability to the continuous function of integrated volatility. In other words, by collecting an asset's price on one day N+1 times, with N being frequent enough, applying the logarithmic return, squaring and then summing all returns, we would reach the latent or non-observed volatility for that day. Formally, considering that an asset's price follows a diffusion process:

$$dp_{t+\tau} = \mu_{t+\tau}d\tau + \sigma_{t+\tau}dW_{t+\tau}, \qquad 0 < \tau < 1\,,\ t = 1,2,\dots,T \tag{1}$$

where p_t is the logarithm of instantaneous price on time $t + \tau$, $\mu_{t+\tau}$ is the *drift* component (equal to zero in this case), $dW_{t+\tau}$ is a standard Brownian motion and $\sigma_{t+\tau}$ is a standard deviation. Thus, it is demonstrated that the instantaneous volatility from t-1 to t is the integral of the standard deviation of the Brownian motion, as follows:

$$\sigma_t = IV = \left(\int_{t-1}^{t} \sigma^2(\omega)d\omega\right)^{\frac{1}{2}} \tag{2}$$

However, this variable is not directly observable and the data collection is discreet. Thus, Merton (1980) and Andersen, Bollerslev, Diebold and Labys (2001) point out that:

$$RV_t = \sqrt{\sum_{n=1}^{N} r_{t+n}^2} \tag{3}$$

will be an approximate measure of integrated volatility when:

$$plim_{N\to\infty} \rightarrow RV_t = \sigma_t \tag{4}$$

2.1.1 Microstructure Noise

As presented in the introduction, the implementation of the Realized Volatility aims at correcting the old microstructure noise, found by Black (1986). In this theory there is a price $P_{t,n}^0$, which is the observable price on day t and on the n-th division, comprised of $p_{t,n+1}^0 = p_{t,n+1}^L + \varepsilon_{t,n+1}$, where $p_{t,n}^L$ is the latent price and $\varepsilon_{t,n}$ is an IID disturbance with $E[\varepsilon_t] = 0,\ E|\varepsilon_t|^4 < \infty$ and not correlated with the latent (or efficient) price. If we follow Merton's (1980) technique and sum the N returns to the square derived from the N+1 partitions of the day, with N being sufficiently large, our estimate will be biased because the error term will accumulate. In other words, when we take the first difference of the logarithm of $P_{t,n}^0$ and square it, we will accumulate the N $\varepsilon_{t,n}$ in the realized volatility. Therefore, the problem is to create a sample that is sufficiently frequent to converge to continuous function, but not so frequent as to incur in a large bias of the microstructure noise. It should be observed that the higher the partition of day t, the larger the microstructure bias. Formally, the logarithm of the observable price is

$$p_{t,n}^{0} = p_{t,n}^{L} + \varepsilon_{t,n} \tag{5}$$

Taking the first difference of (5) and defining $r_{t,n+1}$ as the return, we have:

$$p_{t,n+1}^{O} - p_{t,n}^{0} = p_{t,n+1}^{L} + \varepsilon_{t,n+1} - p_{t,n}^{L} - \varepsilon_{t,n}$$
$$r_{t,n+1}^{o} = r_{t,n+1}^{L} + \varepsilon_{t,n+1} - \varepsilon_{t,n} \tag{6}$$

But $v_{t,n+1} = \varepsilon_{t,n+1} - \varepsilon_{t,n}$,

$$r_{t,n+1}^{o} = r_{t,n+1}^{L} + v_{t,n+1} \tag{7}$$

By squaring it, we have:

$$\left(r_{t,n+1}^{o}\right)^{2} = \left(r_{t,n+1}^{L}\right)^{2} + \left(v_{t,n+1}\right)^{2} + 2\left(r_{t,n+1}^{L} v_{t,n+1}\right) \tag{8}$$

Summing the N returns, using definition of (3) and $\sum_{n=1}^{N}\left(v_{t,n+1}\right)^{2} = \epsilon_{t}^{2}$

$$(RV_{t}^{o})^{2} = (RV_{t}^{L})^{2} + \left(\epsilon_{t,}\right)^{2} + 2\sum_{n=1}^{N}\left(r_{t,n+1}^{L} v_{t,n+1}\right) \tag{9}$$

Assuming that the microstructure problem is IID, with $\mathrm{E}v_{t,n} = 0$, as $p_{t,n}^{0}$ is not stochastically correlated with $v_{t,n}$ and the estimator variance is non-infinite, $E\left|v_{t,n}\right|^{4} < \infty$ and taking into consideration that Var $(v_{t,n})$=$E\left(v_{t,n}\right)^{2}$, we have:

$$E[(RV_{t}^{o})^{2}|(RV_{t}^{L})^{2}] = (RV_{t}^{L})^{2} + 2n(\epsilon_{t})^{2} \tag{10}$$

demonstrating to be clearly a biased sampling process. Following the approach of Zhang, Mykland and Aït-Sahalia (2005), which confirms the derivations below, the conditional variance becomes:

$$var[(RV_t^o)^2|(RV_t^L)^2] = 4nE(\epsilon_t)^4 + (8RV_t^L E(\epsilon_t)^2 - 2var\,(\epsilon_t)^2) + O_p(n^{-\frac{1}{2}}) \quad (11)$$

Therefore, putting N to infinite:

$$n^{-\frac{1}{2}}((RV_t^o)^2 - 2n((\epsilon_t)^2) \xrightarrow{\mathcal{L}} 2(E(\epsilon_t)^4)^{-\frac{1}{2}} Z_{Noise} \quad (12)$$

Z_{Noise} has a normal distribution and originates from the disturbance in RV_t^o. Moreover, the authors find that in addition to the mean and the variance being affected by the microstructure noise, there is also the discretization problem, i.e., the process is not effectively continuous in practice. Hence, they indicate the existence of convergence in distribution to Observed Realized Volatility, as:

$$RV_t^o \overset{d}{\approx} IV + \underbrace{2N_t E(\epsilon_t^2)}_{\text{Bias due to noise}} + \underbrace{\Big[\underbrace{4N_t E(\epsilon_t^4)}_{\text{due to noise}} + \underbrace{\tfrac{2}{N_t}\int_0^1 \sigma_t{}^4 dt}_{\text{due to discretization}}\Big]^{\frac{1}{2}}}_{\text{Total Variance}} N(0,1) \quad (13)$$

2.1.2 Correction methods

In this article two correction methods are applied to ensure that the microstructure noise is not relevant to our forecasts with the HAR model and to our empirical value-at-risk applications. Hence, it is used the procedure of finding the optimal frequency for each stock, which will be presented in the next subsection, and also a correction system for the microstructure noise, developed by Hansen, Large and Lunde (2008).

2.1.2.1 Optimal sampling frequency

The most used correction method is the optimal frequency, whose methodology follows the article of Hansen and Lunde (2006), and Zhang, Mykland and Aït-Sahalia (2005). Both articles demonstrate that this correction method is highly efficient in dealing with the microstructure noise. In this paper, we derive the optimal sampling system of Bandi and Russel (2005a, 2006), which works an approximation of the formula by arbitrating the optimal frequency of the estimator's variance.

Bandi and Russel (2005a, 2006) derive and minimize the function of error caused by microstructure noise, in order to ensure the convergence to a continuous function of integrated volatility. Hence, the mean quadratic error function is given by:

$$MSE_t = 2\left(\frac{1}{N}\right)\left(IQ_t + o_{a.s.}(1)\right) + N\left(2E(v_t^4) - 3\left(E(v_t^2)\right)^2\right) + N^2\left(E(v_t^2)\right)^2 + 4E(v_t^2)\left(\int_{t-1}^{t}\sigma_\tau^4 d\tau\right) - E(v_t^4) + 2\left(E(v^2)\right)^2 \quad (14)$$

with T being the total number of days and IQ_tbeing called the *Integrated Quarticity,* theoretically defined by a diffusion process presented in (1) by:

$$IQ_t = \int_0^1 \sigma^4(t+\tau-1)d\tau$$

Minimizing the MSE_t of equation (14) we have:

$$N^* = 2N^3\left(E(v^2)\right)^2 + N^2(2E(v^4) - 3\left(E(v^2)\right)^2 - 2IQ_t \quad (15)$$

The authors derive that $E(v^2)$ is equal to $\frac{(\sum_{t=1}^{T}\sum_{n=1}^{N} r_{t+n}^2)}{TN}$ and $E(v^4)$ is equal to $\frac{(\sum_{t=1}^{T}\sum_{n=1}^{N} r_{t+n}^4)}{TN}$. Thus, they define the approximation of optimal N:

$$N^* \sim \left(\frac{IQ_t}{\left(E(v^2)\right)^2}\right)^{\frac{1}{3}} = \left(\frac{IQ_t}{\left(\frac{(\sum_{t=1}^{T}\sum_{n=1}^{N} r_{t+n}^2)}{TN}\right)^2}\right)^{\frac{1}{3}} \quad (16)$$

However, IQ_t must be estimated and Bandi and Russel define equation (16) for a 15-minute sampling frequency that would be sufficiently fast to approximate, given the empirical experiments, without incurring in microstructure error. In statistical tests, the

IQ is derived with 15 and 30-minute frequency, being that the result obtained for all series investigated is about 7 minutes.

$$IQ_t = \frac{N_t}{3} \sum_{n=0}^{N_t} r_t^4 \qquad (17)$$

Using this methodology, we find a way to significantly minimize the microstructure noise, like Zhang, Mykland and Aït-Sahalia (2005) point out by simulation. Moreover, Hansen and Lunde (2006) demonstrate that the microstructure error for DJIA stocks is small in sampling frequencies lower than 20 minutes, which indicates that this technique would be sufficient to implement the models like Andersen (2007) addresses in his article.

However, the discussion about how to optimally calculate the *Integrated Quarticity* leads many authors such as Bandi and Russel (2005a, 2006b) to calculate it with a 15-minute sampling frequency, thus avoiding the use of a complex method demonstrated in Zhang, Mykland and Aït-Sahalia (2005). Hence, given the optimal choice limitations, the option is to sample in several frequencies (1, 2, 5, 15 and 30 minutes) in order to solve this problem, although we calculate the estimators' variance as demonstrated above pointing out the optimal choice for each stock.

2.1.2.2 Filter-based estimator

The filter-based estimator was introduced by Ebens (1999) and Andersen, Bollerslev, Diebold and Ebens (2001). The general idea was to estimate an auto-regressive model or moving average from the intraday return, because that process of autocorrelation and partial autocorrelation derived exclusively from a process generated by the microstructure noise. Hence, when estimating an AR or MA model it is possible to identify the part of the intraday return that is a microstructure bias, filtering it through the estimated parameters. However, Bandi and Russel (2005) criticize the model and demonstrate that the technique is not sufficient to stop the tendency. Hence, Hansen, Large and Lunde (2006) demonstrate that it is necessary a larger time lag relative to MA to ensure consistency.

Therefore, considering that the error is correlated to the latent price and assuming serial independence, the price return follows the MA(q) process:

$$r_{t,N} = u_{t,i} - \theta_1 u_{t,i-1} - \theta_2 u_{t,i-2} - \cdots - \theta_q u_{t,i-q}, \qquad i = 0, \ldots, N \tag{18}$$

Where the sequence $\{u_{t,m}\}$ is $IID(0,\sigma_{u,t}^2)$. Thus, the filter becomes:

$$RV^{MA} = \left(\frac{(1-\theta_1-\theta_2-\cdots-\theta_q)^2}{1+\theta_1^2+\theta_2^2+\cdots+\theta_q^2}\right) RV \tag{19}$$

However, as the availability of data with frequency higher than 1 minute is too low for assets series traded at Bovespa, the authors demonstrate that the estimator's consistency problem diminishes with an equidistant interval and considering the volatility to be constant in this interval. In this article, the 20/20 second interval is used, which ensures a variance approximately constant in the subinterval. On the other side, less frequent intervals are allowed in a market not so liquid like NYSE and S&P500, enabling the utilization of the data provided.

2.2 Models Used

2.2.1 Heterogeneous Autoregressive (HAR)

The model of Corsi (2004, 2009), denominated Heterogeneous Autoregressive (HAR), is based on the assumption that markets are heterogeneous as presented by Muller, Dacorogna, Dav, Pictet, Olsen, and Ward (1993). This theory argues that different agents operate in the market, with goals, institutional restrictions, performance horizons, information, knowledge, and other variables of their own, which causes each change in the market to have a different response from these agents. This theory attempts to explain why the assets most searched for have the higher volatility, since if the markets were homogeneous the oscillations out of the efficient (or real) price would be more quickly corrected and less oscillations would occur.

In practical terms, this theory proves that while changes in the long term affect the short-term strategies, changes in the latter do not affect the former. This occurs because, according to Müller, Dacorogna, Dav and Pictet (1993), the change in the type of agents is associated with three performance horizons: short, medium and long term. The short-term group would include the brokers aiming to profit with day or intraday operations; the medium-term group would include the operators of high-risk funds; and the long-term group would include the central banks, commercial banks and pension funds. Hence, when brokers modify their operations by changes in the short term, it does not necessarily follow that the others will change their strategies. However, when the Central Banks and the pension funds modify their positions, they end up affecting the dynamics of the short-term agents.

Therefore, the situation described above establishes a hierarchical relationship of the effects of short and long-term volatility and consequently the form of the estimated equation. Thus, the volatility determined by long-term agents is RV_t^m, with the mean of the realized volatility in the last 22 days following a 1st order autoregressive behavior, such as:

$$\log(RV_t^m) = c^m + \beta_m \log(RV_{t-1}^m) + \widetilde{\omega_t^m} \tag{20}$$

where c^m is the constant and $\widetilde{\omega_{t+1}^m}$ is the contemporaneous and serially independent error. Furthermore, the volatility associated with the medium-term agents is RV_t^w, which is the average volatility of the last 5 days, and is explained by itself in the preceding period and by the expectation of the volatility associated with the long-term agents:

$$\log(RV_t^w) = c^w + \beta_w \log(RV_{t-1}^w) + \gamma_m \mathrm{E_t}[\log(RV_t^m)] + \widetilde{\omega_t^w} \tag{21}$$

Finally, the volatility associated with short-term agents, which is the daily realized volatility, is explained by the same variable_in the preceding period and by the expected volatility of medium and long-term agents for the same period, as follows:

$$\log(RV_t^d) = c^d + \beta_d \log(RV_{t-1}^d) + \gamma_w \mathrm{E_t}[\log(RV_t^w)] + \gamma_m \mathrm{E_t}[\log(RV_t^m)] + \widetilde{\omega_t^d} \tag{22}$$

Putting these three equations in a system and resolving it, we have the equation estimated by Corsi (2004, 2009), with HAR equal to:

$$\log(RV_t^d) = C + \theta_d \log(RV_{t-1}^d) + \theta_w \log(RV_{t-1}^w) + \theta_m \log(RV_{t-1}^m) + \widetilde{\omega_{t+1}^d} \quad (23)$$

The expression above is estimated by OLS and uses the Newey-West covariance correction for serial correlation.

To forecast from (23), we estimated a model for each forecasting horizon h, as follows:

$$\log(RV_t^d) = C + \theta_d \log(RV_{t-h}^d) + \theta_w \log(RV_{t-h}^w) + \theta_m \log(RV_{t-h}^m) + \widetilde{\omega_t^d} \quad (24)$$

2.2.2 Traditional Models

The models used for comparison to the HAR's performance are the Exponential Weighted Moving Average (EWMA), with $\lambda = 0{,}94$, which is the model most used in the market for value-at-risk forecasting, and the traditional GARCH model (1,1), as described below:

The EWMA model is defined by:

$$r_t = \epsilon_t \sigma_t \qquad \sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2 \quad (25)$$

where ϵ_t is an N (0,1) variable (normal distribution with mean zero and variance 1). To proceed with the EWMA estimation, we use the variance of the first two observations as proxy of σ_{t-1}^2 and start at t=2.

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is defined by:

$$r_t = \epsilon_t \sigma_t \qquad \sigma_t^2 = w + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (26)$$

where w is a positive constant, $\sigma_t^2 = Var(r|I_{t-1})$, where I_{t-1} is all the information until $t-1$, and $\epsilon_t \sim IID(0,1)$ is independent from r_{t-1}. We use the Marquardt algorithm to estimate the parameters.

To forecast the models above h-steps-ahead, we use the traditional derivation of GARCH:

$$\sigma_{t+h}^2 = w + (\alpha + \beta)\sigma_t^2 \tag{27}$$

In the case of EWMA, if $\alpha + \beta = (\lambda) + (1-\lambda) = 1 \; e \; w = 0$, $\sigma_{t+h}^2 = w + \big((\lambda) + (1-\lambda)\big)\sigma_t^2$, we have:

$$\sigma_{t+h}^2 = \sigma_t^2 \tag{28}$$

This result emphasizes that the EWMA forecast will always be the same given a set of information from a fixed past.

2.2 Value-at-Risk

In this subsection, it will be presented the value-at-risk and the tests used to find out whether volatility models are good measures to forecast this parameter.

2.3.1 Theoretical approach

The value-at-risk is defined as the maximum financial loss for a given period, given a probability associated with catastrophic events that may affect an asset. Then, probabilistically defining, the VaR is

$$p = \Pr[\Lambda V(h) \leq VaR] = F_h(VaR) \tag{29}$$

where the time indexer is t and we calculate for the time horizon periods ahead, with $\Lambda V(h)$ being the return variation from current time to time h and $F_h(VaR)$ being the probability cumulative function of $\Lambda V(h)$. Interpreting the formula, we ask: what is the probability of the return variation being lower (remember that normally $\Lambda V(h) < 0$) than the VaR? For such, we have to establish the probability to which we want to be exposed, and that is why this procedure focus is to find the probability cumulative function or the quantile[4] corresponding to a probability cumulative function. If F_h were known, the VaR would be equal to the quantile associated with the desired probability; as it is not known, each methodology determines differently.

Therefore, in the standard methodology we select a standard normal distribution and the quantile to a given probability "α" that we wish. As the distribution has a deviation equal to 1, we only have to multiply the estimated standard deviation (assuming that the mean is equal to ZERO) by the value of the cumulative probability distribution to find the maximum variation for the period. Formally, the maximum loss for h-th days ahead at a probability α is:

$$VaR(\alpha, h) = Valor\ do\ Ativo \times (x_\alpha \times \sigma_{t+h}), \tag{30}$$

where x_α is the p-th quantile of a standard normal cumulative distribution and σ_{t+h} is the standard deviation estimated by the GARCH, EWMA or HAR models. We emphasize that for backtesting purposes our asset is equal to 1 (monetary value of the portfolio) and our volatility is always multiplied by -1, so as the $VaR(\alpha, h)$ is violated when $r_t < VaR(\alpha, h)$.

2.3.1 Kupiec Test

In order to observe whether the number of violations of the VaR is consistent with the chosen percentage of loss, we selected the Kupiec Test (1995). This test has as a null hypothesis that the error rate (π) of the VaR is equal to the chosen percentage (α) of losses (10%, 5% and 1%, to us), i.e.,$H_0: \pi = \alpha$. The statistic follows an asymptotic distribution of chi-square with 1 degree of freedom, as follows:

$$TK = -2\ln\left(\frac{\alpha^{t_f}(1-\alpha)^{t_v}}{\pi^{t_f}(1-\pi)^{t_v}}\right) \sim \chi^2(1) \tag{31}$$

[4] O α-th quantile of $F_\alpha(x)$ is defined as: $x_\alpha = \inf[x|\ F_h(x) \geq \alpha]$

Thus we reject the null hypothesis at 5% level of confidence if the values are higher than 3.841, that is, we reject the hypothesis of the VaR(α) fitting what actually occurred. We emphasize that the model must have a number of violations consistent with the probability proposed to be fit, otherwise we may be underestimating or overestimating the loss, which has serious consequences on a portfolio's leverage.

2.3.2 Christoffersen Test

A problem proposed by the Kupiec Test is that we might be correlating the violations of the model such that the percentage found, although similar to the desired, may incur in more errors over a given evaluation period. Therefore, based on Markov chains, Christoffersen (1998) verifies whether the period t-1 is correlated with t and proposes the following test:

$$I_t = \begin{cases} 1 & se\ há\ falha \\ 0 & se\ não\ há\ falha \end{cases}$$

Additionally, we define $n_{i,j}$ as the number of observations in state j after having been in state i the previous day, such that:

	$I_{t-1} = 0$	$I_t = 1$
$I_{t-1} = 0$	n_{00}	n_{01}
$I_t = 1$	n_{10}	n_{11}

Now, defining φ_i as the probability of a violation occurring given that i occurred in the previous day, we have:

$$\varphi_0 = \frac{n_{01}}{n_{00} + n_{01}}$$

$$\varphi_1 = \frac{n_{11}}{n_{10} + n_{11}}$$

$$\varphi = \frac{n_{01} + n_{11}}{n_{00} + n_{01}n_{10} + n_{11}}$$

With these probabilities, the test checks whether statistically $\varphi_0 = \varphi_1$, then verifying whether the probability of failure is equal to that of non failure. In other words, failing today does not increase the probability of failing tomorrow again. Thus, Christoffersen (1998) uses the LM test to check whether there is difference:

$$TC = -2\ln\left(\frac{(1-\varphi)^{n_{00}+n_{01}}\varphi^{n_{10}+n_{11}}}{(1-\varphi_0)^{n_{00}}\varphi_0^{n_{01}}(1-\varphi_1)^{n_{10}}\varphi_1^{n_{11}}}\right) \sim \chi^2(1) \tag{32}$$

2.4 Forecasting methods

The forecasting performance of the models is tested from the traditional evaluation on the Root Mean Square Error (RMSE), the Mean Absolute Error (MAE), the Mean Percentage Absolute Error (MPAE) and by the Mincer-Zarnowits test. However, these criteria are constructed on the basis of the h-step-ahead forecast error of volatility[5]. Also, to generate the EWMA and GARCH models we do not have an endogenous variable for comparison. Thus, we use as volatility in our main evaluation the Realized Volatility (RV), as commonly used in the literature; see Hansen and Lunde (2001), Medeiros and McAleer (2008) and Corsi (2004). However, as this variable is the dependent variable of the HAR models we believe that this represents a natural advantage to these models. Thus, the h-step-ahead forecast error of the GARCH models is shown against a proxy of volatility, normally using the root of the squared return. The tests are presented below:

$$RMSE_h^{Proxy} = \sqrt{\frac{\left(\ln(Proxy) - \ln\left(Modelo(h)\right)\right)^2}{T_{prev}}} \tag{34}$$

[5] The HAR model generates a result to the standard deviation $\sigma$and the GARCH models generate the variance σ^2. In order to compare, we had to take the square root of the squared return and the 1-step ahead predictions of the GARCH models. We did the opposite too and the results did not change.

$$MAE_h^{Proxy} = \frac{|\ln(Proxy) - \ln(modelo(h))|}{T_{prev}} \quad (35)$$

$$MPAE_h^{Proxy} = \frac{\left(\frac{|\ln(Proxy) - \ln(modelo(h))|}{\ln|modelo(h)|}\right)}{T_{prev}} \quad (36)$$

In the formulas 34, 35 and 36, proxy[6] can be RV (Realized Volatility) or the squared return, and model*(h)* is the h-step-ahead forecast of the HAR, GARCH or EWMA models.

Furthermore, we emphasize that forecasts are made from the parameters estimated in the entire sample, that is, our application is for in-sample forecasting tests. However, we emphasize that although using the estimated parameters, the forecast more than 1-step-ahead considers all information available at the time of the forecast. In other words, when forecasting a GARCH model 3 steps ahead, for instance, the GARCH is determined by the parameters estimated in the equation; but the 2-step-ahead forecast is made from the 1-step-ahead one, and the 3-step-ahead forecast is made from the 2-step-ahead one. With HAR, the forecast is always 1-step-ahead but the model uses lagged endogenous variables, that is, we have a model estimated for each horizon.

2.4.2 Mincer-Zarnowitz Test

The Mincer-Zarnowitz test checks when the h-step-ahead forecast explains what is effectively observed. For that purpose, it is used a regression that uses as dependent variable the observed Realized Volatility and as explanatory variable the h-step-ahead volatility forecast, which comes from the forecast models, as in the following equation:

$$\sum_{j=0}^{h} RV_{t+j} = b_0 + b_1 E_{t-h}\left[\sum_{j=0}^{h} RV_{t+j}\right] + erro \quad (37)$$

The measure used to find the explanation level is the adjusted R^2, largely known in statistics and econometrics. In other words, the higher the adjusted R^2, the better the forecasting ability of the model.

[6] We also mention as reference the one-step-ahead estimation of the GARCH and the EWMA, to check the differences of the models.

3. Data

This paper used four stocks traded at BOVESPA, which have the trading codes PETR4 (Petrobrás), USIM5 (Usiminas), GGBR4 (Gerdau) and VALE5 (Vale do Rio Doce), covering the period from 03/03/2006 to 30/04/2010 from the database provided by the Instituto Educacional BM&FBovespa. The assets were chosen based on data availability and liquidity, since this is an important measure for Realized Volatility models due to the microstructure noise. For EWMA and GARCH, we used the closing daily prices, while for Realized Volatility we used data from 10am to 5pm for trading sessions conducted in normal time, and from 11am to 6pm for those in daylight-saving time[7]. We emphasize that the extraction method is important to error correction mechanisms, and for that reason we explain that this is a time-based method. For such, our algorithm searched data on a predetermined day and time, considering as valid the price closest to that time. This is important, as the database provided is not regular with respect to trading, which causes the number of daily samples to significantly change; thus there is not a calendar-based sample model, but a hybrid system between trading and calendar. Finally, we call attention to the fact that Stock Split adjustments were made. The table below presents the number of observations by asset:

Table 01 – Number of observations in each frequency used

Asset	Daily	30 minutes	15 minutes	5 minutes	2 minutes	1 minute
GGBR4	1026	14364	28728	86184	215461	430921
PETR4	1026	14364	28728	86184	215461	430921
USIM5	1026	14364	28728	86184	215461	430921
VALE5	1026	14364	28728	86184	215461	430921

4. Results

4.1 Realized Volatility Estimation

In this paper, the realized volatility was estimated at several sampling frequencies considered high and capable of reducing the microstructure noise, as presented in the methodology section. For such, our sample consists of frequency series

[7] The Bovespa changes the trading time according to the daylight-saving time, but not necessarily the beginning and end of this time will coincide with the time change in the trading session.

of 1, 2, 5, 15 and 30 minutes without correction of the microstructure noise; they are denoted $ALL^{1}, ALL^{2}, ALL^{5}, ALL^{15}$ and ALL^{30}, respectively.

Moreover, according to equation (18), we derive the optimal frequency for our data series in agreement with Bandi and Russel (2005), who estimate the *Integrated Quarticity* (IQ) every 15 minutes. The authors point out that using this frequency even with higher or lower frequencies, but still incurring in little microstructure noise, would be sufficient and would not significantly modify the sampling choice. Hence, since we have a different and less liquid market, we also estimate the IQ every 30 minutes, just for checking. We conclude that the optimal frequency is similar for all assets and the different optimal frequencies are about 7 minutes, with the 30-minute IQ decreasing approximately 1 minute at the optimal frequency. The Realized Volatility estimated by this method was denominated OPT. The table below presents the results.

Table 02 – Estimated optimal frequency

Asset	Sampling Freq. of IQ	Optimal Freq. (min)
GGBR4	15 min	7,7553
	30 min	6,5562
PETR4	15 min	7,7040
	30 min	6,6477
USIM5	15 min	8,2067
	30 min	6,9453
VALE5	15 min	7,5756
	30 min	6,4190

Finally, we estimated the realized volatility with the microstructure correction method of Hansen, Large and Lunde (2006), as presented in the methodology section, at three sampling frequencies: 1, 2 and 5 minutes, denoted HLL^{1}, HLL^{2} and HLL^{5} respectively. The table below presents the four moments of distribution of the logarithm of realized volatility employed in this paper for each asset.

Table 03 – Distribution of Realized Volatility for GGBR4

Model	1st moment	2nd moment	3rd moment	4th moment
ALL 1	-4,421157	0,404033	0,843259	3,954799
ALL 2	-4,458405	0,416578	0,809826	3,875392
ALL 5	-4,501202	0,440506	0,711545	3,734478
ALL 15	-4,563046	0,468618	0,631542	3,704188
ALL 30	-4,610009	0,505225	0,518403	3,579628
HLL 1	-4,345466	0,404033	0,843259	3,954799
HLL 2	-4,391233	0,416578	0,809826	3,875392
HLL 5	-4,451299	0,440506	0,711545	3,734478
OPT	-4,519145	0,451244	0,722456	3,688488

Table 04 – Distribution of Realized Volatility for PETR4

Model	1st moment	2nd moment	3rd moment	4th moment
ALL 1	-4,6853	0,441071	0,99078	4,23273
ALL 2	-4,721184	0,459941	0,892529	3,951214
ALL 5	-4,762177	0,484044	0,845146	3,809193
ALL 15	-4,808497	0,510234	0,666641	3,624253
ALL 30	-4,838894	0,547069	0,544077	3,520688
HLL 1	-4,64162	0,441071	0,99078	4,23273
HLL 2	-4,689048	0,459941	0,892529	3,951214
HLL 5	-4,761531	0,484044	0,845146	3,809193
OPT	-4,781008	0,489373	0,775947	3,661625

Table 05 - Distribution of Realized Volatility for VALE5

Model	1st moment	2nd moment	3rd moment	4th moment
ALL 1	-4,656461	0,465569	0,656686	3,552221
ALL 2	-4,678516	0,481621	0,571408	3,382949
ALL 5	-4,712957	0,505414	0,504084	3,323086
ALL 15	-4,759971	0,525347	0,360199	3,209626
ALL 30	-4,792632	0,555442	0,285751	3,193579
HLL 1	-4,613046	0,465569	0,656686	3,552221
HLL 2	-4,624028	0,481621	0,571408	3,382949
HLL 5	-4,661982	0,505414	0,504084	3,323086
OPT	-4,721151	0,510326	0,465175	3,235819

Table 06 – Distribution of Realized Volatility for USIM5

Model	1st moment	2nd moment	3rd moment	4th moment
ALL 1	-4,421691	0,395451	0,735221	4,056223
ALL 2	-4,462425	0,404723	0,704063	3,938455
ALL 5	-4,51346	0,427799	0,658733	3,791962
ALL 15	-4,58738	0,451209	0,563894	3,573505
ALL 30	-4,631624	0,481613	0,466403	3,560391
HLL 1	-4,318825	0,395451	0,735221	4,056223
HLL 2	-4,364121	0,404723	0,704063	3,938455
HLL 5	-4,454681	0,427799	0,658733	3,791962
OPT	-4,538838	0,43235	0,646835	3,848222

4.2 GARCH Estimation

The GARCH models (1.1) were estimated via maximum likelihood, using the Eviews 5 software, with Marquardt optimization algorithms configured with 500 maximum iterations and convergence 0.001. Moreover, the series presented ARCH effect at least until lag 10, and the estimation residuals became white noises after the estimation of the models, as presented in Table 07. This produced coefficients of significant parameters as presented in Table 08. Finally, we took the natural logarithm of volatility estimated by the GARCH and EWMA, as presented in the methodology section. At this point we observe, according to Tables 09 and 10, that the GARCH models estimates are very similar to the Realized Volatility, only with a little more Kurtosis and Asymmetry.

Table 07 – ARCH Effect (10) in the return series and in the GARCHS residuals

Test\ Asset	GGBR4	PETR4	USIM5	VALE5
Daily return series	0	0	0	0
GARCH Residual	0,5649	0,5353	0,7068	0,7027

Table 08 – Estimated coefficients in the GARCHs

	GGBR4		PETR4	
Variable	Coef.	Prob.	Coef.	Prob.
w	2,15E-06	0,0284	1,79E-06	0,0024
α	0,071252	0	0,074029	0
β	0,915909	0	0,911047	0
	USIM5		VALE5	
Variable	Coef.	Prob.	Coef.	Prob.
w	2,83E-06	0,0071	2,83E-06	0,0071
α	0,077405	0	0,077501	0
β	0,907215	0	0,907111	0

Table 09 – Distribution of the series estimated by the EWMAs

Asset\Moment	Mean	Std. Dev.	Skewness	Kurtosis
GGBR4	-4,34349	0,31212	1,055487	4,242361
PETR4	-4,639748	0,358441	1,095302	4,321611
USIM5	-4,317289	0,299914	1,097908	4,506207
VALE5	-4,610429	0,353679	0,888831	3,98666

Table 10 – Distribution of the series estimated by the GARCHs

Asset\ Moment	Mean	Std. Dev.	Skewness	Kurtosis
GGBR4	-4,437278	0,316071	1,108404	4,239725
PETR4	-4,608215	0,342299	1,229766	4,318741
USIM5	-4,393586	0,294194	1,100405	4,339393
VALE5	-4,393781	0,294312	1,101729	4,337651

Table 11 - Estimated coefficients of the HAR models for GGBR4

	1-step-ahead Model					2-step-ahead Model				
Model	C	LRV(-1)	LRV5(-1)	LRV22(-1	Adjusted R^2	C	LRV(-2)	LRV5(-2)	LRV22(-2)	Adjusted R^2
	-0,384637	0,297192	0,41166	0,208212	0,594554	-0,507228	0,239175	0,398513	0,252155	0,542397
ALL 1	0,0014	0,0000	0,0000	0,0001		0,0011	0,0000	0,0000	0,0001	
	-0,397539	0,29268	0,398263	0,224334	0,577828	-0,523745	0,22472	0,392755	0,27006	0,524502
ALL 2	0,0014	0,0000	0,0000	0,0000		0,0012	0,0000	0,0000	0,0001	
	-0,419269	0,259518	0,391264	0,261562	0,545263	-0,543601	0,1931	0,381718	0,310513	0,49547
ALL 5	0,0013	0,0000	0,0000	0,0000		0,0011	0,0000	0,0000	0,0000	
	-0,462461	0,17481	0,403673	0,327579	0,491801	-0,563734	0,154242	0,377465	0,352402	0,461117
ALL 15	0,0011	0,0002	0,0000	0,0000		0,0009	0,0002	0,0000	0,0000	
	-0,51521	0,137403	0,350238	0,410411	0,436418	-0,606771	0,108205	0,34293	0,427351	0,412463
ALL 30	0,0010	0,0016	0,0000	0,0000		0,0010	0,0044	0,0001	0,0000	
	-0,378359	0,297192	0,41166	0,208212	0,594554	-0,49889	0,239175	0,398513	0,252155	0,542397
HLL 1	0,0014	0,0000	0,0000	0,0001		0,0011	0,0000	0,0000	0,0001	
	-0,391848	0,29268	0,398263	0,224334	0,577828	-0,51619	0,22472	0,392755	0,27006	0,524502
HLL 2	0,0014	0,0000	0,0000	0,0000		0,0011	0,0000	0,0000	0,0001	
	-0,414895	0,259518	0,391264	0,261562	0,545263	-0,537879	0,1931	0,381718	0,310513	0,49547
HLL 5	0,0013	0,0000	0,0000	0,0000		0,0011	0,0000	0,0000	0,0000	
	-0,42721	0,26407	0,38083	0,26631	0,541502	-0,559491	0,170783	0,388135	0,323892	0,486749
OPT	0,0009	0,0000	0,0000	0,0000		0,0009	0,0001	0,0000	0,0000	
	5-step-ahead Model					10-step-ahead Model				
	C	LRV(-5)	LRV5(-5)	LRV22(-5	Adjusted R^2	C	LRV(-10)	LRV5(-10	LRV22(-10	Adjusted R^2
	-0,779876	0,183779	0,336615	0,308219	0,451606	-1,121311	0,120169	0,353799	0,277347	0,366305
ALL 1	0,0004	0,0001	0,0003	0,0005		0,0000	0,0199	0,0069	0,0139	
	-0,79687	0,168458	0,333439	0,324815	0,436916	-1,131658	0,133651	0,339468	0,278231	0,359465
ALL 2	0,0004	0,0003	0,0004	0,0003		0,0000	0,0062	0,0057	0,0116	
	-0,80558	0,139127	0,307338	0,38112	0,414202	-1,104255	0,122165	0,335171	0,303348	0,353297
ALL 5	0,0005	0,0021	0,0011	0,0000		0,0000	0,0127	0,0058	0,0052	
	-0,821889	0,06215	0,366655	0,399347	0,390351	-1,096353	0,083244	0,363505	0,320409	0,339232
ALL 15	0,0004	0,1521	0,0001	0,0000		0,0000	0,0605	0,0016	0,0015	
	-0,832755	0,027997	0,393129	0,408644	0,36515	-1,101762	0,044853	0,412436	0,312965	0,320821
ALL 30	0,0004	0,5043	0,0000	0,0000		0,0000	0,2971	0,0002	0,0026	
	-0,766903	0,183779	0,336615	0,308219	0,451606	-1,102488	0,120169	0,353799	0,277347	0,366305
HLL 1	0,0004	0,0001	0,0003	0,0005		0,0000	0,0199	0,0069	0,0139	
	-0,78523	0,168458	0,333439	0,324815	0,436916	-1,114956	0,133651	0,339468	0,278231	0,359465
HLL 2	0,0004	0,0003	0,0004	0,0003		0,0000	0,0062	0,0057	0,0116	
	-0,796976	0,139127	0,307338	0,38112	0,414202	-1,092313	0,122165	0,335171	0,303348	0,353297
HLL 5	0,0005	0,0021	0,0011	0,0000		0,0000	0,0127	0,0058	0,0052	
	-0,812071	0,116981	0,34256	0,367688	0,413988	-1,10204	0,10620	0,35808	0,29818	0,354713
OPT	0,0003	0,0131	0,0003	0,0000		0,0000	0,0267	0,0029	0,0049	

4.3. **HAR estimation and comparison with the literature**

The HAR estimated with Brazilian data demonstrated to be as fit or better than that found by the literature for North-American data. Our estimated parameters were significant in all models, proving that the structure proposed by the HAR which is using the mean of the realized volatility for the last 5 and 22 days is coherent in our data. Another important point is that the adjusted R^2 found slightly drops as the lags of the variables increase. For instance, in lag 10 the models still have a considerable explanatory level, about 0.35.

Table 12 - Estimated coefficients of the HARs models for PETR4

	1-step-ahead Model					2-step-ahead Model				
Model	C	LRV(-1)	LRV5(-1)	LRV22(-1	Adjusted R	C	LRV(-2)	LRV5(-2)	LRV22(-2	Adjusted R²
	-0,37174	0,278343	0,457708	0,18828	0,654206	-0,46038	0,315876	0,343905	0,245669	0,617902
ALL 1	0,0021	0,0000	0,0000	0,0001		0,0024	0,0000	0,0000	0,0000	
	-0,38127	0,275752	0,448913	0,198714	0,638968	-0,47558	0,307935	0,333927	0,261626	0,600439
ALL 2	0,0019	0,0000	0,0000	0,0000		0,0021	0,0000	0,0000	0,0000	
	-0,4184	0,254511	0,439023	0,223606	0,607899	-0,51874	0,267927	0,3496	0,27864	0,569908
ALL 5	0,0010	0,0000	0,0000	0,0000		0,0011	0,0000	0,0000	0,0000	
	-0,44954	0,210278	0,41737	0,285538	0,551974	-0,55457	0,212339	0,349621	0,329477	0,517591
ALL 15	0,0008	0,0000	0,0000	0,0000		0,0008	0,0000	0,0000	0,0000	
	-0,48148	0,213697	0,348318	0,346919	0,507152	-0,59032	0,176332	0,321058	0,3895	0,471782
ALL 30	0,0009	0,0000	0,0000	0,0000		0,0009	0,0000	0,0000	0,0000	
	-0,36843	0,278343	0,457708	0,18828	0,654206	-0,45625	0,315876	0,343905	0,245669	0,617902
HLL 1	0,0021	0,0000	0,0000	0,0001		0,0024	0,0000	0,0000	0,0000	
	-0,37881	0,275752	0,448913	0,198714	0,638968	-0,47248	0,307935	0,333927	0,261626	0,600439
HLL 2	0,0018	0,0000	0,0000	0,0000		0,0021	0,0000	0,0000	0,0000	
	-0,41835	0,254511	0,439023	0,223606	0,607899	-0,51868	0,267927	0,3496	0,27864	0,569908
HLL 5	0,0010	0,0000	0,0000	0,0000		0,0011	0,0000	0,0000	0,0000	
	-0,41935	0,217668	0,484365	0,215563	0,599783	-0,51012	0,26424	0,368123	0,266143	0,568131
OPT	0,0010	0,0000	0,0000	0,0000		0,0012	0,0000	0,0000	0,0000	

	5-step-ahead Model					10-step-ahead Model				
	C	LRV(-5)	LRV5(-5)	LRV22(-5	Adjusted R	C	LRV(-10)	LRV5(-10	LRV22(-1	Adjusted R²
	-0,73113	0,198767	0,284973	0,364687	0,512371	-1,02652	0,175914	0,263093	0,34623	0,43491
ALL 1	0,0013	0,0000	0,0036	0,0000		0,0001	0,0016	0,0618	0,0016	
	-0,74813	0,19498	0,277426	0,374225	0,498158	-1,0559	0,171907	0,248541	0,360738	0,41996
ALL 2	0,0012	0,0000	0,0042	0,0000		0,0001	0,0015	0,0638	0,0008	
	-0,77862	0,176872	0,289252	0,376204	0,479892	-1,07667	0,157491	0,281765	0,340036	0,410289
ALL 5	0,0007	0,0001	0,0022	0,0000		0,0001	0,0020	0,0258	0,0008	
	-0,81298	0,120634	0,334457	0,383182	0,442227	-1,09468	0,146878	0,308629	0,32311	0,38655
ALL 15	0,0005	0,0038	0,0003	0,0000		0,0000	0,0017	0,0095	0,0012	
	-0,83398	0,074046	0,316192	0,447177	0,406263	-1,10681	0,11026	0,277352	0,39199	0,3559
ALL 30	0,0007	0,0730	0,0016	0,0000		0,0001	0,0152	0,0161	0,0001	
	-0,72451	0,198767	0,284973	0,364687	0,512371	-1,01714	0,175914	0,263093	0,34623	0,43491
HLL 1	0,0013	0,0000	0,0036	0,0000		0,0001	0,0016	0,0618	0,0016	
	-0,7432	0,19498	0,277426	0,374225	0,498158	-1,04886	0,171907	0,248541	0,360738	0,41996
HLL 2	0,0012	0,0000	0,0042	0,0000		0,0001	0,0015	0,0638	0,0008	
	-0,77852	0,176872	0,289252	0,376204	0,479892	-1,07652	0,157491	0,281765	0,340036	0,410289
HLL 5	0,0007	0,0001	0,0022	0,0000		0,0001	0,0020	0,0258	0,0008	
	-0,77603	0,175756	0,295762	0,371986	0,475377	-1,07259	0,162316	0,242316	0,376498	0,402558
OPT	0,0008	0,0000	0,0011	0,0000		0,0001	0,0012	0,0577	0,0002	

Table 13 - Estimated coefficients of the HARs models for USIM5

	1-step-ahead Model					2-step-ahead Model				
Model	C	LRV(-1)	LRV5(-1)	LRV22(-1)	Adjusted R^2	C	LRV(-2)	LRV5(-2)	LRV22(-2)	Adjusted R^2
	-0,402577	0,264354	0,431007	0,217529	0,569846	-0,521304	0,187258	0,456384	0,242792	0,523302
ALL 1	0,0020	0,0000	0,0000	0,0001		0,0017	0,0000	0,0000	0,0003	
	-0,424564	0,25447	0,426473	0,228328	0,551182	-0,546851	0,167603	0,46778	0,246953	0,505437
ALL 2	0,0014	0,0000	0,0000	0,0000		0,0012	0,0001	0,0000	0,0002	
	-0,457356	0,2499	0,36996	0,284232	0,515512	-0,583379	0,174379	0,396166	0,3061	0,471265
ALL 5	0,0014	0,0000	0,0000	0,0000		0,0011	0,0000	0,0000	0,0000	
	-0,514868	0,168903	0,3694	0,356772	0,452546	-0,620747	0,105753	0,407966	0,358712	0,423473
ALL 15	0,0006	0,0000	0,0000	0,0000		0,0005	0,0093	0,0000	0,0000	
	-0,594758	0,13433	0,345594	0,401012	0,398724	-0,695171	0,083046	0,373327	0,403257	0,374819
ALL 30	0,0003	0,0002	0,0000	0,0000		0,0003	0,0273	0,0000	0,0000	
	-0,393617	0,264354	0,431007	0,217529	0,569846	-0,509622	0,187258	0,456384	0,242792	0,523302
HLL 1	0,0020	0,0000	0,0000	0,0001		0,0017	0,0000	0,0000	0,0003	
	-0,415645	0,25447	0,426473	0,228328	0,551182	-0,535284	0,167603	0,46778	0,246953	0,505437
HLL 2	0,0014	0,0000	0,0000	0,0000		0,0012	0,0001	0,0000	0,0002	
	-0,451719	0,2499	0,36996	0,284232	0,515512	-0,576129	0,174379	0,396166	0,3061	0,471265
HLL 5	0,0013	0,0000	0,0000	0,0000		0,0010	0,0000	0,0000	0,0000	
	-0,470285	0,255338	0,335822	0,310873	0,501558	-0,605447	0,145245	0,400396	0,327303	0,45444
OPT	0,0010	0,0000	0,0000	0,0000		0,0007	0,0007	0,0000	0,0000	
	5-step-ahead Model					10-step-ahead Model				
	C	LRV(-5)	LRV5(-5)	LRV22(-5)	Adjusted R^2	C	LRV(-10)	LRV5(-10)	LRV22(-10)	Adjusted R^2
	-0,780004	0,101008	0,475178	0,252117	0,446166	-1,132336	0,114182	0,428378	0,205835	0,365692
ALL 1	0,0009	0,0171	0,0000	0,0046		0,0000	0,0169	0,0016	0,0571	
	-0,808109	0,104431	0,463013	0,256665	0,430961	-1,166474	0,100699	0,447992	0,194861	0,354355
ALL 2	0,0006	0,0126	0,0000	0,0036		0,0000	0,0297	0,0005	0,0693	
	-0,851557	0,06225	0,447128	0,308445	0,399883	-1,184319	0,103478	0,420212	0,219639	0,337839
ALL 5	0,0005	0,1599	0,0000	0,0008		0,0000	0,0175	0,0010	0,0445	
	-0,857019	0,024523	0,462791	0,334075	0,372689	-1,183174	0,071552	0,442823	0,234841	0,31729
ALL 15	0,0003	0,5426	0,0000	0,0002		0,0000	0,0717	0,0003	0,0279	
	-0,939196	0,018741	0,415782	0,372798	0,329211	-1,263065	0,08131	0,44191	0,212354	0,288462
ALL 30	0,0001	0,6418	0,0000	0,0001		0,0000	0,0373	0,0001	0,0327	
	-0,762342	0,101008	0,475178	0,252117	0,446166	-1,106454	0,114182	0,428378	0,205835	0,365692
HLL 1	0,0009	0,0171	0,0000	0,0046		0,0000	0,0169	0,0016	0,0571	
	-0,790818	0,104431	0,463013	0,256665	0,430961	-1,141264	0,100699	0,447992	0,194861	0,354355
HLL 2	0,0006	0,0126	0,0000	0,0036		0,0000	0,0297	0,0005	0,0693	
	-0,840849	0,06225	0,447128	0,308445	0,399883	-1,169232	0,103478	0,420212	0,219639	0,337839
HLL 5	0,0005	0,1599	0,0000	0,0008		0,0000	0,0175	0,0010	0,0445	
	-0,853622	0,04918	0,446468	0,323087	0,392877	-1,177901	0,080285	0,49858	0,167376	0,343747
OPT	0,0004	0,2485	0,0000	0,0002		0,0000	0,0534	0,0001	0,1014	

Table 14 - Estimated coefficients of the HARs models for VALE5

	1-step-ahead Model					2-step-ahead Model				
Model	C	θd	θw	θm	Adjusted R^2	C	θd	θw	θm	Adjusted R^2
	-0,40951	0,213393	0,483422	0,220613	0,576513	-0,510267	0,154805	0,50889	0,232517	0,542111
ALL 1	0,0016	0,0000	0,0000	0,0001		0,0017	0,0001	0,0000	0,0003	
	-0,399859	0,219567	0,480586	0,220063	0,576321	-0,500574	0,175007	0,475388	0,248707	0,539454
ALL 2	0,0016	0,0000	0,0000	0,0001		0,0017	0,0000	0,0000	0,0001	
	-0,432561	0,248168	0,420835	0,245621	0,547856	-0,54952	0,192492	0,40671	0,291182	0,503529
ALL 5	0,0012	0,0000	0,0000	0,0000		0,0012	0,0000	0,0000	0,0000	
	-0,472079	0,224215	0,385063	0,299552	0,489059	-0,595328	0,168357	0,356606	0,358679	0,444502
ALL 15	0,0017	0,0000	0,0000	0,0000		0,0018	0,0000	0,0000	0,0000	
	-0,492079	0,184556	0,371474	0,351552	0,446392	-0,60133	0,131072	0,35143	0,403052	0,410359
ALL 30	0,0016	0,0000	0,0000	0,0000		0,0017	0,0013	0,0000	0,0000	
	-0,405925	0,213393	0,483422	0,220613	0,576513	-0,505761	0,154805	0,50889	0,232517	0,542111
HLL 1	0,0016	0,0000	0,0000	0,0001		0,0017	0,0001	0,0000	0,0003	
	-0,395512	0,219567	0,480586	0,220063	0,576321	-0,495076	0,175007	0,475388	0,248707	0,539454
HLL 2	0,0016	0,0000	0,0000	0,0001		0,0017	0,0000	0,0000	0,0001	
	-0,428209	0,248168	0,420835	0,245621	0,547856	-0,543932	0,192492	0,40671	0,291182	0,503529
HLL 5	0,0012	0,0000	0,0000	0,0000		0,0012	0,0000	0,0000	0,0000	
	-0,439718	0,232306	0,440943	0,240253	0,541651	-0,555113	0,184233	0,417269	0,288093	0,498315
OPT	0,0010	0,0000	0,0000	0,0000		0,0011	0,0000	0,0000	0,0000	
	5-step-ahead Model					10-step-ahead Model				
	C	θd	θw	θm	Adjusted R^2	C	θd	θw	θm	Adjusted R^2
	-0,73327	0,1674	0,389974	0,290805	0,472847	-1,085754	0,160994	0,368113	0,242888	0,390832
ALL 1	0,0010	0,0001	0,0000	0,0006		0,0000	0,0005	0,0062	0,0335	
	-0,728088	0,185355	0,346209	0,318777	0,468571	-1,067543	0,170207	0,322085	0,285119	0,388026
ALL 2	0,0010	0,0000	0,0001	0,0002		0,0000	0,0002	0,0144	0,0116	
	-0,786856	0,159063	0,336367	0,344678	0,434323	-1,13367	0,150157	0,331637	0,284054	0,361842
ALL 5	0,0007	0,0003	0,0002	0,0000		0,0000	0,0008	0,0094	0,0085	
	-0,842227	0,105579	0,322748	0,403736	0,37972	-1,179428	0,155791	0,262446	0,341593	0,320391
ALL 15	0,0009	0,0220	0,0008	0,0000		0,0000	0,0002	0,0294	0,0011	
	-0,835009	0,088601	0,291468	0,456947	0,353492	-1,173302	0,143501	0,296872	0,323981	0,30744
ALL 30	0,0009	0,0350	0,0030	0,0000		0,0000	0,0003	0,0115	0,0026	
	-0,726678	0,1674	0,389974	0,290805	0,472847	-1,075855	0,160994	0,368113	0,242888	0,390832
HLL 1	0,0010	0,0001	0,0000	0,0006		0,0000	0,0005	0,0062	0,0335	
	-0,719934	0,185355	0,346209	0,318777	0,468571	-1,055415	0,170207	0,322085	0,285119	0,388026
HLL 2	0,0010	0,0000	0,0001	0,0002		0,0000	0,0002	0,0144	0,0116	
	-0,778706	0,159063	0,336367	0,344678	0,434323	-1,121734	0,150157	0,331637	0,284054	0,361842
HLL 5	0,0007	0,0003	0,0002	0,0000		0,0000	0,0008	0,0094	0,0085	
	-0,804413	0,168902	0,3186	0,349086	0,427773	-1,15092	0,153438	0,329416	0,279676	0,356851
OPT	0,0005	0,0001	0,0005	0,0000		0,0000	0,0008	0,0094	0,0091	

4.4 Prediction of the GARCH and EWMA models

The volatility forecasting models GARCH and EWMA, constructed with conditional variance, do not have a reference like the OLS estimation models which have their own endogenous variable to measure their performance. Therefore, it has always been very complicated to accurately know if these models are fit. In the article of Hansen (2001), it is applied a realized volatility model which would be the theoretically correct measure for forecast comparison of conditional variance models, becoming a reference to evaluate these models. In this article we used two measures of realized volatility - correction filter of Hansen, Large and Lunde (2006) with 1-minute-frequency; and the optimal frequency choice of Bandi and Russel (2005). Further, it was used the squared return reference and the GARCH and EWMA's one-step-ahead forecast to demonstrate the loss of precision as the number of steps ahead (PAF) increase.

As we can see in Tables 15, 16 and 17, the GARCH and EWMA models have a very similar performance, with the EWMA showing slightly lower RMSE, MAE and MPAE. Varying according to the asset and using the realized volatility as reference, the GARCH model has a one-step-ahead RMSE between 0.09 and 0.17, while the EWMA has it between 0.06 and 0.09. Moreover, the MPAE of the GARCH model is between 5.8% and 7.4%, with the EWMA a little below, between 4.53% and 5.43%. Further, when we extend the forecast to between 2 and 10 steps, the EWMA model's error increases less than the GARCH model's error, as in the 10-step-ahead forecast compared to its own 1-step-ahead forecast the EWMA always has a percentage result lower or equal to that of the GARCH. With respect to the stocks, no one presented a very different behavior; only the VALE5 series showed the RMSE, MAE and MPAE a little higher at all horizons and references and in both models compared.

The Mincer-Zarnovitz tests against the realized volatility of HLL demonstrate that the deterministic volatility models had an explanatory level close to 50% of the realized volatility value with the 1-step-ahead forecast. However, neither model's forecast is able to explain the endogenous variable, which indicates that even showing relatively little error they fail to explain the variations that actually occur at long horizons. In practical terms, this means that the model gets very close to what actually happens because it presents few errors, however without being accurate enough to explain all the volatility variations in each time t. In this criterion the GARCH model showed the best results.

Table 15 – Root Mean Square Error of Forecast of GARCHs and EWMAs

		EWMA				GARCH			
ASSET	PAF	GARCH	RV HLL¹	RV OPT	EWMA	GARCH	RV HLL¹	RV OPT	EWMA
	1	0,0871	0,2564	0,2095	0,0000	0,0000	0,3092	0,3663	0,0871
	2	0,1041	0,2653	0,2204	0,0442	0,0468	0,3149	0,3736	0,0975
	5	0,1377	0,2802	0,2408	0,0905	0,0908	0,3260	0,3878	0,1208
GGBR4	10	0,1764	0,3004	0,2669	0,1376	0,1317	0,3365	0,4031	0,1546
	1	0,1212	0,2084	0,2364	0,0000	0,0000	0,2939	0,3520	0,1186
	2	0,1280	0,2142	0,2428	0,0522	0,0530	0,3008	0,3621	0,1216
	5	0,1488	0,2180	0,2263	0,0862	0,1020	0,3127	0,3786	0,1378
USIM5	10	0,2086	0,2781	0,3060	0,1603	0,1449	0,3202	0,3947	0,1660
	1	0,1186	0,2262	0,2150	0,0000	0,0000	0,2834	0,3689	0,1212
	2	0,1127	0,2300	0,2196	0,0474	0,0508	0,2949	0,3811	0,1249
	5	0,1495	0,2495	0,2423	0,0967	0,1030	0,3203	0,4067	0,1459
PETR4	10	0,1936	0,2714	0,2676	0,1506	0,1507	0,3394	0,4294	0,1827
	1	0,1222	0,3049	0,3892	0,0000	0,0000	0,4233	0,5237	0,1381
	2	0,1347	0,3091	0,3932	0,0472	0,0529	0,4278	0,5292	0,1389
	5	0,1655	0,3236	0,4066	0,0960	0,1021	0,4460	0,5476	0,1541
VALE5	10	0,2054	0,3501	0,4286	0,1487	0,1453	0,4677	0,5671	0,1866

Table 16 – Mean absolute percentage error of forecast of GARCHs and EWMAs

		EWMA				GARCH			
ASSET	PAF	GARCH	RV HLL¹	RV OPT	EWMA	GARCH	RV HLL¹	RV OPT	EWMA
	1	1,36%	4,53%	3,71%	0,00%	0,00%	5,80%	6,37%	1,34%
	2	1,65%	4,71%	3,90%	0,69%	0,78%	5,90%	6,51%	1,58%
	5	2,30%	5,02%	4,28%	1,55%	1,66%	6,05%	6,83%	2,12%
GGBR4	10	3,12%	5,42%	4,82%	2,49%	2,50%	6,23%	7,18%	2,81%
	1	1,79%	3,34%	3,97%	0,00%	0,00%	5,61%	6,13%	1,71%
	2	1,82%	3,43%	4,09%	0,69%	0,88%	5,68%	6,31%	1,90%
	5	2,32%	3,58%	3,79%	1,15%	1,85%	5,84%	6,69%	2,40%
USIM5	10	3,28%	4,55%	5,29%	2,58%	2,67%	5,97%	6,96%	2,99%
	1	1,75%	3,99%	3,71%	0,00%	0,00%	4,92%	6,27%	1,76%
	2	1,60%	4,05%	3,83%	0,73%	0,79%	5,09%	6,47%	1,94%
	5	2,38%	4,45%	4,26%	1,64%	1,75%	5,58%	6,95%	2,47%
PETR4	10	3,32%	4,87%	4,81%	2,62%	2,62%	5,99%	7,39%	3,19%
	1	1,88%	5,43%	6,87%	0,00%	0,00%	7,39%	8,88%	2,25%
	2	2,12%	5,52%	6,92%	0,72%	0,88%	7,47%	8,96%	2,33%
	5	2,79%	5,80%	7,15%	1,64%	1,86%	7,81%	9,31%	2,65%
VALE5	10	3,62%	6,28%	7,55%	2,61%	2,68%	8,21%	9,66%	3,33%

Table 17 – Mean absolute error of forecast of GARCHs e EWMAs

		EWMA				GARCH			
ASSET	PAF	GARCH	RV HLL[1]	RV OPT	EWMA	GARCH	RV HLL[1]	RV OPT	EWMA
	1	0,0615	0,1950	0,1662	0,0000	0,0000	0,2416	0,2749	0,0615
	2	0,0736	0,2025	0,1746	0,0307	0,0341	0,2454	0,2812	0,0723
	5	0,1015	0,2150	0,1914	0,0686	0,0730	0,2514	0,2955	0,0962
GGBR4	10	0,1366	0,2311	0,2152	0,1100	0,1095	0,2582	0,3109	0,1266
	1	0,0842	0,1524	0,1879	0,0000	0,0000	0,2339	0,2801	0,0776
	2	0,0852	0,1564	0,1933	0,0319	0,0380	0,2365	0,2883	0,0860
	5	0,1083	0,1632	0,1784	0,0529	0,0799	0,2429	0,3051	0,1075
USIM5	10	0,1484	0,2054	0,2499	0,1179	0,1156	0,2483	0,3174	0,1330
	1	0,0776	0,1719	0,1677	0,0000	0,0000	0,2217	0,3000	0,0842
	2	0,0711	0,1745	0,1733	0,0318	0,0356	0,2290	0,3097	0,0925
	5	0,1036	0,1906	0,1925	0,0720	0,0794	0,2511	0,3323	0,1164
PETR4	10	0,1431	0,2076	0,2173	0,1144	0,1185	0,2698	0,3534	0,1492
	1	0,0823	0,2520	0,3282	0,0000	0,0000	0,3456	0,4316	0,1006
	2	0,0928	0,2559	0,3307	0,0317	0,0378	0,3499	0,4357	0,1038
	5	0,1208	0,2685	0,3413	0,0717	0,0802	0,3648	0,4519	0,1177
VALE5	10	0,1557	0,2898	0,3595	0,1137	0,1158	0,3830	0,4681	0,1469

Table 18 – Mincer-Zarnowitz Test of GARCHs and EWMAs

Assets	PETR4		GGBR4		USIM5		VALE5	
PAF\Model	GARCH	EWMA	GARCH	EWMA	GARCH	EWMA	GARCH	EWMA
1	0,5954	0,5534	0,4760	0,4258	0,4897	0,4777	0,4020	0,3574
2	0,5633	0,5208	0,4531	0,4025	0,4618	0,4282	0,3913	0,3430
5	0,4886	0,5135	0,4015	0,3607	0,4080	0,3698	0,3348	0,3003
10	0,4305	0,3920	0,3460	0,3073	0,3654	0,3210	0,2659	0,2382

4.5 HAR Models Forecasting

In general, the models performed well with low mean absolute percentage error at long horizons and presented good adaptation in the Mincer-Zarnowitz tests. Naturally, the best fit models showed the best forecasting ability - the HAR models with RV HLL, with correction filter; and with RV ALL, without microstructure error correction, both with 1-minute frequency. However, it can be observed that the fit and forecasting ability of the models drop as the sampling frequency decrease, independently from any correction methods. That is perhaps the reason why some authors that work with the methodology of *Jumps* jointly tend to ignore the microstructure error and directly apply the realized volatility. It is interesting to note that the microstructure error should increase as the frequency increases, causing the performance of these models to fall since there would be a “non-modeled” disturbance component inside the estimates. This could indicate that the error in Brazilian models would be small and irrelevant to the estimation of models like the HAR or that the correction methods used have non-feasible hypotheses with regard to local data.

The 1-minute frequency models reached the 1-step-ahead forecast: 60% in the Mincer-Zarnowitz test with mean squared error of forecast of 0.25 and percentage mean absolute error of 4.6%. However, the advantages over the others decrease as the horizon increases, reaching approximately 37% in the Mincer-Zarnowitz test at the 10-step-ahead horizon. It is also interesting that the mean percentage error increases only 1.2 percentage points for all models as the horizon increases, indicating high forecasting ability of this technique.

Table 19 – Forecasting performance of the HARs for GGBR4

	1-step-ahead model				2-step-ahead Model			
Asset	RMSE	MAE	MAPE	Mincer-Z.	RMSE	MAE	MAPE	Mincer-Z.
ALL 1	0,2569	0,1990	4,64%	0,5954	0,2730	0,2099	4,91%	0,5433
ALL 2	0,2702	0,2102	4,85%	0,5787	0,2869	0,2217	5,13%	0,5255
ALL 5	0,2966	0,2303	5,27%	0,5462	0,3125	0,2427	5,55%	0,4965
ALL 15	0,3335	0,2607	5,87%	0,4928	0,3446	0,2688	6,06%	0,4622
ALL 30	0,3786	0,3012	6,71%	0,4375	0,3867	0,3059	6,82%	0,4136
HLL 1	0,2569	0,1990	4,73%	0,5954	0,2730	0,2099	5,00%	0,5433
HLL 2	0,2702	0,2102	4,93%	0,5787	0,2869	0,2217	5,21%	0,5255
HLL 5	0,2966	0,2303	5,33%	0,5462	0,3125	0,2427	5,62%	0,4965
OPT	0,3051	0,2365	5,39%	0,5424	0,3228	0,2502	5,71%	0,4878
	5-step-ahead Model				10-step-ahead Model			
	RMSE	MAE	MAPE	Mincer-Z.	RMSE	MAE	MAPE	Mincer-Z.
ALL 1	0,2992	0,2310	5,41%	0,4527	0,3222	0,2501	5,87%	0,3676
ALL 2	0,3126	0,2433	5,65%	0,4380	0,3341	0,2591	6,03%	0,3608
ALL 5	0,3371	0,2625	6,02%	0,4154	0,3549	0,2765	6,36%	0,3546
ALL 15	0,3659	0,2852	6,45%	0,3916	0,3817	0,2967	6,73%	0,3406
ALL 30	0,4025	0,3181	7,12%	0,3664	0,4171	0,3289	7,38%	0,3222
HLL 1	0,2992	0,2310	5,51%	0,4527	0,3222	0,2501	5,98%	0,3676
HLL 2	0,3126	0,2433	5,74%	0,4380	0,3341	0,2591	6,13%	0,3608
HLL 5	0,3371	0,2625	6,10%	0,4154	0,3549	0,2765	6,43%	0,3546
OPT	0,3453	0,2692	6,16%	0,4152	0,3630	0,2818	6,47%	0,3560

Table 20 – Forecasting performance of the HARs for PETR4

	1-step-ahead Model				2-step-ahead Model			
Asset	RMSE	MAE	MAPE	Mincer-Z.	RMSE	MAE	MAPE	Mincer-Z.
ALL 1	0,2590	0,1950	4,32%	0,6549	0,2724	0,2065	4,59%	0,6187
ALL 2	0,2874	0,2172	4,78%	0,6098	0,2904	0,2197	4,83%	0,6012
ALL 5	0,3026	0,2302	5,02%	0,6087	0,3171	0,2407	5,26%	0,5708
ALL 15	0,3410	0,2669	5,72%	0,5529	0,3540	0,2739	5,89%	0,5186
ALL 30	0,3834	0,3005	6,38%	0,5081	0,3971	0,3097	6,59%	0,4728
HLL 1	0,2590	0,1950	4,37%	0,6549	0,2724	0,2065	4,63%	0,6187
HLL 2	0,2759	0,2082	4,60%	0,6397	0,2904	0,2197	4,87%	0,6012
HLL 5	0,3026	0,2302	5,02%	0,6087	0,3171	0,2407	5,26%	0,5708
OPT	0,3090	0,2358	5,10%	0,6006	0,3212	0,2445	5,30%	0,5690
	5-step-ahead Model				10-step-ahead Model			
	RMSE	MAE	MAPE	Mincer-Z.	RMSE	MAE	MAPE	Mincer-Z.
ALL 1	0,3080	0,2338	5,20%	0,5133	0,3321	0,2570	5,72%	0,4360
ALL 2	0,3258	0,2489	5,48%	0,4992	0,3508	0,2732	0,60%	0,4211
ALL 5	0,3490	0,2675	5,85%	0,4809	0,3721	0,2891	6,33%	0,4115
ALL 15	0,3809	0,2965	6,39%	0,4433	0,3997	0,3111	6,72%	0,3878
ALL 30	0,4312	0,3307	7,05%	0,4075	0,4387	0,3437	7,36%	0,3572
HLL 1	0,3080	0,2338	5,26%	0,5133	0,3321	0,2570	5,78%	0,4360
HLL 2	0,3258	0,2489	5,52%	0,4992	0,3507	0,2732	6,07%	0,4211
HLL 5	0,3490	0,2675	5,85%	0,4809	0,3721	0,2891	6,33%	0,4115
OPT	0,3542	0,2721	5,91%	0,4764	0,3783	0,2947	6,41%	0,4038

Table 21 – Forecasting performance of the HARs for USIM5

	1-step-ahead Model				2-step-ahead Model			
Asset	RMSE	MAE	MAPE	Mincer-Z.	RMSE	MAE	MAPE	Mincer-Z.
ALL 1	0,2589	0,1999	4,62%	0,5707	0,2726	0,2113	4,89%	0,5243
ALL 2	0,2707	0,2090	4,79%	0,5521	0,2843	0,2216	5,08%	0,5064
ALL 5	0,2973	0,2316	5,25%	0,5165	0,3107	0,2419	5,50%	0,4723
ALL 15	0,3332	0,2658	5,93%	0,4536	0,3420	0,2732	6,11%	0,4246
ALL 30	0,3728	0,2993	6,63%	0,3999	0,3803	0,3054	6,77%	0,3761
HLL 1	0,2589	0,1999	4,73%	0,5707	0,2726	0,2113	5,01%	0,5243
HLL 2	0,2707	0,2090	4,90%	0,5521	0,2843	0,2216	5,20%	0,5064
HLL 5	0,2973	0,2316	5,32%	0,5165	0,3107	0,2419	5,57%	0,4723
OPT	0,3047	0,2401	5,42%	0,5026	0,3190	0,2523	5,70%	0,4555
	5 step-ahead Model				10-step-ahead Model			
	RMSE	MAE	MAPE	Mincer-Z.	RMSE	MAE	MAPE	Mincer-Z.
ALL 1	0,2943	0,2294	5,33%	0,4473	0,3154	0,2455	5,71%	0,3670
ALL 2	0,3053	0,2386	5,49%	0,4321	0,3256	0,2533	5,85%	0,3557
ALL 5	0,3315	0,2583	5,89%	0,4011	0,3488	0,2684	6,13%	0,3392
ALL 15	0,3573	0,2852	6,39%	0,3739	0,3735	0,2943	6,61%	0,3187
ALL 30	0,3943	0,3256	7,01%	0,3306	0,4083	0,3218	7,17%	0,2852
HLL 1	0,2943	0,2294	5,46%	0,4473	0,3154	0,2455	5,86%	0,3670
HLL 2	0,3053	0,2386	5,62%	0,4321	0,3256	0,2533	5,99%	0,3557
HLL 5	0,3315	0,2583	5,97%	0,4011	0,3488	0,2684	6,22%	0,3392
OPT	0,3369	0,2646	6,00%	0,3941	0,3509	0,2727	6,20%	0,3451

Table 22 – Forecasting performance of the HARs for VALE5

	1-step-ahead Model				2-step-ahead Model			
Asset	RMSE	MAE	MAPE	Mincer-Z.	RMSE	MAE	MAPE	Mincer-Z.
ALL 1	0,3025	0,2400	5,30%	0,5774	0,3147	0,2474	5,47%	0,5430
ALL 2	0,3130	0,2490	5,46%	0,5772	0,3265	0,2583	5,68%	0,5404
ALL 5	0,3393	0,2706	5,88%	0,5488	0,3557	0,2819	6,14%	0,5045
ALL 15	0,3749	0,3000	6,44%	0,4901	0,3911	0,3098	6,66%	0,4456
ALL 30	0,4127	0,3321	7,07%	0,4475	0,4260	0,3412	7,27%	0,4115
HLL 1	0,3025	0,2400	5,35%	0,5774	0,3147	0,2474	5,53%	0,5430
HLL 2	0,3130	0,2490	5,53%	0,5772	0,3265	0,2583	5,74%	0,5404
HLL 5	0,3393	0,2706	5,95%	0,5488	0,3557	0,2819	6,21%	0,5045
OPT	0,3450	0,2766	6,00%	0,5426	0,3611	0,2866	6,23%	0,4993
	5 step-ahead Model				10-step-ahead-model			
	RMSE	MAE	MAPE	Mincer-Z.	RMSE	MAE	MAPE	Mincer-Z.
ALL 1	0,3378	0,2694	5,97%	0,4739	0,3634	0,2845	6,34%	0,3921
ALL 2	0,3507	0,2802	6,17%	0,4696	0,3766	0,2965	6,56%	0,3893
ALL 5	0,3797	0,3029	6,62%	0,4355	0,4035	0,3181	6,98%	0,3631
ALL 15	0,4132	0,3285	7,08%	0,3810	0,4328	0,3422	7,41%	0,3218
ALL 30	0,4462	0,3581	7,65%	0,3548	0,4621	0,3689	7,91%	0,3088
HLL 1	0,3378	0,2694	6,04%	0,4739	0,3634	0,2845	6,41%	0,3921
HLL 2	0,3507	0,2802	6,24%	0,4696	0,3766	0,2965	6,64%	0,3893
HLL 5	0,3797	0,3029	6,69%	0,4355	0,4035	0,3181	7,06%	0,3631
OPT	0,3852	0,3077	6,71%	0,4289	0,4088	0,3223	7,06%	0,3581

4.6 HAR vs GARCH and EWMA: forecasting ability

The performance comparison between GARCH, EWMA and HAR is based on the realized volatility measure, which is the method used by Hansen and Lunde (2001) and adopted by the literature as the main way of comparing the forecasting performance of these models. Otherwise, the only aspect we can evaluate is that both models provide consistent forecasts at larger forecasting horizons such as 5 and 10 steps. The HAR increases less the forecast error relative to RV than the EWMA and GARCH do in relation to their own 1-step-ahead forecast. Hence, using the realized volatility as reference, it can be observed that the mean squared error of the 1-step-ahead forecasts of the EWMA and the GARCH is higher than that of the HAR: 0.31 to 0.42 and 0.30 to 0.36 against 0.25 to 0.39, respectively, varying according to the chosen asset. Furthermore, the MPAE also presents higher values: 4.32% to 5.97% of the HAR against 6.42% to 7.46% and 5.8% to 8.2% of the EWMA and GARCH, respectively. With respect to the Mincer-Zarnowitz test, the HAR obtained 0.65 to 0.44, while the

EWMA obtained 0.55 to 0.35 and 0.59 to 0.40, demonstrating a higher ability of the HAR models.

When the criterion for comparison is the forecast more than one step ahead, the EWMA and GARCH increase the root mean squared error between 0.13 and 0.16 in relation to their own 1-step-ahead forecast, while the HAR increases 0.05 and 0.08, demonstrating that the loss of precision occurs more often in the conditional variance models. In addition to that aspect, the other tests show that the RMSE, MAE, MPSE and the Mincer-Zarnowitz test for the HAR are superior to those of the EWMA and GARCH. Thus, since the differences were not significant, we believe that the HAR models' ability is slightly superior to that of the conditional variance models. The table below summarizes the results:

Table 23 – HAR HLL, GARCH and EWMA – Forecasting ability

		RMSE			MAE			MPSE			Micer-Zarnowitz		
PAF	Asset	GARCH	EWMA	HAR HLL[1]	GARCH	EWMA	HAR HLL[1]	GARCH	EWMA	HAR HLL[1]	GARCH	EWMA	HAR HLL[1]
	GGBR4	0,3092	0,3497	0,2569	0,2416	0,2729	0,1990	0,0580	6,52%	4,73%	47,60%	0,4258	0,5954
	PETR4	0,3149	0,3089	0,2590	0,2454	0,2355	0,1950	0,0590	5,24%	4,37%	59,54%	0,5534	0,6549
	USIM5	0,3260	0,3186	0,2589	0,2514	0,2521	0,1999	0,0605	6,02%	4,73%	48,97%	0,4777	0,5707
1	VALE5	0,3365	0,4236	0,3025	0,2582	0,3456	0,2400	0,0623	7,46%	5,35%	40,20%	0,3574	0,5774
	GGBR4	0,2939	0,3583	0,2730	0,2339	0,2778	0,2099	0,0561	6,64%	5,00%	45,31%	0,4025	0,5433
	PETR4	0,3008	0,3220	0,2724	0,2365	0,2453	0,2065	0,0568	5,47%	4,63%	56,33%	0,5208	0,6187
	USIM5	0,3127	0,3345	0,2726	0,2429	0,2627	0,2113	0,0584	6,28%	5,01%	46,18%	0,4282	0,5243
2	VALE5	0,3202	0,4286	0,3147	0,2483	0,3501	0,2474	0,0597	7,56%	5,53%	39,13%	0,3430	0,5430
	GGBR4	0,2834	0,3721	0,2992	0,2217	0,2872	0,2310	0,0492	6,88%	5,51%	40,15%	0,3607	0,4527
	PETR4	0,2949	0,3275	0,3080	0,2290	0,2502	0,2338	0,0509	5,61%	5,26%	48,86%	0,5135	0,5133
	USIM5	0,3203	0,3533	0,2943	0,2511	0,2734	0,2294	0,0558	6,55%	5,46%	40,80%	0,3698	0,4473
5	VALE5	0,3394	0,4433	0,3378	0,2698	0,3611	0,2694	0,0599	7,82%	6,04%	33,48%	0,3003	0,4739
	GGBR4	0,4233	0,3901	0,3222	0,3456	0,3005	0,2501	0,0739	7,22%	5,98%	34,60%	0,3073	0,3676
	PETR4	0,4278	0,3735	0,3321	0,3499	0,2861	0,2570	0,0747	6,41%	5,78%	43,05%	0,3920	0,4360
	USIM5	0,4460	0,3693	0,3154	0,3648	0,2838	0,2455	0,0781	6,81%	5,86%	36,54%	0,3210	0,3670
10	VALE5	0,4677	0,4650	0,3634	0,3830	0,3793	0,2845	0,0821	8,24%	6,41%	26,59%	0,2382	0,3921

4.7 Value-at-Risk

In this section, it is presented the performance of the HAR models and the conditional variance models, EWMA and GARCH, in the Value-at-Risk models. The criteria used to evaluate the performance are the Kupiec Test and the Christoffersen

Test, both described in the methodology section. Finally, we will make the comparison between the models in the subsection 4.7.1 and 4.7.2 and general comments about the results found.

4.7.1 Heterogeneous Auto-Regressive (HAR)

Overall, the HAR model performed well in the value-at-risk for GGBR4, USIM5 and VALE5 only, since the result for PETR4 was poor. In the case of GGBR4 and USIM5, the HAR model constructed without microstructure noise correction with 1-minute frequency was approved in the Kupiec Test at all forecasting horizons and fitted the maximum loss of 10%, 5%, 2,5% and 1%[8]. In the Christoffersen Test, we perceived that the errors are independent, i.e., the fact that an error occurred on time t does not mean that the probability of another violation increases on t+1. However, we emphasize that in the independence criterion, the USIM5 series provided forecasts adequate to the VaR at short horizons, such as 1 and 2 steps ahead. Table 24 presents the data:

Table 24 – Performance of the HAR ALL¹ model in the Value-at-Risk for USIM5 and GGBR4

		USIM5				GGBR4			
	Test\PAF	1	2	5	10	1	2	5	10
10%	Violations	**94**	**99**	**92**	**100**	**96**	**96**	**100**	**98**
	% Violations	**9,40%**	**9,90%**	**9,20%**	**10,00%**	**9,60%**	**9,60%**	**10,00%**	**9,80%**
	Kupiec	**0,4073**	**0,0111**	**0,7287**	**-**	**0,1799**	**0,1799**	**0,00**	**0,0447**
	Christoffersen	**3,6248**	5,5785	12,2844	11,2971	**0,2579**	**0,5783**	**1,7920**	**0,7304**
5%	Violations	**53**	**58**	**52**	**61**	**47**	**40**	**49**	**57**
	% Violations	**5,30%**	**5,80%**	**5,20%**	**6,10%**	**4,70%**	**4,00%**	**4,90%**	**5,70%**
	Kupiec	**0,1860**	**1,2843**	**0,0832**	**2,3877**	**0,1932**	**2,2534**	**0,0212**	**0,9889**
	Christoffersen	**3,3300**	**3,2163**	11,6095	12,3945	**0,2161**	**2,3537**	**1,0168**	**1,9127**
2,5%	Violations	**29**	**29**	**33**	**38**	**18**	15	**25**	**31**
	% Violations	**2,90%**	**2,90%**	**3,30%**	**3,80%**	**1,80%**	1,50%	**2,50%**	**3,10%**
	Kupiec	**0,6248**	**0,6248**	**2,3895**	**5,9961**	**2,2240**	4,7774	**0,0000**	**1,3739**
	Christoffersen	**0,65**	**0,65**	9,90	21,94	**0,00**	0,00	**0,20**	**2,30**
1%	Violations	**15**	**14**	**15**	**22**	**11**	**10**	**14**	**15**
	% Violations	**1,50%**	**1,40%**	**1,50%**	**2,20%**	**1,10%**	**1,00%**	**1,40%**	**1,50%**
	Kupiec	**2,1892**	**1,4374**	**2,1892**	**10,8382**	**0,0978**	**-**	**1,4374**	**2,1892**
	Christoffersen	**0,0000**	**0,0000**	**0,0000**	29,9335	**0,0000**	**0,0000**	**0,0000**	**0,0000**

In the case of the VALE5 series, the low volatility presented by the model caused the number of violations of the Value-at-Risk to slightly increase, as we can see in Table 25. In this case, since the model is estimated by OLS, the forecast directly

[8] We want to remind that the reference value of the Kupiec Test and the Christoffersen Test is 3.87 to 5% of confidence.

generated by the equation is the most likely source of error, but it presents standard deviations that end up constructing an interval of possibilities, thus the most distant from the center, the less likely it becomes. Considering that, we added a standard deviation to the HAR model's forecast for the VALE5 series. Thus, the model performed satisfactorily in the Kupiec Test for 5%, 2.5% and 1% of maximum loss and at all horizons. The model did not fit in 10%. In the Christoffersen Test, the model also presented good results and passed all the tests, being rejected at the 10-step-ahead horizon only. In other words, the HAR model fully performed the activity for more accurate VaRs (5%, 2.5% and 1%) at 1- to 5-step-ahead horizons at least.

Table 25 – Performance of the HAR ALL[1] model in the Value-at-Risk for VALE%

	PAF	1		2		5		10	
	Test	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 DP	Without SD
10%	Violations	**84**	144	**91**	141	79	139	76	151
	% Violations	**8,40%**	14,40%	**9,10%**	14,10%	7,90%	13,90%	7,60%	15,10%
	Kupiec	**2,9914046**	19,2042659	**0,9251578**	16,7891012	5,24210404	15,2613959	6,91998522	25,4033048
	Christoffersen	5,04080362	22,1143405	5,55190822	19,6714595	0,91295951	15,5415761	8,38326141	25,8900969
5%	Violations	**44**	99	**42**	101	**41**	98	**40**	95
	% Violations	**4,40%**	9,90%	**4,20%**	10,10%	**4,10%**	9,80%	**4,00%**	9,50%
	Kupiec	**0,7884785**	39,8251532	**1,4214956**	42,813945	**1,8120182**	38,3642907	**2,2534116**	34,1182944
	Christoffersen	**0,0021541**	6,50568575	**0,7925603**	0,64482277	**2,582174**	3,29243511	5,2275551	2,9610076
2,5%	Violations	**22**	69	**21**	66	**22**	68	**23**	71
	% Violations	**2,20%**	6,90%	**2,10%**	6,60%	**2,20%**	6,80%	**2,30%**	7,10%
	Kupiec	**0,384553**	54,1180371	**0,6935456**	47,8916079	**0,384553**	52,0108564	**0,1685458**	58,425391
	Christoffersen	**0,8280432**	54,3477052	**1,2428039**	45,5946819	**0,8280432**	49,2711348	6,20457364	58,003175
1%	Violations	**7**	35	**10**	32	**9**	40	**15**	44
	% Violations	**0,70%**	3,50%	**1,00%**	3,20%	**0,90%**	4,00%	**1,50%**	4,40%
	Kupiec	**1,0156325**	38,3301031	**0**	30,9342029	**0,1045205**	51,8219642	**2,1892484**	63,5624781
	Christoffersem	**0**	38,7757514	**0**	31,7210201	**0**	46,5951126	13,2976301	61,3656862

The HAR models did not perform well for the PETR4 series estimation. Only the HAR model based on the 1-minute RV HLL performed reasonably in the Kupiec Test, and even so, only at 1- to 5-step-ahead horizons. In addition, it did not fit all maximum loss percentages of the test; further, the model performed poorly in the Christoffersen independence test, demonstrating that it presents errors in a correlated way. Ultimately, it can be said that the model presented errors in a correlated way in the past, which means that the maximum loss for more than one day ahead may be higher than suggested by the percentage of violations estimated. Attached are all the tables with the value-at-risk estimates for all the HAR models estimated, with the forecast center and a standard deviation.

4.7.2 GARCH and EWMA

The results of the GARCH and EWMA models indicate that none of the models works properly for all the stocks used in the paper. Moreover, the main problem is that these models fail the independence test of Christoffersen, demonstrating that they violate the value-at-risk in a non-random way.

The EWMA model was approved in the Kupiec Test in 10%, 5% and 2.5% at all horizons for the GGBR4, USIM5 and VALE5 stock series. Further, in 1% it was only approved for USIM5. However, the Christoffersen test accepts GGBR4 and VALE5 only, rejecting almost all the USIM5 forecasts. In short, the EWMA model worked at all horizons in 10%, 5% and 2.5% for GGBR4 and VALE5.

On the other hand, the GARCH models were approved in the Kupiec Test at all forecasting horizons with 10%, 5%, 2.5% and 1% of maximum loss for the next period for PETR4, USIM5 and VALE5. However, when the maximum loss for the next period is 10% for the PETR4 price series, and 10% and 5% for the USIM5 price series, the Christoffersen test rejects the independence of the model's violation, putting into doubt the model's reliability with this configuration. In other words, the GARCH models showed superiority to the EWMA models, since they can provide safe forecasts for PETR4, USIM5 and VALE5 in maximum losses for the next period in the more accurate 2.5% and 1% systems.

Table 26 – Performance in the Value-at-Risk of the GARCHs and EWMAs for GGBR4

Model		EWMA				GARCH			
	PAF	Violations	% Viol.	Kupiec	Christoffersen	Violations	% Viol.	Kupiec Test	Christ. Test
	1	**106**	**10,60%**	**0,3931**	**1,4512**	**99**	**9,91%**	**0,0090**	**0,5737**
	2	**109**	**10,90%**	**0,8770**	**1,6522**	**106**	**10,61%**	**0,4066**	11,0774
	5	**109**	**10,90%**	**0,8770**	4,7189	**108**	**10,81%**	**0,7129**	8,6710
10%	10	**110**	**11,00%**	**1,0798**	**2,2771**	**111**	**11,11%**	**1,3276**	11,1719
	1	**55**	**5,50%**	**0,5105**	**0,3208**	**49**	**4,90%**	**0,0191**	**1,1612**
	2	**55**	**5,50%**	**0,5105**	**2,6187**	**62**	**6,21%**	**2,8515**	13,5336
	5	**61**	**6,10%**	**2,3877**	**2,6764**	**61**	**6,11%**	**2,4110**	11,0710
5%	10	**64**	**6,40%**	**3,8054**	**3,4317**	**60**	**6,01%**	**2,0054**	13,8398
	1	**29**	**2,90%**	**0,6248**	**0,0298**	**25**	**2,50%**	**0,0000**	**0,2002**
	2	**28**	**2,80%**	**0,3556**	**0,0577**	40	4,00%	7,8633	15,9755
	5	**32**	**3,20%**	**1,8494**	**0,7868**	39	3,90%	6,9165	15,5454
2,5%	10	**35**	**3,50%**	**3,6560**	**0,0476**	38	3,80%	6,0230	15,1918
	1	18	1,80%	5,2251	**0,0000**	11	1,10%	0,0999	**0,0000**
	2	16	1,60%	3,0766	**0,0000**	19	1,90%	6,4908	26,0279
	5	20	2,00%	7,8272	**0,0000**	19	1,90%	6,4908	26,0279
1%	10	20	2,00%	7,8272	**0,0000**	19	1,90%	6,4908	26,0279

Table 27 – Performance in the Value-at-Risk of the GARCHs and EWMAs for PETR4

Model		EWMA				GARCH			
	PAF	Violations	% Viol.	Kupiec	Christoffersen	Violations	% Viol.	Kupiec Test	Christ. Test
	1	**112**	**11,20%**	**1,5463**	4,9460	**99**	**9,91%**	**0,0090**	5,5411
	2	**112**	**11,20%**	**1,5463**	11,1145	**101**	**10,11%**	**0,0134**	9,5868
	5	**113**	**11,30%**	**1,8099**	10,5754	**103**	**10,31%**	**0,1059**	7,0654
10%	10	**118**	**11,80%**	**3,4238**	13,2259	**103**	**10,31%**	**0,1059**	7,0654
	1	**65**	**6,50%**	**4,3455**	**1,7783**	**55**	**5,51%**	**0,5211**	**0,8375**
	2	**64**	**6,40%**	**3,8054**	7,2738	**55**	**5,51%**	**0,5211**	**1,7395**
	5	71	7,10%	8,2609	6,3864	**59**	**5,91%**	**1,6353**	**3,3529**
5%	10	72	7,20%	9,0221	2,7501	**58**	**5,81%**	**1,3013**	**2,0833**
	1	**32**	**3,20%**	**1,8494**	**0,0007**	**27**	**2,70%**	**0,1641**	**0,2583**
	2	**32**	**3,20%**	**1,8494**	**0,0007**	**29**	**2,90%**	**0,6330**	**0,0000**
	5	39	3,90%	6,8875	1,2349	**28**	**2,80%**	**0,3618**	**0,4187**
2,5%	10	40	4,00%	7,8323	5,2202	**31**	**3,10%**	**1,3863**	**2,3072**
	1	17	1,70%	4,0910	**0,0000**	**14**	**1,40%**	**1,4455**	**0,0000**
	2	18	1,80%	5,2251	**0,0000**	**12**	**1,20%**	**0,3838**	**0,0000**
	5	20	2,00%	7,8272	**0,0000**	**15**	**1,50%**	**2,1994**	**0,0000**
1%	10	22	2,20%	10,8382	0,4435	17	1,70%	4,1052	5,2233

Table 28 – Performance in the Value-at-Risk of the GARCHs and EWMAs for USIM5

		EWMA				GARCH			
	PAF	Violations	% Viol.	Kupiec	Christoffersen	Violations	% Viol.	Kupiec Test	Christ. Test
	1	**98**	**9,80%**	**0,0447**	6,7810	**92**	**9,21%**	**0,7111**	4,4237
	2	**100**	**10,00%**	**0,0000**	11,2971	**89**	**8,91%**	**1,3667**	**3,5471**
	5	**102**	**10,20%**	**0,0442**	9,0407	**88**	**8,81%**	**1,6342**	5,2401
10%	10	**94**	**9,40%**	**0,4073**	9,2025	**86**	**8,61%**	**2,2443**	8,8878
	1	**58**	**5,80%**	**1,2843**	6,4468	**45**	**4,50%**	**0,5334**	4,0078
	2	**56**	**5,60%**	**0,7308**	12,0687	**44**	**4,40%**	**0,7759**	**1,3151**
	5	**60**	**6,00%**	**1,9842**	8,5864	**49**	**4,90%**	**0,0191**	**0,9936**
5%	10	**61**	**6,10%**	**2,3877**	10,0068	**53**	**5,31%**	**0,1924**	11,2251
	1	**26**	**2,60%**	**0,0405**	**1,8308**	**22**	**2,20%**	**0,3784**	**0,8154**
	2	**28**	**2,80%**	**0,3556**	**1,4233**	**23**	**2,30%**	**0,1645**	**0,5092**
	5	**34**	**3,40%**	**2,9923**	11,6661	**24**	**2,40%**	**0,0395**	**0,0000**
2,5%	10	37	3,70%	5,1594	10,4229	**33**	**3,30%**	**2,4060**	11,7758
	1	**15**	**1,50%**	**2,1892**	**1,5151**	**11**	**1,10%**	**0,0999**	**0,0000**
	2	**14**	**1,40%**	**1,4374**	**1,7458**	**12**	**1,20%**	**0,3838**	**0,0000**
	5	**15**	**1,50%**	**2,1892**	10,0658	**12**	**1,20%**	**0,3838**	**0,0000**
1%	10	22	2,20%	10,8382	14,4233	**16**	**1,60%**	**3,0887**	16,7392

Table 29 – Performance in the Value-at-Risk of the GARCHs and EWMAs for VALE5

	Model	EWMA				GARCH			
	PAF	Violations	% Viol.	Kupiec	Christoffersen	Violations	% Viol.	Kupiec Test	Christ. Test
	1	**106**	**10,60%**	**0,3931**	**1,4512**	**85**	**8,50%**	**2,6204**	**3,7676**
	2	**109**	**10,90%**	**0,8770**	**1,6522**	**88**	**8,80%**	**1,6606**	**2,3853**
	5	**109**	**10,90%**	**0,8770**	4,7189	**90**	**9,00%**	**1,1458**	**3,1559**
10%	10	**110**	**11,00%**	**1,0798**	**2,2771**	**87**	**8,70%**	**1,9553**	**3,6161**
	1	**55**	**5,50%**	**0,5105**	**0,3208**	35	3,50%	5,2684	5,7123
	2	**55**	**5,50%**	**0,5105**	2,6187	**46**	**4,60%**	**0,3457**	**0,3533**
	5	**61**	**6,10%**	**2,3877**	2,6764	**44**	**4,40%**	**0,7885**	**1,3393**
5%	10	**64**	**6,40%**	**3,8054**	3,4317	**51**	**5,10%**	**0,0209**	8,4600
	1	**29**	**2,90%**	**0,6248**	**0,0298**	**15**	**1,50%**	**4,7774**	**0,0000**
	2	**28**	**2,80%**	**0,3556**	**0,0577**	**25**	**2,50%**	**0,0000**	**2,0569**
	5	**32**	**3,20%**	**1,8494**	**0,7868**	**26**	**2,60%**	**0,0405**	**0,0000**
2,5%	10	**35**	**3,50%**	**3,6560**	**0,0476**	**26**	**2,60%**	**0,0405**	**1,8685**
	1	18	1,80%	5,2251	**0,0000**	2	0,20%	9,6267	**0,0000**
	2	**16**	**1,60%**	**3,0766**	**0,0000**	**12**	**1,20%**	**0,3798**	**0,0000**
	5	20	2,00%	7,8272	**0,0000**	**13**	**1,30%**	**0,8306**	**0,0000**
1%	10	20	2,00%	7,8272	**0,0000**	**14**	**1,40%**	**1,4374**	7,6100

4.7.3 GARCH compared to the HAR

In our assessment, we concluded that none of the models showed superiority to the others, with each of them presenting advantages and disadvantages. The GARCH models succeeded with three stocks when the loss estimated for the next period was 1%

and 2.5% for all forecasting horizons. On the other hand, for three stocks the HAR models performed better at short forecasting horizons - 1 and 2 steps ahead - for all the maximum loss percentages for the next day. For comparison purposes, we can only point out that none of the models was superior; however, the HAR could not model the value-at-risk of PETR4 and was inferior to the GARCH for VALE5. In turn, the GARCH could not model the value-at-risk of GGBR4, had an equivalent performance for USIM5 and was superior for VALE5. In that case, we may conclude that the models are equivalent, with a small advantage of the GARCH.

Comparing our results with the international literature, it is possible to highlight that the HAR models actually provide better forecasts, as pointed out by almost all the literature on HAR. However, as noted by Giot and Laurent (2001, 2003), still using realized volatility modeled with ARFIMA, they do not outperform better the RV models. Further, Kruse (2006) tests the realized volatility and does not prove that these models are statistically superior to the GARCHs. In that sense, we believe that further studies are required to analyze these models, as we know that comparisons can always be criticized due to the reference issue. However they should be investigated, as the RV models have all the theory on their side, and the GARCHs have all their history and successful empirical applications.

The difference between the group of stocks that succeeded and the one that did not may have two explanations: (i) the HAR specification should be improved to find models with better fit levels, like the Thresold-HAR of Medeiros and McAller (2008); and (ii) it may be due to the microstructure noise that the hypotheses of the model of Hansen, Large and Lunde (2006) did not fit the VaR in Brazilian data. Since the market liquidity is higher for PETR4 and VALE5 series, perhaps they require correction methods more accurate if compared to the GGBR4 and USIM5 series, which showed good results. Models with Jumps - Andersen (2007), Zhang, Mykland and Sahalia (2005), or Barndorff-Nielsen, Hansen, Lunde, Shephard (2006a, 2006b, 2008a, 2008b) - have different and more restrictive hypotheses that can provide better results. We still highlight the contribution of this article from the perspective of the realized volatility. Unlike the USA market, the best models had the highest frequencies, suggesting that the microstructure noise is very small and does not significantly grows as the sampling frequency increases, at least for USIM5 and GGBR4.

5. Conclusion

This article applies the recent realized volatility techniques and the Heterogeneous Auto-Regressive (HAR) models to data of four stocks traded at the São Paulo Stock Exchange with an effective participation at the Ibovespa index. The goal is to find out whether these models have a superior forecasting ability to the traditional GARCH (1,1) and the Exponential Weighted Moving-Average (EWMA). Furthermore, we performed an empirical application to the Value-at-Risk with the different volatility estimates generated from the models above.

For that purpose, we had to deal with the microstructure noise in connection with observation of the intraday price to create a realized volatility measure that was really consistent and not biased. Hence, we used the optimal frequency of Bandi and Russel (2005), the microstructure correction filter of Hansen, Large and Lunde (2006), and a variety of frequencies between 30 and 15 minutes. In the next stage, we estimated the HAR models for all the different estimates and compared the results to the forecasts of the GARCH and EWMA models. Then we concluded that the HAR model fits Brazilian data as well or better than in the international literature, being that the higher the sampling frequency of intraday data, the better the model fits. Moreover, in the forecasting comparison with 1- to 10-step-ahead horizons against the GARCH and the EWMA, we found out that the HAR model's forecast was slightly superior, especially at 10-day horizons.

Finally, we used the realized volatility prediction to forecast the maximum loss for the next period at a 1- to 10-day horizon, with different expected loss percentages (10%, 5%, 2.5% and 1%). Our results, based on the Kupiec Test and the Christoffersen Test, point out that the techniques complement each other by offering qualities and disadvantages. The HAR model performed well in three stocks at 1- and 2-step-ahead horizons, independently from the maximum loss percentage attributed in the value-at-risk. Furthermore, it outperformed the GARCH for the GGBR4 series. On the other hand, the GARCH models were also fit to 3 stocks, at all forecasting horizons, but only when the value-at-risk expected losses were small: 2.5% and 1%. The GARCH was not fit to GGBR4 and the HAR was not sufficient to properly model the PETR4 series. Hence, we found out that the models do not offer any advantages over the others and therefore there is not a clear evidence of superiority relative to the utilization for Value-at-Risk.

Thus, the contribution of this article is basically to present evidence that the microstructure noise is small or the filter-based correction method hypotheses do not fit Brazilian data, since the models with high sampling frequency and with no correction method were more successful. Moreover, we demonstrated that the HAR models are fit to Brazilian data, showing superiority with data of S&P500 and DJIA. Further, we found out that this technique's forecasting ability is slightly superior to that of traditional techniques. With respect to the VaR, the application of the technique does not show advantages in any of the econometric models.

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ANNEXES

Table A01 – HAR Models for GGBR4 with VaR (10%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD
ALL 1MIN	N. Violations	45	96	37	96	43	100	47	98
	% N. Violations	4,50%	9,60%	3,70%	9,60%	4,30%	10,00%	4,70%	9,80%
	Kupiec Test	41,42896971	0,179928551	56,73588873	0,179928551	44,95468819	0	38,08946455	0,044710522
	Christ. Test	41,42936647	0,257860597	57,01170713	0,578313715	47,05513233	1,79204065	41,03415679	0,730412291
ALL 2MIN	N. Violations	49	101	43	105	45	109	50	106
	% N. Violations	4,90%	10,10%	4,30%	10,50%	4,50%	10,90%	5,00%	10,60%
	Kupiec Test	34,92861811	0,011078355	44,95468819	0,273763837	41,42896971	0,87703929	33,41300236	0,393097709
	Christ. Test	35,00761527	0,38250697	45,620423	1,20674869	41,88071054	1,32778941	35,61376036	2,679693683
ALL 5MIN	N. Violations	47	109	42	112	48	114	52	107
	% N. Violations	4,70%	10,90%	4,20%	11,20%	4,80%	11,40%	5,20%	10,70%
	Kupiec Test	38,08946455	0,87703929	46,78992838	1,546341399	36,48715607	2,09335969	30,5073124	0,533536922
	Christ. Test	38,11235207	1,00392835	47,57979747	2,670827208	39,16970635	2,467696019	32,28040392	2,596619154
ALL 15MIN	N. Violations	50	123	48	120	57	122	58	123
	% N. Violations	5,00%	12,30%	4,80%	12,00%	5,70%	12,20%	5,80%	12,30%
	Kupiec Test	33,41300236	5,518335419	36,48715607	4,204947331	23,94092875	5,06182356	22,74185085	5,518335419
	Christ. Test	33,53230652	5,805720141	36,69926023	4,424406926	26,08872528	5,860478671	24,67383458	9,189149923
ALL 30MIN	N. Violations	53	134	50	132	55	131	62	134
	% N. Violations	5,30%	13,40%	5,00%	13,20%	5,50%	13,10%	6,20%	13,40%
	Kupiec Test	29,11570388	11,73638809	33,41300236	10,44628598	26,4513402	9,827363003	18,30588902	11,73638809
	Christ. Test	29,61733451	12,39087674	35,61376036	10,92833377	27,67859004	10,41904611	22,33959043	14,24795525
HLL 1MIN	N. Violations	29	77	26	79	28	81	34	88
	% N. Violations	2,90%	7,70%	2,60%	7,90%	2,80%	8,10%	3,40%	8,80%
	Kupiec Test	75,6626584	6,332660119	83,87639085	5,242104043	78,32618612	4,261510827	63,36678798	1,660620206
	Christ. Test	76,90561175	6,333494993	85,70720696	5,782172748	79,74946752	7,305136172	65,61140691	1,892536933
HLL 2MIN	N. Violations	38	89	32	93	32	92	40	94
	% N. Violations	3,80%	8,90%	3,20%	9,30%	3,20%	9,20%	4,00%	9,40%
	Kupiec Test	54,63989245	1,390880737	68,08926505	0,556154971	68,08926505	0,728685298	50,61070203	0,407335507
	Christ. Test	54,84694609	1,559267433	68,87608225	1,277388553	70,84423116	4,521504037	55,83088999	1,010036182
HLL 5MIN	N. Violations	43	100	36	107	37	105	45	99
	% N. Violations	4,30%	10,00%	3,60%	10,70%	3,70%	10,50%	4,50%	9,90%
	Kupiec Test	44,95468819	0	58,8880225	0,533536922	56,73588873	0,273763837	41,42896971	0,011144201
	Christ. Test	44,96750364	1,027442393	60,68560293	1,810149122	58,33208037	1,911224682	44,94257544	0,581152331
HAR OPT	N. Violations	48	108	41	113	48	116	53	113
	% N. Violations	4,80%	10,80%	4,10%	11,30%	4,80%	11,60%	5,30%	11,30%
	Kupiec Test	36,48715607	0,694907638	48,67487979	1,809873476	36,48715607	2,719586373	29,11570388	1,809873476
	Christ. Test	36,53396956	0,87853172	49,60074629	2,279501501	39,16970635	3,301892335	30,69440054	6,413686121

*The model did not have any sequential errors

Table A02 –HAR Models for GGBR4 with VaR (5%)

99	**0,01**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD
ALL 1MIN	N. Violations	3	11	2	10	1	14	6	15
	% N. Violations	0,30%	1,10%	0,20%	1,00%	0,10%	1,40%	0,60%	1,50%
	Kupiec	6,825541879	0,097834397	9,626721383	0	13,47640118	1,437406052	1,886232408	2,189248389
	Christoffersen	0*	0*	0*	0*	0*	0*	0*	0*
ALL 2MIN	N. Violations	3	11	2	12	1	16	7	18
	% N. Violations	0,30%	1,10%	0,20%	1,20%	0,10%	1,60%	0,70%	1,80%
	Kupiec Test	6,825541879	0,097834397	9,626721383	0,379760491	13,47640118	3,076553458	1,015632525	5,22514124
	Christ. Test	0*	0*	0*	0*	0*	0*	0*	6,178614482
ALL 5MIN	N. Violations	3	12	2	13	1	17	7	19
	% N. Violations	0,30%	1,20%	0,20%	1,30%	0,10%	1,70%	0,70%	1,90%
	Kupiec Test	6,825541879	0,379760491	9,626721383	0,830570982	13,47640118	4,090972555	1,015632525	6,472514923
	Christ. Test	0*	0*	0*	0*	0*	0*	0*	0*
ALL 15MIN	N. Violations	2	14	2	15	2	21	6	29
	% N Violations	0,20%	1,40%	0,20%	1,50%	0,20%	2,10%	0,60%	2,90%
	Kupiec Test	9,626721383	1,437406052	9,626721383	2,189248389	9,626721383	9,284045908	1,886232408	24,12022461
	Christ. Test	0*	0*	0*	0*	0*	0*	0*	25,36317796
ALL 30MIN	N. Violations	2	17	3	19	1	24	6	32
	% N. Violations	0,20%	1,70%	0,30%	1,90%	0,10%	2,40%	0,60%	3,20%
	Kupiec Test	9,626721383	4,090972555	6,825541879	6,472514923	13,47640118	14,22141908	1,886232408	30,9342029
	Christ. Test	0*	0*	0*	0*	0*	0*	0*	33,68916901
HLL 1MIN	N. Violations	2	7	2	6	1	10	5	11
	% N. Violations	0,20%	0,70%	0,20%	0,60%	0,10%	1,00%	0,50%	1,10%
	Kupiec Test	9,626721383	1,015632525	9,626721383	1,886232408	13,47640118	0	3,093738314	0,097834397
	Christ. Test	0*	0*	0*	0*	0*	0*	0*	0*
HLL 2MIN	N. Violations	2	8	2	7	1	14	6	14
	% N. Violations	0,20%	0,80%	0,20%	0,70%	0,10%	1,40%	0,60%	1,40%
	Kupiec Test	9,626721383	0,433740865	9,626721383	1,015632525	13,47640118	1,437406052	1,886232408	1,437406052
	Christ. Test	0*	0*	0*	0*	0*	0*	0*	0*
HLL 5MIN	N. Violations	2	12	2	11	1	15	6	14
	% N. Violations	0,20%	1,20%	0,20%	1,10%	0,10%	1,50%	0,60%	1,40%
	Kupiec Test	9,626721383	0,379760491	9,626721383	0,097834397	13,47640118	2,189248389	1,886232408	1,437406052
	Christ. Test	0*	0*	0*	0*	0*	0*	0*	0*
HAR OPT	N. Violations	3	13	2	13	1	20	7	20
	% N. Violations	0,30%	1,30%	0,20%	1,30%	0,10%	2,00%	0,70%	2,00%
	Kupiec Test	6,825541879	0,830570982	9,626721383	0,830570982	13,47640118	7,827239153	1,015632525	7,827239153
	Teste de Christ.	0*	0*	0*	0*	0*	0*	0*	8,496061442 *o

* the model did not have any sequential errors

Table A03 – HAR Models for GGBR4 with VaR (2,5%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD
ALL 1MIN	N. Violations	45	96	37	96	43	100	47	98
	% N. Violations	4,50%	9,60%	3,70%	9,60%	4,30%	10,00%	4,70%	9,80%
	Kupiec	41,42896971	0,179928551	56,73588873	0,179928551	44,95468819	0	38,08946455	0,044710522
	Christoffersen	41,42936647	0,257860597	57,01170713	0,578313715	47,05513233	1,79204065	41,03415679	0,730412291
ALL 2MIN	N. Violations	49	101	43	105	45	109	50	106
	% N. Violations	4,90%	10,10%	4,30%	10,50%	4,50%	10,90%	5,00%	10,60%
	Kupiec Test	34,92861811	0,011078355	44,95468819	0,273763837	41,42896971	0,87703929	33,41300236	0,393097709
	Christ. Test	35,00761527	0,38250697	45,620423	1,20674869	41,88071054	1,32778941	35,61376036	2,679693683
ALL 5MIN	N. Violations	47	109	42	112	48	114	52	107
	% N. Violations	4,70%	10,90%	4,20%	11,20%	4,80%	11,40%	5,20%	10,70%
	Kupiec Test	38,08946455	0,87703929	46,78992838	1,546341399	36,48715607	2,09335969	30,5073124	0,533536922
	Christ. Test	38,11235207	1,00392835	47,57979747	2,670827208	39,16970635	2,467696019	32,28040392	2,596619154
ALL 15MIN	N. Violations	50	123	48	120	57	122	58	123
	% N. Violations	5,00%	12,30%	4,80%	12,00%	5,70%	12,20%	5,80%	12,30%
	Kupiec Test	33,41300236	5,518335419	36,48715607	4,204947331	23,94092875	5,06182356	22,74185085	5,518335419
	Christ. Test	33,53230652	5,805720141	36,69926023	4,424406926	26,08872528	5,860478671	24,67383458	9,189149923
ALL 30MIN	N. Violations	53	134	50	132	55	131	62	134
	% N. Violations	5,30%	13,40%	5,00%	13,20%	5,50%	13,10%	6,20%	13,40%
	Kupiec Test	29,11570388	11,73638809	33,41300236	10,44628598	26,4513402	9,827363003	18,30588902	11,73638809
	Christ. Test	29,61733451	12,39087674	35,61376036	10,92833377	27,67859004	10,41904611	22,33959043	14,24795525
HLL 1MIN	N. Violations	29	77	26	79	28	81	34	88
	% N. Violations	2,90%	7,70%	2,60%	7,90%	2,80%	8,10%	3,40%	8,80%
	Kupiec Test	75,6626584	6,332660119	83,87639085	5,242104043	78,32618612	4,261510827	63,36678798	1,660620206
	Christ. Test	76,90561175	6,333494993	85,70720696	5,782172748	79,74946752	7,305136172	65,61140691	1,892536933
HLL 2MIN	N. Violations	38	89	32	93	32	92	40	94
	% N. Violations	3,80%	8,90%	3,20%	9,30%	3,20%	9,20%	4,00%	9,40%
	Kupiec Test	54,63989245	1,390880737	68,08926505	0,556154971	68,08926505	0,728685298	50,61070203	0,407335507
	Christ. Test	54,84694609	1,559267433	68,87608225	1,277388553	70,84423116	4,521504037	55,83088899	1,010036182
HLL 5MIN	N. Violations	43	100	36	107	37	105	45	99
	% N. Violations	4,30%	10,00%	3,60%	10,70%	3,70%	10,50%	4,50%	9,90%
	Kupiec Test	44,95468819	0	58,8880225	0,533536922	56,73588873	0,273763837	41,42896971	0,011144201
	Christ. Test	44,96750364	1,027442393	60,68560293	1,810149122	58,33208037	1,911224682	44,94257544	0,581152331
HAR OPT	N. Violations	48	108	41	113	48	116	53	113
	% N. Violations	4,80%	10,80%	4,10%	11,30%	4,80%	11,60%	5,30%	11,30%
	Kupiec Test	36,48715607	0,694907638	48,67487979	1,809873476	36,48715607	2,719586373	29,11570388	1,809873476
	Christ. Test	36,53396956	0,87853172	49,60074629	2,279501501	39,16970635	3,301892335	30,69440054	6,413686121

*the model did not have any sequential errors

Table A04 – HAR Models for GGBR4 with VaR (1%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD
ALL 1MIN	N. Violations	45	96	37	96	43	100	47	98
	% N. Violations	4,50%	9,60%	3,70%	9,60%	4,30%	10,00%	4,70%	9,80%
	Kupiec	41,42896971	0,179928551	56,73588873	0,179928551	44,95468819	0	38,08946455	0,044710522
	Christoffersen	41,42936647	0,257860597	57,01170713	0,578313715	47,05513233	1,79204065	41,03415679	0,730412291
ALL 2MIN	N. Violations	49	101	43	105	45	109	50	106
	% N. Violations	4,90%	10,10%	4,30%	10,50%	4,50%	10,90%	5,00%	10,60%
	Kupiec Test	34,92861811	0,011078355	44,95468819	0,273763837	41,42896971	0,87703929	33,41300236	0,393097709
	Christ. Test	35,00761527	0,38250697	45,620423	1,20674869	41,88071054	1,32778941	35,61376036	2,679693683
ALL 5MIN	N. Violations	47	109	42	112	48	114	52	107
	% N. Violations	4,70%	10,90%	4,20%	11,20%	4,80%	11,40%	5,20%	10,70%
	Kupiec Test	38,08946455	0,87703929	46,78992838	1,546341399	36,48715607	2,09335969	30,5073124	0,533536922
	Christ. Test	38,11235207	1,00392835	47,57979747	2,670827208	39,16970635	2,467696019	32,28040392	2,596619154
ALL 15MIN	N. Violations	50	123	48	120	57	122	58	123
	% N. Violations	5,00%	12,30%	4,80%	12,00%	5,70%	12,20%	5,80%	12,30%
	Kupiec Test	33,41300236	5,518335419	36,48715607	4,204947331	23,94092875	5,06182356	22,74185085	5,518335419
	Christ. Test	33,53230652	5,805720141	36,69926023	4,424406926	26,08872528	5,860478671	24,67383458	9,189149923
ALL 30MIN	N. Violations	53	134	50	132	55	131	62	134
	% N. Violations	5,30%	13,40%	5,00%	13,20%	5,50%	13,10%	6,20%	13,40%
	Kupiec Test	29,11570388	11,73638809	33,41300236	10,44628598	26,4513402	9,827363003	18,30588902	11,73638809
	Christ. Test	29,61733451	12,39087674	35,61376036	10,92833377	27,67859004	10,41904611	22,33959043	14,24795525
HLL 1MIN	N. Violations	29	77	26	79	28	81	34	88
	% N. Violations	2,90%	7,70%	2,60%	7,90%	2,80%	8,10%	3,40%	8,80%
	Kupiec Test	75,6626584	6,332660119	83,87639085	5,242104043	78,32618612	4,261510827	63,36678798	1,660620206
	Christ. Test	76,90561175	6,333494993	85,70720696	5,782172748	79,74946752	7,305136172	65,61140691	1,892536933
HLL 2MIN	N. Violations	38	89	32	93	32	92	40	94
	% N. Violations	3,80%	8,90%	3,20%	9,30%	3,20%	9,20%	4,00%	9,40%
	Kupiec Test	54,63989245	1,390880737	68,08926505	0,556154971	68,08926505	0,728685298	50,61070203	0,407335507
	Christ. Test	54,84694609	1,559267433	68,87608225	1,277388553	70,84423116	4,521504037	55,83088999	1,010036182
HLL 5MIN	N. Violations	43	100	36	107	37	105	45	99
	% N. Violations	4,30%	10,00%	3,60%	10,70%	3,70%	10,50%	4,50%	9,90%
	Kupiec Test	44,95468819	0	58,8880225	0,533536922	56,73588873	0,273763837	41,42896971	0,011144201
	Christ. Test	44,96750364	1,027442393	60,68560293	1,810149122	58,33208037	1,911224682	44,94257544	0,581152331
HAR OPT	N. Violations	48	108	41	113	48	116	53	113
	% N. Violations	4,80%	10,80%	4,10%	11,30%	4,80%	11,60%	5,30%	11,30%
	Kupiec Test	36,48715607	0,694907638	48,67487979	1,809873476	36,48715607	2,719586373	29,11570388	1,809873476
	Christ. Test	36,53396956	0,87853172	49,60074629	2,279501501	39,16970635	3,301892335	30,69440054	6,413686121

*the model did not have any sequential errors

Table A05 – HAR Models for PETR4 with VaR (10%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD
ALL 1MIN	N. Violations	68	112	66	121	65	119	65	121
	% N. Violations	6,80%	11,20%	6,60%	12,10%	6,50%	11,90%	6,50%	12,10%
	Kupiec	12,67443035	1,546341399	14,42053012	4,623983232	15,34202326	3,804851017	15,34202326	4,623983232
	Christoffersen	15,06491698	7,825604185	23,03393158	14,42261076	15,49497831	11,45624132	18,49342596	12,85688104
ALL 2MIN	N. Violations	71	122	70	124	72	128	71	128
	% N. Violations	7,10%	12,20%	7,00%	12,40%	7,20%	12,80%	7,10%	12,80%
	Kupiec Test	10,29094259	5,06182356	11,0545783	5,993388121	9,557629677	8,07646801	10,29094259	8,07646801
	Christ. Test	14,8546236	14,35818178	22,38501249	14,32875316	9,565724649	14,66021217	12,03517586	16,0256457
ALL 5MIN	N. Violations	73	125	71	128	75	133	68	136
	% N. Violations	7,30%	12,50%	7,10%	12,80%	7,50%	13,30%	6,80%	13,60%
	Kupiec Test	8,854250609	6,486853137	10,29094259	8,07646801	7,53579138	11,08266477	12,67443035	13,0954318
	Christ. Test	9,413279892	12,98216489	16,67732073	14,66021217	8,586867412	18,19442333	15,06491698	20,22816815
ALL 15MIN	N. Violations	76	138	71	133	80	138	70	141
	% N. Viol.ations	7,60%	13,80%	7,10%	13,30%	8,00%	13,80%	7,00%	14,10%
	Kupiec Test	6,919985224	14,52256042	10,29094259	11,08266477	4,738220152	14,52256042	11,0545783	16,78910122
	Christ. Test	9,979109964	19,66320318	13,30014466	16,93303829	8,054844411	23,46673949	15,96063334	23,08794021
ALL 30MIN	N. Violations	70	139	71	137	72	143	74	146
	% N. Violations	7,00%	13,90%	7,10%	13,70%	7,20%	14,30%	7,40%	14,60%
	Kupiec Test	11,0545783	15,26139589	10,29094259	13,80053803	9,557629677	18,3828592	8,180427156	20,89566246
	Christ. Test	15,96063334	21,16684668	12,03517586	19,30822745	13,79309037	26,33900307	15,27922241	26,39493792
HLL 1MIN	N. Violations	60	104	56	116	56	114	55	117
	% N. Violations	6,00%	10,40%	5,60%	11,60%	5,60%	11,40%	5,50%	11,70%
	Kupiec Test	20,45293559	0,17571251	25,17721706	2,719586373	25,17721706	2,09335969	26,4513402	3,062029457
	Christ. Test	21,99114785	6,761824418	25,42328609	11,76734213	25,42328609	10,4967824	27,67859004	10,1276638
HLL 2MIN	N. Violations	65	110	64	119	64	120	61	123
	% N. Violations	6,50%	11,00%	6,40%	11,90%	6,40%	12,00%	6,10%	12,30%
	Kupiec Test	15,34202326	1,079764492	16,29642589	3,804851017	16,29642589	4,204947331	19,36195317	5,518335419
	Christ. Test	18,49342596	6,750245392	23,57017756	12,99170881	16,29923569	11,41934958	22,03835227	14,32698678
HLL 5MIN	N. Violations	73	125	71	128	75	133	68	134
	% N. Violations	7,30%	12,50%	7,10%	12,80%	7,50%	13,30%	6,80%	13,40%
	Kupiec Test	8,854250609	6,486853137	10,29094259	8,07646801	7,53579138	11,08266477	12,67443035	11,73638809
	Christ. Test	9,413279892	12,98216489	16,67732073	14,66021217	8,586867412	18,19442333	15,06491698	19,75673287
HAR OPT	N. Violations	76	129	73	133	80	134	73	141
	% N. Violations	7,60%	12,90%	7,30%	13,30%	8,00%	13,40%	7,30%	14,10%
	Kupiec Test	6,919985224	8,642338306	8,854250609	11,08266477	4,738220152	11,73638809	8,854250609	16,78910122
	Christ. Test	9,979109964	13,62029819	12,77538612	15,78611225	6,828827007	18,42653564	11,35817476	23,08794021

Table A06 –HAR Models for PETR4 with VaR (5%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD
ALL 1MIN	N. Violations	68	112	66	121	65	119	65	121
	% N. Violations	6,80%	11,20%	6,60%	12,10%	6,50%	11,90%	6,50%	12,10%
	Kupiec	12,67443035	1,546341399	14,42053012	4,623983232	15,34202326	3,804851017	15,34202326	4,623983232
	Christoffersen	15,06491698	7,825604185	23,03393158	14,42261076	15,49497831	11,45624132	18,49342596	12,85688104
ALL 2MIN	N. Violations	71	122	70	124	72	128	71	128
	% N. Violations	7,10%	12,20%	7,00%	12,40%	7,20%	12,80%	7,10%	12,80%
	Kupiec Test	10,29094259	5,06182356	11,0545783	5,993388121	9,557629677	8,07646801	10,29094259	8,07646801
	Christ. Test	14,8546236	14,35818178	22,38501249	14,32875316	9,565724649	14,66021217	12,03517586	16,0256457
ALL 5MIN	N. Violations	73	125	71	128	75	133	68	136
	% N. Violations	7,30%	12,50%	7,10%	12,80%	7,50%	13,30%	6,80%	13,60%
	Kupiec Test	8,854250609	6,486853137	10,29094259	8,07646801	7,53579138	11,08266477	12,67443035	13,0954318
	Christ. Test	9,413279892	12,98216489	16,67732073	14,66021217	8,586867412	18,19442333	15,06491698	20,22816815
ALL 15MIN	N. Violations	76	138	71	133	80	138	70	141
	% N. Violations	7,60%	13,80%	7,10%	13,30%	8,00%	13,80%	7,00%	14,10%
	Kupiec Test	6,919985224	14,52256042	10,29094259	11,08266477	4,738220152	14,52256042	11,0545783	16,78910122
	Christ. Test	9,979109964	19,66320318	13,30014466	16,93303829	8,054844411	23,46673949	15,96063334	23,08794021
ALL 30MIN	N. Violations	70	139	71	137	72	143	74	146
	% N. Violations	7,00%	13,90%	7,10%	13,70%	7,20%	14,30%	7,40%	14,60%
	Kupiec Test	11,0545783	15,26139589	10,29094259	13,80053803	9,557629677	18,3828592	8,180427156	20,89566246
	Christ. Test	15,96063334	21,16684668	12,03517586	19,30822745	13,79309037	26,33900307	15,27922241	26,39493792
HLL 1MIN	N. Violations	60	104	56	116	56	114	55	117
	% N. Violations	6,00%	10,40%	5,60%	11,60%	5,60%	11,40%	5,50%	11,70%
	Kupiec Test	20,45293559	0,17571251	25,17721706	2,719586373	25,17721706	2,09335969	26,4513402	3,062029457
	Christ. Test	21,99114785	6,761824418	25,42328609	11,76734213	25,42328609	10,4967824	27,67859004	10,1276638
HLL 2MIN	N. Violations	65	110	64	119	64	120	61	123
	% N. Violations	6,50%	11,00%	6,40%	11,90%	6,40%	12,00%	6,10%	12,30%
	Kupiec Test	15,34202326	1,079764492	16,29642589	3,804851017	16,29642589	4,204947331	19,36195317	5,518335419
	Christ. Test	18,49342596	6,750245392	23,57017756	12,99170881	16,29923569	11,41934958	22,03835227	14,32698678
HLL 5MIN	N. Violations	73	125	71	128	75	133	68	134
	% N. Violations	7,30%	12,50%	7,10%	12,80%	7,50%	13,30%	6,80%	13,40%
	Kupiec Test	8,854250609	6,486853137	10,29094259	8,07646801	7,53579138	11,08266477	12,67443035	11,73638809
	Christ. Test	9,413279892	12,98216489	16,67732073	14,66021217	8,586867412	18,19442333	15,06491698	19,75673287
HAR OPT	N. Violations	76	129	73	133	80	134	73	141
	% N. Violations	7,60%	12,90%	7,30%	13,30%	8,00%	13,40%	7,30%	14,10%
	Kupiec Test	6,919985224	8,642338306	8,854250609	11,08266477	4,738220152	11,73638809	8,854250609	16,78910122
	Christ. Test	9,979109964	13,62029819	12,77538612	15,78611225	6,828827007	18,42653564	11,35817476	23,08794021

*the model did not have any sequential errors

Table A07 – HAR Models for PETR4 with VaR (2,5%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD
ALL 1MIN	N. Violations	68	112	66	121	65	119	65	121
	% N. Violations	6,80%	11,20%	6,60%	12,10%	6,50%	11,90%	6,50%	12,10%
	Kupiec	12,67443035	1,546341399	14,42053012	4,623983232	15,34202326	3,804851017	15,34202326	4,623983232
	Christoffersen	15,06491698	7,825604185	23,03393158	14,42261076	15,49497831	11,45624132	18,49342596	12,85688104
ALL 2MIN	N. Violations	71	122	70	124	72	128	71	128
	% N. Violations	7,10%	12,20%	7,00%	12,40%	7,20%	12,80%	7,10%	12,80%
	Kupiec Test	10,29094259	5,06182356	11,0545783	5,993388121	9,557629677	8,07646801	10,29094259	8,07646801
	Teste de Christ.	14,8546236	14,35818178	22,38501249	14,32875316	9,565724649	14,66021217	12,03517586	16,0256457
ALL 5MIN	N. Violations	73	125	71	128	75	133	68	136
	% N. Violations	7,30%	12,50%	7,10%	12,80%	7,50%	13,30%	6,80%	13,60%
	Kupiec Test	8,854250609	6,486853137	10,29094259	8,07646801	7,53579138	11,08266477	12,67443035	13,0954318
	Teste de Christ.	9,413279892	12,98216489	16,67732073	14,66021217	8,586867412	18,19442333	15,06491698	20,22816815
ALL 15MIN	N. Violations	76	138	71	133	80	138	70	141
	% N. Violations	7,60%	13,80%	7,10%	13,30%	8,00%	13,80%	7,00%	14,10%
	Kupiec Test	6,919985224	14,52256042	10,29094259	11,08266477	4,738220152	14,52256042	11,0545783	16,78910122
	Teste de Christ.	9,979109964	19,66320318	13,30014466	16,93303829	8,054844411	23,46673949	15,96063334	23,08794021
ALL 30MIN	N. Violations	70	139	71	137	72	143	74	146
	% N. Violations	7,00%	13,90%	7,10%	13,70%	7,20%	14,30%	7,40%	14,60%
	Kupiec Test	11,0545783	15,26139589	10,29094259	13,80053803	9,557629677	18,3828592	8,180427156	20,89566246
	Teste de Christ.	15,96063334	21,16684668	12,03517586	19,30822745	13,79309037	26,33900307	15,27922241	26,39493792
HLL 1MIN	N. Violations	60	104	56	116	56	114	55	117
	% N. Violations	6,00%	10,40%	5,60%	11,60%	5,60%	11,40%	5,50%	11,70%
	Kupiec Test	20,45293559	0,17571251	25,17721706	2,719586373	25,17721706	2,09335969	26,4513402	3,062029457
	Teste de Christ.	21,99114785	6,761824418	25,42328609	11,76734213	25,42328609	10,4967824	27,67859004	10,1276638
HLL 2MIN	N. Violations	65	110	64	119	64	120	61	123
	% N. Violations	6,50%	11,00%	6,40%	11,90%	6,40%	12,00%	6,10%	12,30%
	Kupiec Test	15,34202326	1,079764492	16,29642589	3,804851017	16,29642589	4,204947331	19,36195317	5,518335419
	Teste de Christ.	18,49342596	6,750245392	23,57017756	12,99170881	16,29923569	11,41934958	22,03835227	14,32698678
HLL 5MIN	N. Violations	73	125	71	128	75	133	68	134
	% N. Violations	7,30%	12,50%	7,10%	12,80%	7,50%	13,30%	6,80%	13,40%
	Kupiec Test	8,854250609	6,486853137	10,29094259	8,07646801	7,53579138	11,08266477	12,67443035	11,73638809
	Teste de Christ.	9,413279892	12,98216489	16,67732073	14,66021217	8,586867412	18,19442333	15,06491698	19,75673287
HAR OPT	N. Violations	76	129	73	133	80	134	73	141
	% N. Violations	7,60%	12,90%	7,30%	13,30%	8,00%	13,40%	7,30%	14,10%
	Kupiec Test	6,919985224	8,642338306	8,854250609	11,08266477	4,738220152	11,73638809	8,854250609	16,78910122
	Teste de Christ.	9,979109964	13,62029819	12,77538612	15,78611225	6,828827007	18,42653564	11,35817476	23,08794021

*the model did not have any sequential errors

Table A08 – HAR Models for PETR4 with VaR (1%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD
ALL 1MIN	N. Violations	68	112	66	121	65	119	65	121
	% N. Violations	6,80%	11,20%	6,60%	12,10%	6,50%	11,90%	6,50%	12,10%
	Kupiec	12,67443035	1,546341399	14,42053012	4,623983232	15,34202326	3,804851017	15,34202326	4,623983232
	Christoffersen	15,06491698	7,825604185	23,03393158	14,42261076	15,49497831	11,45624132	18,49342596	12,85688104
ALL 2MIN	N. Violations	71	122	70	124	72	128	71	128
	% N. Violations	7,10%	12,20%	7,00%	12,40%	7,20%	12,80%	7,10%	12,80%
	Kupiec Test	10,29094259	5,06182356	11,0545783	5,993388121	9,557629677	8,07646801	10,29094259	8,07646801
	Christ. Test	14,8546236	14,35818178	22,38501249	14,32875316	9,565724649	14,66021217	12,03517586	16,0256457
ALL 5MIN	N. Violations	73	125	71	128	75	133	68	136
	% N. Violations	7,30%	12,50%	7,10%	12,80%	7,50%	13,30%	6,80%	13,60%
	Kupiec Test	8,854250609	6,486853137	10,29094259	8,07646801	7,53579138	11,08266477	12,67443035	13,0954318
	Christ. Test	9,413279892	12,98216489	16,67732073	14,66021217	8,586867412	18,19442333	15,06491698	20,22816815
ALL 15MIN	N. Violations	76	138	71	133	80	138	70	141
	% N. Violations	7,60%	13,80%	7,10%	13,30%	8,00%	13,80%	7,00%	14,10%
	Kupiec Test	6,919985224	14,52256042	10,29094259	11,08266477	4,738220152	14,52256042	11,0545783	16,78910122
	Christ. Test	9,979109964	19,66320318	13,30014466	16,93303829	8,054844411	23,46673949	15,96063334	23,08794021
ALL 30MIN	N. Violations	70	139	71	137	72	143	74	146
	% N. Violations	7,00%	13,90%	7,10%	13,70%	7,20%	14,30%	7,40%	14,60%
	Kupiec Test	11,0545783	15,26139589	10,29094259	13,80053803	9,557629677	18,3828592	8,180427156	20,89566246
	Christ. Test	15,96063334	21,16684668	12,03517586	19,30822745	13,79309037	26,33900307	15,27922241	26,39493792
HLL 1MIN	N. Violations	60	104	56	116	56	114	55	117
	% N. Violations	6,00%	10,40%	5,60%	11,60%	5,60%	11,40%	5,50%	11,70%
	Kupiec Test	20,45293559	0,17571251	25,17721706	2,719586373	25,17721706	2,09335969	26,4513402	3,062029457
	Christ. Test	21,99114785	6,761824418	25,42328609	11,76734213	25,42328609	10,4967824	27,67859004	10,1276638
HLL 2MIN	N. Violations	65	110	64	119	64	120	61	123
	% N. Violations	6,50%	11,00%	6,40%	11,90%	6,40%	12,00%	6,10%	12,30%
	Kupiec Test	15,34202326	1,079764492	16,29642589	3,804851017	16,29642589	4,204947331	19,36195317	5,518335419
	Christ. Test	18,49342596	6,750245392	23,57017756	12,99170881	16,29923569	11,41934958	22,03835227	14,32698678
HLL 5MIN	N. Violations	73	125	71	128	75	133	68	134
	% N. Violations	7,30%	12,50%	7,10%	12,80%	7,50%	13,30%	6,80%	13,40%
	Kupiec Test	8,854250609	6,486853137	10,29094259	8,07646801	7,53579138	11,08266477	12,67443035	11,73638809
	Christ. Test	9,413279892	12,98216489	16,67732073	14,66021217	8,586867412	18,19442333	15,06491698	19,75673287
HAR OPT	N. Violations	76	129	73	133	80	134	73	141
	% N. Violations	7,60%	12,90%	7,30%	13,30%	8,00%	13,40%	7,30%	14,10%
	Kupiec Test	6,919985224	8,642338306	8,854250609	11,08266477	4,738220152	11,73638809	8,854250609	16,78910122
	Christ. Test	9,979109964	13,62029819	12,77538612	15,78611225	6,828827007	18,42653564	11,35817476	23,08794021

** the model did not have any sequential errors

Table A09 – HAR Models for USIM5 with VaR (10%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD
ALL 1MIN	N. Violations	52	94	54	99	45	92	52	100
	% N. Violations	5,20%	9,40%	5,40%	9,90%	4,50%	9,20%	5,20%	10,00%
	Kupiec	30,5073124	0,407335507	27,76394538	0,011144201	41,42896971	0,728685298	30,5073124	0
	Christoffersen	33,93530648	3,624764697	30,63833739	5,578532802	53,61864929	12,28443306	44,53814768	11,29706696
ALL 2MIN	N. Violations	55	107	60	104	50	102	55	106
	% N. Violations	5,50%	10,70%	6,00%	10,40%	5,00%	10,20%	5,50%	10,60%
	Kupiec Test	26,4513402	0,533536922	20,45293559	0,17571251	33,41300236	0,044183709	26,4513402	0,393097709
	Christ. Test	30,88024005	5,958547211	23,38641273	4,053800343	43,5248834	12,0536938	38,9830742	12,16772873
ALL 5MIN	N. Violations	61	114	60	114	54	109	58	118
	% N. Violations	6,10%	11,40%	6,00%	11,40%	5,40%	10,90%	5,80%	11,80%
	Kupiec Test	19,36195317	2,09335969	20,45293559	2,09335969	27,76394538	0,87703929	22,74185085	3,423831773
	Christ. Test	22,03835227	6,368165718	23,38641273	5,276605479	38,45893424	12,84390805	33,93647769	15,78816123
ALL 15MIN	N. Violations	64	127	64	136	63	127	66	133
	% N. Violations	6,40%	12,70%	6,40%	13,60%	6,30%	12,70%	6,60%	13,30%
	Kupiec Test	16,29642589	7,528516451	16,29642589	13,0954318	17,28421654	7,528516451	14,42053012	11,08266477
	Christ. Test	19,72807827	9,323216006	19,72807827	16,83032207	25,01428802	17,29207527	26,36467642	18,663858
ALL 30MIN	N. Violations	64	136	68	141	66	141	70	146
	% N. Violations	6,40%	13,60%	6,80%	14,10%	6,60%	14,10%	7,00%	14,60%
	Kupiec Test	16,29642589	13,0954318	12,67443035	16,78910122	14,42053012	16,78910122	11,0545783	20,89566246
	Christ. Test	19,72807827	16,83032207	16,54046815	21,94454932	22,75120615	25,74729793	23,25471212	29,05810895
HLL 1MIN	N. Violations	35	74	35	76	37	75	38	82
	% N. Violations	3,50%	7,40%	3,50%	7,60%	3,70%	7,50%	3,80%	8,20%
	Kupiec Test	61,09779365	8,180427156	61,09779365	6,919985224	56,73588873	7,53579138	54,63989245	3,811669765
	Christ. Test	61,54344189	13,41611219	65,48595379	11,46098463	64,97620442	16,04837939	70,58796415	10,43251816
HLL 2MIN	N. Violations	39	84	43	89	39	81	44	91
	% N. Violations	3,90%	8,40%	4,30%	8,90%	3,90%	8,10%	4,40%	9,10%
	Kupiec Test	52,59861282	2,991404633	44,95468819	1,390880737	52,59861282	4,261510827	43,1680537	0,925157827
	Christ. Test	52,74723812	9,622102289	47,05513233	7,705853332	62,29154886	13,74651335	58,77332931	9,79705156
HLL 5MIN	N. Violations	52	102	51	106	45	102	51	107
	% N. Violations	5,20%	10,20%	5,10%	10,60%	4,50%	10,20%	5,10%	10,70%
	Kupiec Test	30,5073124	0,044183709	31,93949454	0,393097709	41,42896971	0,044183709	31,93949454	0,533536922
	Christ. Test	32,28040392	4,558123889	35,66613391	4,859711279	53,61864929	13,77512841	46,50858575	13,464474
HAR OPT	N. Violations	64	121	63	117	60	116	61	121
	% N. Violations	6,40%	12,10%	6,30%	11,70%	6,00%	11,60%	6,10%	12,10%
	Kupiec Test	16,29642589	4,623983232	17,28421654	3,062029457	20,45293559	2,719586373	19,36195317	4,623983232
	Christ. Test	19,72807827	7,867058457	19,48604726	7,522827727	29,03937877	13,06602348	29,36876009	14,26124369 **

* the model did not have any sequential errors

Table A10 – HAR Models for USIM5 with VaR (5%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Sem DP	Com 1 DP	Sem DP	Com 1 DP	Sem DP	Com 1 DP	Sem DP	Com 1 DP
ALL 1MIN	N. Violations	52	94	54	99	45	92	52	100
	% N. Violations	5,20%	9,40%	5,40%	9,90%	4,50%	9,20%	5,20%	10,00%
	Kupiec	30,5073124	0,407335507	27,76394538	0,011144201	41,42896971	0,728685298	30,5073124	0
	Christoffersen	33,93530648	3,624764697	30,63833739	5,578532802	53,61864929	12,28443306	44,53814768	11,29706696
ALL 2MIN	N. Violations	55	107	60	104	50	102	55	106
	% N. Violations	5,50%	10,70%	6,00%	10,40%	5,00%	10,20%	5,50%	10,60%
	Kupiec Test	26,4513402	0,533536922	20,45293559	0,17571251	33,41300236	0,044183709	26,4513402	0,393097709
	Christ. Test	30,88024005	5,958547211	23,38641273	4,053800343	43,5248834	12,0536938	38,9830742	12,16772873
ALL 5MIN	N. Violations	61	114	60	114	54	109	58	118
	% N. Violations	6,10%	11,40%	6,00%	11,40%	5,40%	10,90%	5,80%	11,80%
	Kupiec Test	19,36195317	2,09335969	20,45293559	2,09335969	27,76394538	0,87703929	22,74185085	3,423831773
	Christ. Test	22,03835227	6,368165718	23,38641273	5,276605479	38,45893424	12,84390805	33,93647769	15,78816123
ALL 15MIN	N. Violations	64	127	64	136	63	127	66	133
	% N. Violations	6,40%	12,70%	6,40%	13,60%	6,30%	12,70%	6,60%	13,30%
	Kupiec Test	16,29642589	7,528516451	16,29642589	13,0954318	17,28421654	7,528516451	14,42053012	11,08266477
	Christ. Test	19,72807827	9,323216006	19,72807827	16,83032207	25,01428802	17,29207527	26,36467642	18,663858
ALL 30MIN	N. Violations	64	136	68	141	66	141	70	146
	% N. Violations	6,40%	13,60%	6,80%	14,10%	6,60%	14,10%	7,00%	14,60%
	Kupiec Test	16,29642589	13,0954318	12,67443035	16,78910122	14,42053012	16,78910122	11,0545783	20,89566246
	Christ. Test	19,72807827	16,83032207	16,54046815	21,94454932	22,75120615	25,74729793	23,25471212	29,05810895
HLL 1MIN	N. Violations	35	74	35	76	37	75	38	82
	% N. Violations	3,50%	7,40%	3,50%	7,60%	3,70%	7,50%	3,80%	8,20%
	Kupiec Test	61,09779365	8,180427156	61,09779365	6,919985224	56,73588873	7,53579138	54,63989245	3,811669765
	Christ. Test	61,54344189	13,41611219	65,48595379	11,46098463	64,97620442	16,04837939	70,58796415	10,43251816
HLL 2MIN	N. Violations	39	84	43	89	39	81	44	91
	% N. Violations	3,90%	8,40%	4,30%	8,90%	3,90%	8,10%	4,40%	9,10%
	Kupiec Test	52,59861282	2,991404633	44,95468819	1,390880737	52,59861282	4,261510827	43,1680537	0,925157827
	Christ. Test	52,74723812	9,622102289	47,05513233	7,705853332	62,29154886	13,74651335	58,77332931	9,79705156
HLL 5MIN	N. Violations	52	102	51	106	45	102	51	107
	% N. Violations	5,20%	10,20%	5,10%	10,60%	4,50%	10,20%	5,10%	10,70%
	Kupiec Test	30,5073124	0,044183709	31,93949454	0,393097709	41,42896971	0,044183709	31,93949454	0,533536922
	Christ. Test	32,28040392	4,558123889	35,66613391	4,859711279	53,61864929	13,77512841	46,50858575	13,464474
HAR OPT	N. Violations	64	121	63	117	60	116	61	121
	% N. Violations	6,40%	12,10%	6,30%	11,70%	6,00%	11,60%	6,10%	12,10%
	Kupiec Test	16,29642589	4,623983232	17,28421654	3,062029457	20,45293559	2,719586373	19,36195317	4,623983232
	Christ. Test	19,72807827	7,867058457	19,48604726	7,522827727	29,03937877	13,06602348	29,36876009	14,26124369

*the model did not have any sequential errors

Table A11 – HAR Models for USIM5 with VaR (2,5%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD
ALL 1MIN	N. Violations	52	94	54	99	45	92	52	100
	% N. Violations	5,20%	9,40%	5,40%	9,90%	4,50%	9,20%	5,20%	10,00%
	Kupiec	30,5073124	0,407335507	27,76394538	0,011144201	41,42896971	0,728685298	30,5073124	0
	Christoffersen	33,93530648	3,624764697	30,63833739	5,578532802	53,61864929	12,28443306	44,53814768	11,29706696
ALL 2MIN	N. Violations	55	107	60	104	50	102	55	106
	% N. Violations	5,50%	10,70%	6,00%	10,40%	5,00%	10,20%	5,50%	10,60%
	Kupiec Test	26,4513402	0,533536922	20,45293559	0,17571251	33,41300236	0,044183709	26,4513402	0,393097709
	Christ. Test	30,88024005	5,958547211	23,38641273	4,053800343	43,5248834	12,0536938	38,9830742	12,16772873
ALL 5MIN	N. Violations	61	114	60	114	54	109	58	118
	% N. Violations	6,10%	11,40%	6,00%	11,40%	5,40%	10,90%	5,80%	11,80%
	Kupiec Test	19,36195317	2,09335969	20,45293559	2,09335969	27,76394538	0,87703929	22,74185085	3,423831773
	Christ. Test	22,03835227	6,368165718	23,38641273	5,276605479	38,45893424	12,84390805	33,93647769	15,78816123
ALL 15MIN	N. Violations	64	127	64	136	63	127	66	133
	% N. Violations	6,40%	12,70%	6,40%	13,60%	6,30%	12,70%	6,60%	13,30%
	Kupiec Test	16,29642589	7,528516451	16,29642589	13,0954318	17,28421654	7,528516451	14,42053012	11,08266477
	Christ. Test	19,72807827	9,323216006	19,72807827	16,83032207	25,01428802	17,29207527	26,36467642	18,663858
ALL 30MIN	N. Violations	64	136	68	141	66	141	70	146
	% N. Violations	6,40%	13,60%	6,80%	14,10%	6,60%	14,10%	7,00%	14,60%
	Kupiec Test	16,29642589	13,0954318	12,67443035	16,78910122	14,42053012	16,78910122	11,0545783	20,89566246
	Christ. Test	19,72807827	16,83032207	16,54046815	21,94454932	22,75120615	25,74729793	23,25471212	29,05810895
HLL 1MIN	N. Violations	35	74	35	76	37	75	38	82
	% N. Violations	3,50%	7,40%	3,50%	7,60%	3,70%	7,50%	3,80%	8,20%
	Kupiec Test	61,09779365	8,180427156	61,09779365	6,919985224	56,73588873	7,53579138	54,63989245	3,811669765
	Christ. Test	61,54344189	13,41611219	65,48595379	11,46098463	64,97620442	16,04837939	70,58796415	10,43251816
HLL 2MIN	N. Violations	39	84	43	89	39	81	44	91
	% N. Violations	3,90%	8,40%	4,30%	8,90%	3,90%	8,10%	4,40%	9,10%
	Kupiec Test	52,59861282	2,991404633	44,95468819	1,390880737	52,59861282	4,261510827	43,1680537	0,925157827
	Christ. Test	52,74723812	9,622102289	47,05513233	7,705853332	62,29154886	13,74651335	58,77332931	9,79705156
HLL 5MIN	N. Violations	52	102	51	106	45	102	51	107
	% N. Violations	5,20%	10,20%	5,10%	10,60%	4,50%	10,20%	5,10%	10,70%
	Kupiec Test	30,5073124	0,044183709	31,93949454	0,393097709	41,42896971	0,044183709	31,93949454	0,533536922
	Christ. Test	32,28040392	4,558123889	35,66613391	4,859711279	53,61864929	13,77512841	46,50858575	13,464474
HAR OPT	N. Violations	64	121	63	117	60	116	61	121
	% N. Violations	6,40%	12,10%	6,30%	11,70%	6,00%	11,60%	6,10%	12,10%
	Kupiec Test	16,29642589	4,623983232	17,28421654	3,062029457	20,45293559	2,719586373	19,36195317	4,623983232
	Christ. Test	19,72807827	7,867058457	19,48604726	7,522827727	29,03937877	13,06602348	29,36876009	14,26124369

* the model did not have any sequential errors

Table A12 – HAR Models for USIM5 with VaR (1%)

90	**0,1**	1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD	Without SD	With 1 SD
ALL 1MIN	N. Violations	52	94	54	99	45	92	52	100
	% N. Violations	5,20%	9,40%	5,40%	9,90%	4,50%	9,20%	5,20%	10,00%
	Kupiec	30,5073124	0,407335507	27,76394538	0,011144201	41,42896971	0,728685298	30,5073124	0
	Christoffersen	33,93530648	3,624764697	30,63833739	5,578532802	53,61864929	12,28443306	44,53814768	11,29706696
ALL 2MIN	N. Violations	55	107	60	104	50	102	55	106
	% N. Violations	5,50%	10,70%	6,00%	10,40%	5,00%	10,20%	5,50%	10,60%
	Kupiec Test	26,4513402	0,533536922	20,45293559	0,17571251	33,41300236	0,044183709	26,4513402	0,393097709
	Christ. Test	30,88024005	5,958547211	23,38641273	4,053800343	43,5248834	12,0536938	38,9830742	12,16772873
ALL 5MIN	N. Violations	61	114	60	114	54	109	58	118
	% N. Violations	6,10%	11,40%	6,00%	11,40%	5,40%	10,90%	5,80%	11,80%
	Kupiec Test	19,36195317	2,09335969	20,45293559	2,09335969	27,76394538	0,87703929	22,74185085	3,423831773
	Christ. Test	22,03835227	6,368165718	23,38641273	5,276605479	38,45893424	12,84390805	33,93647769	15,78816123
ALL 15MIN	N. Violations	64	127	64	136	63	127	66	133
	% N. Violations	6,40%	12,70%	6,40%	13,60%	6,30%	12,70%	6,60%	13,30%
	Kupiec Test	16,29642589	7,528516451	16,29642589	13,0954318	17,28421654	7,528516451	14,42053012	11,08266477
	Christ. Test	19,72807827	9,323216006	19,72807827	16,83032207	25,01428802	17,29207527	26,36467642	18,663858
ALL 30MIN	N. Violations	64	136	68	141	66	141	70	146
	% N. Violations	6,40%	13,60%	6,80%	14,10%	6,60%	14,10%	7,00%	14,60%
	Kupiec Test	16,29642589	13,0954318	12,67443035	16,78910122	14,42053012	16,78910122	11,0545783	20,89566246
	Christ. Test	19,72807827	16,83032207	16,54046815	21,94454932	22,75120615	25,74729793	23,25471212	29,05810895
HLL 1MIN	N. Violations	35	74	35	76	37	75	38	82
	% N. Violations	3,50%	7,40%	3,50%	7,60%	3,70%	7,50%	3,80%	8,20%
	Kupiec Test	61,09779365	8,180427156	61,09779365	6,919985224	56,73588873	7,53579138	54,63989245	3,811669765
	Christ. Test	61,54344189	13,41611219	65,48595379	11,46098463	64,97620442	16,04837939	70,58796415	10,43251816
HLL 2MIN	N. Violations	39	84	43	89	39	81	44	91
	% N. Violations	3,90%	8,40%	4,30%	8,90%	3,90%	8,10%	4,40%	9,10%
	Kupiec Test	52,59861282	2,991404633	44,95468819	1,390880737	52,59861282	4,261510827	43,1680537	0,925157827
	Christ. Test	52,74723812	9,622102289	47,05513233	7,705853332	62,29154886	13,74651335	58,77332931	9,79705156
HLL 5MIN	N. Violations	52	102	51	106	45	102	51	107
	% N. Violations	5,20%	10,20%	5,10%	10,60%	4,50%	10,20%	5,10%	10,70%
	Kupiec Test	30,5073124	0,044183709	31,93949454	0,393097709	41,42896971	0,044183709	31,93949454	0,533536922
	Christ. Test	32,28040392	4,558123889	35,66613391	4,859711279	53,61864929	13,77512841	46,50858575	13,464474
HAR OPT	N. Violations	64	121	63	117	60	116	61	121
	% N. Violations	6,40%	12,10%	6,30%	11,70%	6,00%	11,60%	6,10%	12,10%
	Kupiec Test	16,29642589	4,623983232	17,28421654	3,062029457	20,45293559	2,719586373	19,36195317	4,623983232
	Christ. Test	19,72807827	7,867058457	19,48604726	7,522827727	29,03937877	13,06602348	29,36876009	14,26124369

*the model did not have any sequential errors

Table A13 – HAR Models for VALE5 with VaR (1%)

		1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD
ALL 1MIN	N. Violations	84	144	91	141	79	139	76	151
	% N. Violations	8,40%	14,40%	9,10%	14,10%	7,90%	13,90%	7,60%	15,10%
	Kupiec	2,991404633	19,20426586	0,925157827	16,78910122	5,242104043	15,26139589	6,919985224	25,40330478
	Christoffersen	5,040803622	22,11434051	5,55190822	19,67145947	0,912959508	15,5415761	8,383261411	25,89009693
ALL 2MIN	N. Violations	89	147	92	144	81	144	79	151
	% N. Violations	8,90%	14,70%	9,20%	14,40%	8,10%	14,40%	7,90%	15,10%
	Kupiec Test	1,390880737	21,76546762	0,728685298	19,20426586	4,261510827	19,20426586	5,242104043	25,40330478
	Christ. Test	6,989334993	23,55674587	0,085170057	23,36302665	-0,213929642	18,99245918	5,633699822	25,89009693
ALL 5MIN	N. Violations	91	158	90	152	80	156	79	161
	% N. Violations	9,10%	15,80%	9,00%	15,20%	8,00%	15,60%	7,90%	16,10%
	Kupiec Test	0,925157827	32,36701421	1,145809041	26,3519815	4,738220152	30,30102662	5,242104043	35,57855823
	Christ. Test	5,55190822	33,46360474	1,094572054	27,47472478	1,046057929	31,10030147	5,633699822	34,98519508
ALL 15MIN	N. Violations	92	168	90	159	83	167	78	173
	% N. Violations	9,20%	16,80%	9,00%	15,90%	8,30%	16,70%	7,80%	17,30%
	Kupiec Test	0,728685298	43,58700994	1,145809041	33,42259732	3,388398202	42,39946338	5,773476618	49,73803438
	Christ. Test	6,851700207	42,10914556	2,626304295	32,58676855	-1,192215922	41,77535212	4,879114779	49,04343201
ALL 30MIN	N. Violations	93	172	93	164	86	175	80	182
	% N. Violations	9,30%	17,20%	9,30%	16,40%	8,60%	17,50%	8,00%	18,20%
	Kupiec Test	0,556154971	48,47961164	0,556154971	38,92336412	2,275120227	52,29675374	4,738220152	61,68543628
	Christ. Test	8,240638599	47,37286465	2,544202452	38,31998724	-1,386105454	51,15717216	2,043168259	61,46604843
HLL 1MIN	N. Violations	80	140	79	136	73	129	74	143
	% N. Violations	8,00%	14,00%	7,90%	13,60%	7,30%	12,90%	7,40%	14,30%
	Kupiec Test	4,738220152	16,01694284	5,242104043	13,0954318	8,854250609	8,642338306	8,180427156	18,3828592
	Christ. Test	6,529349015	20,61074376	7,4271905	14,32078571	5,221166002	9,972266703	8,533245034	20,5062502
HLL 2MIN	N. Violations	80	141	78	139	72	132	68	143
	% N. Violations	8,00%	14,10%	7,80%	13,90%	7,20%	13,20%	6,80%	14,30%
	Kupiec Test	4,738220152	16,78910122	5,773476618	15,26139589	9,557629677	10,44628598	12,67443035	18,3828592
	Christ. Test	6,529349015	20,94109914	8,367325477	17,70990206	6,079619591	10,8115235	13,2515552	19,42432712
HLL 5MIN	N. Violations	81	147	76	142	75	139	75	150
	% N. Violations	8,10%	14,70%	7,60%	14,20%	7,50%	13,90%	7,50%	15,00%
	Kupiec Test	4,261510827	21,76546762	6,919985224	17,57777239	7,53579138	15,26139589	7,53579138	24,4702289
	Christ. Test	7,553533583	23,55674587	10,3768413	18,95101024	3,62667907	14,6252054	7,556185557	24,35696243
HAR OPT	N. Violations	92	159	89	152	83	155	78	164
	% N. Violations	9,20%	15,90%	8,90%	15,20%	8,30%	15,50%	7,80%	16,40%
	Kupiec Test	0,728685298	33,42259732	1,390880737	26,3519815	3,388398202	29,2907788	5,773476618	38,92336412
	Christ. Test	4,892441959	34,2030174	3,2457644	27,47472478	-0,656629613	31,41600514	4,879114779	39,93766654

*the model did not have any sequential errors

Table A14 – HAR Models for VALE5 with VaR (5%)

		1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD
ALL 1MIN	N. Violations	84	144	91	141	79	139	76	151
	% N. Violations	8,40%	14,40%	9,10%	14,10%	7,90%	13,90%	7,60%	15,10%
	Kupiec	2,991404633	19,20426586	0,925157827	16,78910122	5,242104043	15,26139589	6,919985224	25,40330478
	Christoffersen	5,040803622	22,11434051	5,55190822	19,67145947	0,912959508	15,5415761	8,383261411	25,89009693
ALL 2MIN	N. Violations	89	147	92	144	81	144	79	151
	% N. Violations	8,90%	14,70%	9,20%	14,40%	8,10%	14,40%	7,90%	15,10%
	Kupiec Test	1,390880737	21,76546762	0,728685298	19,20426586	4,261510827	19,20426586	5,242104043	25,40330478
	Christ. Test	6,989334993	23,55674587	0,085170057	23,36302665	-0,213929642	18,99245918	5,633699822	25,89009693
ALL 5MIN	N. Violations	91	158	90	152	80	156	79	161
	% N. Violations	9,10%	15,80%	9,00%	15,20%	8,00%	15,60%	7,90%	16,10%
	Kupiec Test	0,925157827	32,36701421	1,145809041	26,3519815	4,738220152	30,30102662	5,242104043	35,57855823
	Christ. Test	5,55190822	33,46360474	1,094572054	27,47472478	1,046057929	31,10030147	5,633699822	34,98519508
ALL 15MIN	N. Violations	92	168	90	159	83	167	78	173
	% N. Violations	9,20%	16,80%	9,00%	15,90%	8,30%	16,70%	7,80%	17,30%
	Kupiec Test	0,728685298	43,58700994	1,145809041	33,42259732	3,388398202	42,39946338	5,773476618	49,73803438
	Christ. Test	6,851700207	42,10914556	2,626304295	32,58676855	-1,192215922	41,77535212	4,879114779	49,04343201
ALL 30MIN	N. Violations	93	172	93	164	86	175	80	182
	% N. Violations	9,30%	17,20%	9,30%	16,40%	8,60%	17,50%	8,00%	18,20%
	Kupiec Test	0,556154971	48,47961164	0,556154971	38,92336412	2,275120227	52,29675374	4,738220152	61,68543628
	Christ. Test	8,240638599	47,37286465	2,544202452	38,31998724	-1,386105454	51,15717216	2,043168259	61,46604843
HLL 1MIN	N. Violations	80	140	79	136	73	129	74	143
	% N. Violations	8,00%	14,00%	7,90%	13,60%	7,30%	12,90%	7,40%	14,30%
	Kupiec Test	4,738220152	16,01694284	5,242104043	13,0954318	8,854250609	8,642338306	8,180427156	18,3828592
	Christ. Test	6,529349015	20,61074376	7,4271905	14,32078571	5,221166002	9,972266703	8,533245034	20,5062502
HLL 2MIN	N. Violations	80	141	78	139	72	132	68	143
	% N. Violations	8,00%	14,10%	7,80%	13,90%	7,20%	13,20%	6,80%	14,30%
	Kupiec Test	4,738220152	16,78910122	5,773476618	15,26139589	9,557629677	10,44628598	12,67443035	18,3828592
	Christ. Test	6,529349015	20,94109914	8,367325477	17,70990206	6,079619591	10,8115235	13,2515552	19,42432712
HLL 5MIN	N. Violations	81	147	76	142	75	139	75	150
	% N. Violations	8,10%	14,70%	7,60%	14,20%	7,50%	13,90%	7,50%	15,00%
	Kupiec Test	4,261510827	21,76546762	6,919985224	17,57777239	7,53579138	15,26139589	7,53579138	24,4702289
	Christ. Test	7,553533583	23,55674587	10,3768413	18,95101024	3,62667907	14,6252054	7,556185557	24,35696243
HAR OPT	N. Violations	92	159	89	152	83	155	78	164
	% N. Violations	9,20%	15,90%	8,90%	15,20%	8,30%	15,50%	7,80%	16,40%
	Kupiec Test	0,728685298	33,42259732	1,390880737	26,3519815	3,388398202	29,2907788	5,773476618	38,92336412
	Christ. Test	4,892441959	34,2030174	3,2457644	27,47472478	-0,656629613	31,41600514	4,879114779	39,93766654

*the model did not have any sequential errors

Table A15 – HAR Models for VALE5 with VaR (2,5%)

		1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD	Without SD	With1 SD
ALL 1MIN	N. Violations	84	144	91	141	79	139	76	151
	% N. Violations	8,40%	14,40%	9,10%	14,10%	7,90%	13,90%	7,60%	15,10%
	Kupiec	2,991404633	19,20426586	0,925157827	16,78910122	5,242104043	15,26139589	6,919985224	25,40330478
	Christoffersen	5,040803622	22,11434051	5,55190822	19,67145947	0,912959508	15,5415761	8,383261411	25,89009693
ALL 2MIN	N. Violations	89	147	92	144	81	144	79	151
	% N. Violations	8,90%	14,70%	9,20%	14,40%	8,10%	14,40%	7,90%	15,10%
	Kupiec Test	1,390880737	21,76546762	0,728685298	19,20426586	4,261510827	19,20426586	5,242104043	25,40330478
	Christ. Test	6,989334993	23,55674587	0,085170057	23,36302665	-0,213929642	18,99245918	5,633699822	25,89009693
ALL 5MIN	N. Violations	91	158	90	152	80	156	79	161
	% N. Violations	9,10%	15,80%	9,00%	15,20%	8,00%	15,60%	7,90%	16,10%
	Kupiec Test	0,925157827	32,36701421	1,145809041	26,3519815	4,738220152	30,30102662	5,242104043	35,57855823
	Christ. Test	5,55190822	33,46360474	1,094572054	27,47472478	1,046057929	31,10030147	5,633699822	34,98519508
ALL 15MIN	N. Violations	92	168	90	159	83	167	78	173
	% N. Violations	9,20%	16,80%	9,00%	15,90%	8,30%	16,70%	7,80%	17,30%
	Kupiec Test	0,728685298	43,58700994	1,145809041	33,42259732	3,388398202	42,39946338	5,773476618	49,73803438
	Christ. Test	6,851700207	42,10914556	2,626304295	32,58676855	-1,192215922	41,77535212	4,879114779	49,04343201
ALL 30MIN	N. Violations	93	172	93	164	86	175	80	182
	% N. Violations	9,30%	17,20%	9,30%	16,40%	8,60%	17,50%	8,00%	18,20%
	Kupiec Test	0,556154971	48,47961164	0,556154971	38,92336412	2,275120227	52,29675374	4,738220152	61,68543628
	Christ. Test	8,240638599	47,37286465	2,544202452	38,31998724	-1,386105454	51,15717216	2,043168259	61,46604843
HLL 1MIN	N. Violations	80	140	79	136	73	129	74	143
	% N. Violations	8,00%	14,00%	7,90%	13,60%	7,30%	12,90%	7,40%	14,30%
	Kupiec Test	4,738220152	16,01694284	5,242104043	13,0954318	8,854250609	8,642338306	8,180427156	18,3828592
	Christ. Test	6,529349015	20,61074376	7,4271905	14,32078571	5,221166002	9,972266703	8,533245034	20,5062502
HLL 2MIN	N. Violations	80	141	78	139	72	132	68	143
	% N. Violations	8,00%	14,10%	7,80%	13,90%	7,20%	13,20%	6,80%	14,30%
	Kupiec Test	4,738220152	16,78910122	5,773476618	15,26139589	9,557629677	10,44628598	12,67443035	18,3828592
	Christ. Test	6,529349015	20,94109914	8,367325477	17,70990206	6,079619591	10,8115235	13,2515552	19,42432712
HLL 5MIN	N. Violations	81	147	76	142	75	139	75	150
	% N. Violations	8,10%	14,70%	7,60%	14,20%	7,50%	13,90%	7,50%	15,00%
	Kupiec Test	4,261510827	21,76546762	6,919985224	17,57777239	7,53579138	15,26139589	7,53579138	24,4702289
	Christ. Test	7,553533583	23,55674587	10,3768413	18,95101024	3,62667907	14,6252054	7,556185557	24,35696243
HAR OPT	N. Violations	92	159	89	152	83	155	78	164
	% N. Violations	9,20%	15,90%	8,90%	15,20%	8,30%	15,50%	7,80%	16,40%
	Kupiec Test	0,728685298	33,42259732	1,390880737	26,3519815	3,388398202	29,2907788	5,773476618	38,92336412
	Christ. Test	4,892441959	34,2030174	3,2457644	27,47472478	-0,656629613	31,41600514	4,879114779	39,93766654

*the model did not have any sequential errors

Table A16 – HAR for VALE5 with VaR (1%)

		1-step-ahead Model		2-step-ahead Model		5-step-ahead Model		10-step-ahead Model	
Model	Test	Sem DP	Com 1 DP	Sem DP	Com 1 DP	Sem DP	Com 1 DP	Sem DP	Com 1 DP
ALL 1MIN	N. Violations	84	144	91	141	79	139	76	151
	% N. Violations	8,40%	14,40%	9,10%	14,10%	7,90%	13,90%	7,60%	15,10%
	Kupiec	2,991404633	19,20426586	0,925157827	16,78910122	5,242104043	15,26139589	6,919985224	25,40330478
	Christoffersen	5,040803622	22,11434051	5,55190822	19,67145947	0,912959508	15,5415761	8,383261411	25,89009693
ALL 2MIN	N. Violations	89	147	92	144	81	144	79	151
	% N. Violations	8,90%	14,70%	9,20%	14,40%	8,10%	14,40%	7,90%	15,10%
	Kupiec Test	1,390880737	21,76546762	0,728685298	19,20426586	4,261510827	19,20426586	5,242104043	25,40330478
	Christ. Test	6,989334993	23,55674587	0,085170057	23,36302665	-0,213929642	18,99245918	5,633699822	25,89009693
ALL 5MIN	N. Violations	91	158	90	152	80	156	79	161
	% N. Violations	9,10%	15,80%	9,00%	15,20%	8,00%	15,60%	7,90%	16,10%
	Kupiec Test	0,925157827	32,36701421	1,145809041	26,3519815	4,738220152	30,30102662	5,242104043	35,57855823
	Christ. Test	5,55190822	33,46360474	1,094572054	27,47472478	1,046057929	31,10030147	5,633699822	34,98519508
ALL 15MIN	N. Violations	92	168	90	159	83	167	78	173
	% N. Violations	9,20%	16,80%	9,00%	15,90%	8,30%	16,70%	7,80%	17,30%
	Kupiec Test	0,728685298	43,58700994	1,145809041	33,42259732	3,388398202	42,39946338	5,773476618	49,73803438
	Christ. Test	6,851700207	42,10914556	2,626304295	32,58676855	-1,192215922	41,77535212	4,879114779	49,04343201
ALL 30MIN	N. Violations	93	172	93	164	86	175	80	182
	% N. Violations	9,30%	17,20%	9,30%	16,40%	8,60%	17,50%	8,00%	18,20%
	Kupiec Test	0,556154971	48,47961164	0,556154971	38,92336412	2,275120227	52,29675374	4,738220152	61,68543628
	Christ. Test	8,240638599	47,37286465	2,544202452	38,31998724	-1,386105454	51,15717216	2,043168259	61,46604843
HLL 1MIN	N. Violations	80	140	79	136	73	129	74	143
	% N. Violations	8,00%	14,00%	7,90%	13,60%	7,30%	12,90%	7,40%	14,30%
	Kupiec Test	4,738220152	16,01694284	5,242104043	13,0954318	8,854250609	8,642338306	8,180427156	18,3828592
	Christ. Test	6,529349015	20,61074376	7,4271905	14,32078571	5,221166002	9,972266703	8,533245034	20,5062502
HLL 2MIN	N. Violations	80	141	78	139	72	132	68	143
	% N. Violations	8,00%	14,10%	7,80%	13,90%	7,20%	13,20%	6,80%	14,30%
	Kupiec Test	4,738220152	16,78910122	5,773476618	15,26139589	9,557629677	10,44628598	12,67443035	18,3828592
	Christ. Test	6,529349015	20,94109914	8,367325477	17,70990206	6,079619591	10,8115235	13,2515552	19,42432712
HLL 5MIN	N. Violations	81	147	76	142	75	139	75	150
	% N. Violations	8,10%	14,70%	7,60%	14,20%	7,50%	13,90%	7,50%	15,00%
	Kupiec Test	4,261510827	21,76546762	6,919985224	17,57777239	7,53579138	15,26139589	7,53579138	24,4702289
	Christ. Test	7,553533583	23,55674587	10,3768413	18,95101024	3,62667907	14,6252054	7,556185557	24,35696243
HAR OPT	N. Violations	92	159	89	152	83	155	78	164
	% N. Violations	9,20%	15,90%	8,90%	15,20%	8,30%	15,50%	7,80%	16,40%
	Kupiec Test	0,728685298	33,42259732	1,390880737	26,3519815	3,388398202	29,2907788	5,773476618	38,92336412
	Christ. Test	4,892441959	34,2030174	3,2457644	27,47472478	-0,656629613	31,41600514	4,879114779	39,93766654

*the model did not have any sequential errors