Selection of a Portfolio of Pairs Based on Cointegration: The Brazilian Case

João F. Caldeira
Department of Economics, Federal University of Rio Grande do Sul, Brazil

Guilherme V. Moura
Graduate Program in Economics, Federal University of Santa Catarina, Brazil

---

Abstract

This paper applies cointegration tests to identify stocks to be used in pairs trading strategies. In addition to estimating long-term equilibrium and to model the resulting residuals, we select stock pairs to compose a pairs trading portfolio based on an indicator of profitability evaluated in-sample. The profitability of the strategy is assessed with data from the São Paulo stock exchange ranging from January 2005 to December 2010. Empirical analysis shows that our arbitrage strategy generates high average returns of 16.01% per year, high Sharpe Ratio of 1.28 and low correlation with the market.

Keywords: statistical arbitrage, pairs trading, cointegration, market neutral strategy.

JEL C53, E43, G17

---

1. Introduction

Pairs trading is a statistical arbitrage strategy designed to exploit short-term deviations from a long-run equilibrium between two stocks. Traditional methods of pairs trading have sought to identify trading pairs based on correlation and other non-parametric decision rules. This study selects trading pairs based on the presence of a cointegrating relationship between two stocks. Cointegration enables us to combine the two stocks in a certain linear combination so that the combined portfolio is a stationary process. If two cointegrated stocks share a long-run equilibrium relationship, then deviations from this equilibrium are only short-term and are expected to die out in future periods. To profit from this relative misspricing, a long position in the portfolio is opened when its value falls sufficiently below its long-run equilibrium and is closed out once the value of the portfolio reverts to its expected value. Similarly, profits may be earnt when the portfolio is trading sufficiently above its equilibrium value by shorting the portfolio until it reverts to its expected value (see Pole, 2007, for a comprehensive review on statistical arbitrage and cointegration).

In order to reduce risk in pairs-trading strategies, it is interesting to open many trades all with a very short holding time, hoping to spread the risk of each trade and to profit in expectation through the law of large numbers. According to Avellaneda & Lee (2010), the pairs trading strategy is the "ancestor" of statistical arbitrage. The term "statistical arbitrage" encompasses a variety of investment strategies whose principal characteristic is the use of statistical tools to generate excess returns. Desired characteristics of this class of strategies is market neutrality (low market correlations), and signal generation based on rules rather than fundamentals.

It is well known that pairs trading is a common strategy among many hedge funds. However, there is not a significant amount of academic literature devoted to it due to its proprietary nature. For a review of some of the existing academic models, see Poterba & Summers (1988); Lo & MacKinlay (1990); Gatev *et al.* (2006) and Elliot *et al.* (2005). In a recent paper, Khandani & Lo (2007) discuss the performance of the Lo-MacKinlay contrarian strategies in the context of the liquidity crisis of 2007. These strategies have several common features with the ones developed in this paper. Khandani & Lo (2007) market-neutrality is enforced by ranking stock returns by quantiles and trading "winners-versus-losers", in a dollar-neutral fashion. On the parametric side, Poterba & Summers (1988) study mean-reversion using auto-regressive models in the context of international equity markets. Zebedee & Kasch-Haroutounian (2009) analyzes the impact of pairs-trading at the microstructure level within the airline industry. Avellaneda & Lee (2010) use Principal Component Analysis or sector ETFs in their statistical arbitrage strategy. In both cases, they model the residuals, or idiosyncratic components of stock returns, as mean-reverting processes.

The main objective of this paper is to investigate the risk and return of a proposed pairs trading strategy for Brazilian stock market. The data used contains the daily closing prices of stocks of the Bovespa index (Ibovespa), from January 2005 through December 2010, summing up to 1.512 daily observations. We analyse the profitability of the pairs trading strategy implemented in an out-of-sample exercise. With an average profit of 16.01% per year (net cost), a Sharpe Ratio of 1.28, low market correlation and relatively low levels of volatility, the results reinforce the usefulness of cointegration in quantitative strategies.

The remainder of this paper is summarized as follows. In section 2, the concepts of statistical arbitrage and pairs trading strategies are presented in greater detail. Section 3 contextualizes the use of cointegration within this class of strategies. In section 4, we describe the model proposed for strategy implementation. In section 5 the data are discussed and the results obtained from the out-of-sample simulations are empirically verified. In section 6, a conclusion based on the empirical results is presented, along with suggestions of steps that could boost the results of the strategy.

2. Pairs Trading Strategy; Statistical Arbitrage

Pairs Trading is a trading or investment strategy used to exploit financial markets that are out of equilibrium. Litterman (2003) explains the philosophy of Goldman Sachs Asset Management as one of assuming that while markets may not be in equilibrium, over time they move to an equilibrium, and the trader has an interest to take maximum advantage from deviations from equilibrium.

Pairs-trading was pioneered by Nunzio Tartaglias quant group at Morgan Stanley in the 1980's. It remains an important statistical arbitrage technique used by hedge funds. They found that certain securities were correlated in their day-to-day price movements, (see Vidyamurthy, 2004). One of the trading techniques used involved trading with pairs of stocks. The process involved identifying pairs of stocks whose prices moved together. When an anomaly was identified in the relationship, the pair was traded with the idea that the anomaly would correct itself. This came to be known as pairs trading. Tartaglia and his group used the pairs trading strategy with great success throughout 1987. However, the group was dismantled in 1989, after two years of bad results, the pair trading strategy became increasingly popular among individual traders, institutional investors and hedge-funds.

Recently, due to the financial market crisis, it was widely reported in the specialized media that the year 2007 was especially challenging for quantitative hedge funds (see Khandani & Lo, 2007; Avellaneda & Lee, 2010), in particular for the statistical arbitrage strategies. The strategy proposed here is analyzed in the period in question and the results found corroborate with those of other authors.

Jacobs *et al.* (1993) define long-short stock strategy as being market neutral. Market neutral strategies maintain even exposure to market risks for long and short positions at all times. This approach eliminates exposure to directional risk from the market, such that the obtained return should not present correlation with the market reference index, which is the equivalent to a beta-zero portfolio. The portfolio returns are generated by the isolation of the alfa, adjusted by risk. According to Fung & Hsieh (1999), a strategy is said to be market neutral if its return is independent from the market's relative return. Market neutral funds actively seek to avoid systematic risk factors, betting on relative price movements.

The process of asset pricing can be seen in absolute or relative terms. In absolute terms, asset pricing is made by way of fundamentals, such as discounted future cash flow, for example. Relative pricing means that two assets that are close substitutes for each other should sell for similar prices - it doesn't say what the price should be. Reversion to the mean requires a driving mechanism; pairs trading would not work if prices were truly random. The Law of One Price (LOP) is the proposition that two investments with the same payoff in every state of nature must have the same current value. Thus, the price spread between close substitute assets should have a long term stable equilibrium over time. Hendry & Juselius (2001), use this principal to show that short term deviations from these equivalent pricing conditions can create arbitrage opportunities depending on the duration of price deviation.

When a deviation in the spread of the long term equilibrium price relationship is identified to be substantially greater than the slipage[1] due to the bid-ask spread, a position is opened simultaneously, buying the relatively undervalued stock and selling the relatively overvalued stock. The position is closed when the prices return to the spread level of long term equilibrium. The net profit of the operation is the sum of the profits from the long and short parts, calculated as the difference between the open prices and closed prices (ignoring transaction costs).

The natural extrapolation of pairs trading strategies consists of the operation of a group of stocks against another group of stocks, or generalized pair trading, (see Alexander & Dimitriu, 2005a; Dunis & Ho, 2005; Avellaneda & Lee, 2010; Caldeira & Portugal, 2010).

3. Cointegration-based strategies

The applicability of the cointegration technique to asset allocation was pioneered by Lucas (1997) and Alexander (1999). Its key characteristics, i.e. mean reverting tracking error, enhanced weights stability and better use of the information comprised in the stock prices, allow a flexible design of various funded and self-financing trading strategies, from index and enhanced index tracking, to long-short market neutral and alpha transfer techniques.

[1]Slipage is defined as the difference between the prices that trigger the order and the prices at which the order is executed.

3.1. The cointegration approach

The cointegration approach described by Vidyamurthy (2004), is an attempt to parameterize pairs trading strategies exploring the possibility of cointegration. Cointegration is a statistical feature, where two time series that are integrated of order 1, $I(1)$, can be linearly combined to produce one time series which is stationary, or $I(0)$. In general, linear combination of non-stationary time series are also non-stationary, thus not all possible pair of stocks cointegrate.

Definition. A $(n \times 1)$ time series vector $\mathbf{y}_t$ is cointegrated [2] if

- each of its elements individually are non-stationary and
- there exists a nonzero vector β such that $\beta \mathbf{y}_t$ is stationary.

In the previous decade the concept of cointegration was increasingly applied in financial econometrics, in connection to time series analysis and macroeconomics, (see Alexander & Dimitriu, 2002). It is an extremely powerful technique, which allows dynamic modeling of non-stationary time-series. The fundamental observation that justifies the application of the concept of cointegration in the analysis of stock prices is that a system involving non-stationary stock prices in levels can have a common stochastic trend (see Stock & Watson, 1988). When compared to the concept of correlation, the main advantage of cointegration is that it enables the use of the information contained in the levels of financial variables. Alexander & Dimitriu (2005a,b); Gatev *et al.* (2006); Caldeira & Portugal (2010), suggest that cointegration methodology offers a more adequate structure for financial arbitrage strategies.

The idea of pairs trading is to invests an equal amount in asset A and asset B, $\alpha p_t^A = p_t^B$, making this a cashless investment. This can be done by borrowing a number of shares of assets B, immediately selling these and investing the amount in α shares of asset A. Thus, we define the logarithm of the investment equation as follows:

$$0 = \log(\alpha) + \log(p_t^A) - \log(p_t^B). \tag{1}$$

The minus sign reflects the fact that asset B is sold short. The log-return on this investment over a small horizon $(t-1, t)$ is

$$\log\left(\frac{p_t^A}{p_{t-1}^A}\right) - \log\left(\frac{p_t^B}{p_{t-1}^B}\right). \tag{2}$$

Thus, to make profit the investor doesn't need to predict the behavior of p_t^A and p_t^B , but only that of the difference $\log(p_t^A) - \log(p_t^B)$. If we assume that $\left\{\log(p_t^A), \log(p_t^B)\right\}$ in (1) is a non-stationary $VAR(p)$ process, and there exists a value γ such that $\log(p_t^A) - \gamma \log(p_t^B)$ is stationary, we will have a cointegrated pair. The investment equation will then become

$$0 = \log(\alpha) + \log(p_t^A) - \gamma \log(p_t^B). \tag{3}$$

The value of γ will be determined by the cointegration, and the long run equilibrium relationship between the assets determines α. The return on the investment will be

$$\log\left(\frac{p_t^A}{p_{t-1}^A}\right) - \gamma \log\left(\frac{p_t^B}{p_{t-1}^B}\right). \tag{4}$$

If $\gamma = 1$, the investor is able to profit from the trade, even though the investment has an initial value of 0. A γ close to zero requires funds to invest in A. A large γ exposes the investor to risk of going short on B.

3.2. The Model

The investment strategy we aim at implementing is market neutral, thus we will try to find shares with similar correlation to the market, where we believe one stock will outperform the other one in the short term.

[2]For more details about cointegration analysis, see Johansen (1995); Hamilton (1994)

By simultaneously taking both a long and short position the correlation of the pair with the market will tend to equal zero.

In order to implement the strategy we need a couple of trading rules to follow, i.e. to determine when to open and when to close a trade. Our basic rule will be to open a position when the ratio of two share prices hits the 2 rolling standard deviation and close it when the ratio returns to the mean. Furthermore, there will be some additional rules to prevent us from loosing too much money on one single trade. If the ratio develops in an unfavourable way, we will use a stop-loss and close the position if we have lost 7%. Finally, we will never keep a position for more that 50 days. This should be enough time for the pairs to revert, but also a short enough time not to loose time value. On average, the mean reversion will occur in approximately 10 days , and there is no reason to wait for a pair to revert fully, if there is very little return to be earned. The potential return to be earned must always be higher than the return earned on the benchmark or in the fixed income market. The rules described are totally based on statistics and predetermined numbers.

The first step of the strategy consists of identifying potential stock pairs. Having defined the stock universe to be considered, we first check if all the series are integrated of the same order, $I(1)$. This is done by way of the Augmented Dickey Fuller Test (ADF). Having passed the ADF, cointegration tests are done on all possible combination of pairs. To test for cointegration we adopts Engle and Grangers 2-step approach and Johansen test[3].

As an additional criteria for selecting the pairs to be used in the composition of the portfolio, the Sharpe Ratio (SR) is calculated and the pairs are ranked based on:

$$SR = \frac{R^A}{\sigma^A} \tag{5}$$

where R^A is the annualized return σ^A is the annualized volatility of the strategy A. The 20 pairs that present the greatest SR in the in-sample simulations are used to compose a pairs trading portfolio to be employed out-of-sample.

The strategy adopted here seeks to be beta-neutral, this way the financial values allocated to long and short stocks might not be equal[4]. As soon as the spread distances itself from its long term mean, one can bet that the spread will return to its long term mean, however we do not know if we will gain more on long or on short positions[5]. Once an operation is initiated, the portfolio is not rebalanced. Therefore, after the opening of a position, even when prices move and the position may no longer be neutral, the portfolio is not rebalanced. We only have two types of transactions that are admitted by the strategy's methodology; move into a new position, or the total liquidation of a previously opened position.

4. Data and Empirical Results

4.1. Data

The data used in the study consists of the daily closing prices of the 50 stocks with largest weights in the Ibovespa index from Sao Paulo Stock Exchange in the last quarter of the period analysed. All of the stocks used are listed in Bovespa, which means they are among the most liquid stocks traded on the Brazilian market. This characteristic is important for pairs trading, since it often diminishes the slipage effect. Moreover, using low liquidity stocks may involve greater operational costs (bid and ask spread) and difficulty in renting a stock. The data were obtained from Bloomberg, taken from the period of January 2005 to December 2010. The data are adjusted for dividends and splits.

It is common in pairs trading strategies to require that the stock pairs belong to the same sector, for example in Chan (2009) and Dunis *et al.* (2010). Here, we do not adopt this restriction. Therefore, stock

[3]All of the procedures are implemented on MATLAB software, version 7.0. The functions cadf and johansen are used, and are available at www.spatial-econometrics.com.

[4]One alternative is to define the investment as financially-neutral, assuming equal financial volumes of long and short shares.

[5]We don't know which case occurs first: if the stocks return to their long term equilibrium because the overvalued stock price falls more or because the undervalued stock price climbs more, or if they both present the same performance.

pairs from companies belonging to different sectors can be traded, as long as they satisfy the cointegration criterion.

4.2. Estimation and Out-of-sample Results

Initially, we divide the sample into training, and testing periods. The training period is a preselected period where the parameters of the experiment are computed. Immediately after the training period, the testing period follows, where we run the experiments using the parameters computed in the first period. Note that pairs are also treated as parameters in our trading system. We use one year for training and four months for testing. Since the price series of the 50 stocks are employed, there exist 1,225 possible pairs. Cointegration tests are performed (Johansen and Engle-Granger) on all combinations. Of the 1,225 possible pairs, an average of 90 cointegrated pairs was obtained in each period. The pairs that passed the cointegration tests are then ranked based on the in-sample SR, following Gatev *et al.* (2006) and Andrade *et al.* (2005). After selecting 20 pairs with highest SR, four months of pairs trading are carried out. At the end of each trading period the operations that were opened are closed, and a new training period ending on the last observation of the previous trading period is initiated. Now pairs can be substituted and all parameters are re-estimated. This procedure continues in a rolling window fashion until the end of the sample.

The return from each period is calculated as follows:

$$R_t = ln\left(P_{x,t}/P_{x,t-1}\right) - ln\left(P_{y,t}/P_{y,t-1}\right) \tag{6}$$

where P_x are prices of the stocks held long, and P_y are the prices of the shorted stocks. Transaction costs are considered to be 0.5% of the total price (made up of brokerage fees and other costs).

Table 1 shows the pairs used in the strategy during the last quarter of 2009. Of the 1,225 possible pairs, 97 passed the cointegration tests of Johansen and Engle-Granger. Of those 97, the 20 pairs that presented the greatest in-sample SR were selected to be used out-of-sample. Even though there weren't restrictions requiring stocks within a pair to come from companies from the same sector, the majority of pairs are comprised of stocks from companies that are in some way related. One also notes that many of the pairs present a half life of less than 10 days, reinforcing the mean-reversion characteristic, which is desirable for the strategy. Although all pairs present positive SR in-sample simulations, not all obtained positive return in the out-of-sample trading period. During the period in question, 6 of the 20 pairs that comprise the portfolio showed negative results, and on average a net return of 3.82% per pair was obtained.

Table 2 summarizes the excess returns for the pairs portfolios. The results presented refer to the out-of-sample analysis (from January, 2006 to December, 2010). The profitability shown has already been discounted for transaction costs[6]. One can also note that the strategy presents a relatively low volatility of 12.49% in annualized terms, and a correlation coefficient with the market of -0.103, indicating that the strategy can be considered market neutral.

We also present the maximum drawdown of the strategy in the analyzed period, which was 24.49%. This is a simple measure of the fall in percentage terms with respect to the peak of the cumulative return, and can be used as a measurement of how aggressively the strategy's leverage can be increased. It can be seen in figure 1, which conveys the strategy's cumulative profit and volatility, that the maximum drawdown occurred in the first semester of 2008.

Figure 1 compares the cumulative excess returns and volatility of the strategy with the cumulative excess returns of the Ibovespa index. The smooth path of the pairs trading portfolio contrasts dramatically with the volatility of the stock market. It can be noted in the second panel of Figure 1, that the pairs trading strategy presented a relatively low and stable standard of volatility for practically the entire analyzed period, running at levels below 15% in annualized terms, for nearly the entire analyzed period. Even in the most acute period of the international financial crisis, when the volatility on the domestic stock market surpassed 120%, the volatility of the strategy never reached 40%.

[6]The costs considered are 0.5% in opening and 0.5% in closing the position, summing up to 0.10% per operation. Costs related to renting stocks sold short were considered to be 2% per year.

Table 1: Descriptive Statistics of the Pairs. Sample Period 2009:09 to 2009:12.

	Stock 1	**Stock 2**	**EG (ADF)**	**JH (λ_{tr})**	SR**(*in-sample*)**	**Half-Life**	**Net Ret.**
Pairs	Itub4	Itsa4	-3.67	18.32	4.33	4.58	6.60
	Usim5	Usim3	-3.47	18.00	3.29	16.82	9.03
	Vale3	Brap4	-4.25	24.23	3.13	12.19	7.96
	Ambv4	Natu3	-4.27	19.42	2.69	6.00	10.19
	Ambv4	Jbss3	-4.35	20.63	2.57	9.01	4.00
	Csna3	Brap4	-3.78	19.27	2.39	11.62	10.10
	Bbas3	Lren3	-3.47	16.86	1.90	6.31	-19.68
	Cyre3	Gfsa3	-4.17	18.20	1.70	4.81	24.19
	Vale3	Ccro3	-3.39	14.24	1.61	10.87	-4.00
	Bbas3	Usim3	-3.69	19.58	1.60	8.61	-10.69
	Brfs3	Pcar5	-4.15	19.20	1.59	6.41	-6.59
	Netc4	Jbss3	-3.87	17.23	1.58	8.39	-8.28
	Cple6	Pcar5	-4.05	17.26	1.57	12.22	13.78
	Cple6	Ccro3	-3.58	15.97	1.56	13.25	5.37
	Lame4	Ambv4	-3.85	17.06	1.46	14.85	8.91
	Itub4	Ccro3	-3.92	17.60	1.45	6.18	13.45
	Usim5	Bbas3	-4.39	22.13	1.40	6.51	0.40
	Ambv4	Ccro3	-3.82	18.76	1.24	6.98	22.09
	Brfs3	Cple6	-3.55	19.38	1.15	7.04	-14.38
	Bvmf3	Netc4	-3.98	19.39	1.10	17.01	4.02

Note: half-life is expressed in days and net return in %. The 95% critical values for Johansen test is 13.43 and for ADF is 3.38.

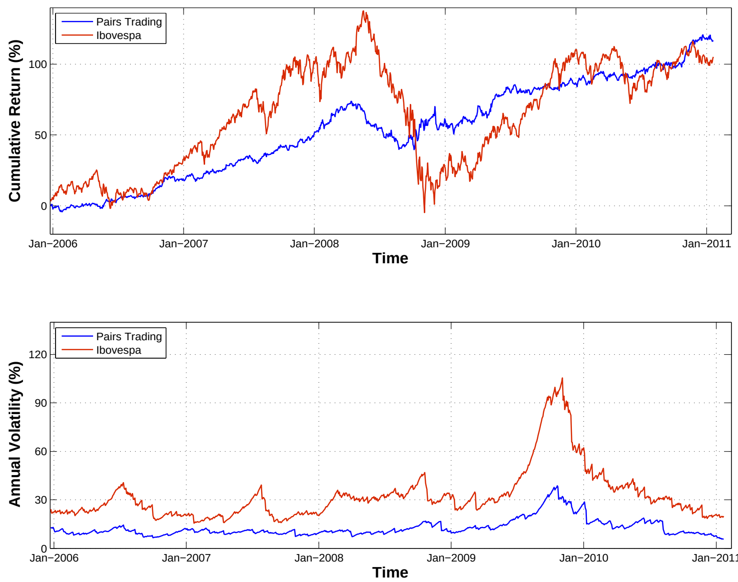
Table 2: **Statistics of excess returns of unrestricted pairs trading strategies, 2006:01 to 2010:12**

Summary Statistics of the Pairs Trading Strategy	
# of observations in the sample	1512
# of days in the training window	250
# of days in the trading period	84
# of trading periods	15
# of pairs in each trading period	20
Average annualized return	16.01%
Annualized volatility	12.42%
Annualized Sharpe Ratio	1.282
Largest daily return	6.25%
Lowest daily return	-4.80%
Cumulative profit	113.89%
Spearman correlation coefficient	-0.10
Skewness	0.51
Kurtosis	6.21
Maximum *Drawdown*	24.49%

Note: Summary statistics of the daily and monthly excess returns on portfolios of pairs between Jan 2006 and December 2010 (60 and 1,008 observations, respectively). The sample period is from January 2005 through December 2010, where as the out-of-sample simulations were performed from January 2006 through December 2010. In particular, we report the minimum, median, mean, the skewness and kurtosis, and the maximum of three important performance measures: the accumulative profit, the Sharpe Ratio, and the maximum drawdown.

Figure 1: **Cumulative excess return and Volatility of top 20 pairs and Ibovespa. Period 2006:01 - 2010:12.**

Note: In the first panel, cumulative profit of the pairs trading strategy and Bovespa, in the second annualized volatilities (EWMA Vol with $\lambda = 0.94$).



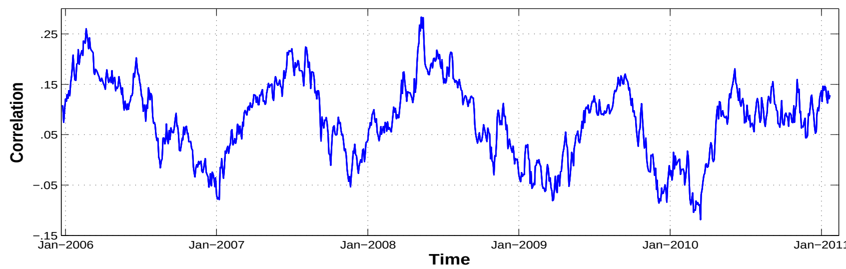
Another relevant part of the evaluation of the pairs trading strategy is the analysis of the correlation with the main reference market index, since one of the goals of the strategy is market neutrality (see Alexander & Dimitriu, 2002). The strategy showed a correlation with Ibovespa of less than 0.15, and for a good part of the sample it was less than 0.10, as can be observed in Figure 2. The estimated β of the portfolio[7], is also presented, which corroborates with the strategy's market neutrality, presenting greater instability only during the turbulent year of 2008.

Table 3 summarizes annual statistics of pairs trading strategies. It can be observed that the strategy showed its worst performance in the year 2008, accumulating a net profit of 2.85% and volatility slightly higher than in other years. As highlighted by Khandani & Lo (2007) and Avellaneda & Lee (2010), the second semester of 2007 and first semester of 2008 were quite complicated for quantitative investment funds. Particularly for statistical arbitrage strategies that experienced significant losses during the period, with subsequent recovery in some cases. Many managers suffered losses and had to deleverage their portfolios, not benefiting from the subsequent recovery. We obtain results which are consistent with Khandani & Lo (2007) and Khandani & Lo (2007) and validate their unwinding theory for the quant fund drawdown. Note that in Figure 1, the proposed pairs trading strategy presented significant losses in the first semester of 2008, starting its recovery in the second semester. Khandani & Lo (2007) and Avellaneda & Lee (2010) suggest that the events of 2007-2008 may be a consequence of a lack of liquidity, caused by funds that had to undo their positions.

[7] β estimated with the Kalman filter, with the goal of verifying its stability over time.

Figure 2: **Spearman correlation coefficient and β of the Strategy**.

The Spearman correlation coefficient calculated based on a sliding window of 84 observations. β estimated by the Kalman filter.



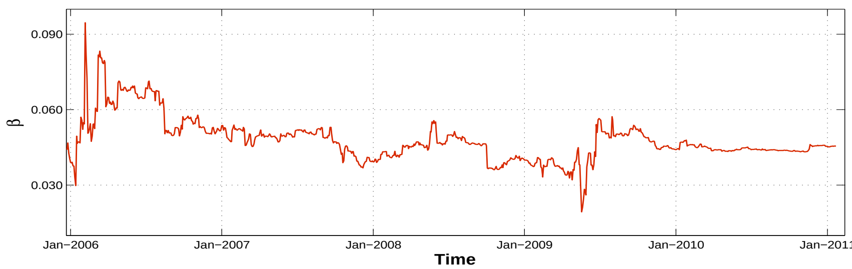


Table 3: **The P&L (in %) of the Statistical Arbitrage Strategy for 5 Years**

Year	Max	Min	Median	Mean	Std	Kurt	Skew	Accum	Sharpe	MDD
2006	7.41	-3.11	1.46	1.44	0.12	0.04	5.57	17.37	1.72	5.57
2007	4.32	-1.48	2.92	2.13	0.12	0.23	4.71	24.94	2.62	4.71
2008	12.41	-6.19	-0.59	0.29	0.14	0.68	24.98	2.85	0.15	24.98
2009	3.89	-1.76	2.08	1.77	0.04	0.34	5.24	21.25	1.86	5.24
2010	6.99	-0.88	0.66	1.14	0.17	0.07	3.66	13.71	1.46	3.66
All Time	12.41%	-6.19%	1.37%	1.19%	0.124	0.51	6.21	113.89%	1.28	24.98

Note: Summary statistics for the annual percentage excess (net) returns on portfolios of top 20 pairs between 2006 and 2010.

5. Conclusions

In this paper we have proposed a statistical arbitrage strategy known as pairs trading for stocks of Sao Paulo stock exchange. The strategy is implemented based on cointegration, exploring the the mean-reversion of pairs. Cointegration tests are applied to all possible pair combinations in order to identify stock pairs that share a long term equilibrium relationship. Of 1,225 possible pairs, on average, 90 cointegrated pairs from each formation period were obtained. Subsequently, we calculated the standardized spread between the stocks and we simulated trades in-sample. From there, a diversified portfolio containing 20 pairs that displayed the greatest SR in-sample were selected to be traded out-of-sample.

The cumulative net profit from the four year period of rolling window out-of-sample tests was of 113.89%, with an annual mean of 16.01%. In addition, the pairs trading here implemented showed relatively low levels of volatility and no significant correlation to Ibovespa, confirming its market neutrality. The results are attractive when compared to other strategies employed by hedge funds (see Soerensen, 2006). Specially if we take into account that the strategy is practically cashless. In future research projects we will try to enhance profitability and to mitigate risks, through a method to identify the stability of the cointegration parameters. Another goal is to apply the proposed methodology to high frequency data. The results presented reinforce the use of the concept of cointegration as an important tool in the quantitative management of funds.

References

Alexander, C. 1999. Optimal hedging using cointegration. *Philosophical Transactions of the Royal Society Series A*, **357**, 2039–2058.

Alexander, Carol, & Dimitriu, A. 2002. *The Cointegration Alpha: Enhanced Index Tracking and Long-Short Equity Market Neutral Strategies.* ISMA Discussion Papers in Finance 08. ISMA Finance.

Alexander, Carol, & Dimitriu, Anca. 2005a. Indexing and statistical arbitrage: tracking error or cointegration? *Journal of Portfolio Management*, **31**, 50–63.

Alexander, Carol, & Dimitriu, Anca. 2005b. Indexing, cointegration and equity market regimes. *International Journal of Finance and Economics*, **10**, 213–231.

Andrade, S. C., Pietro, V., & Seasholes, M. S. 2005. *Understanding the Profitability of Pairs Trading.* UC Berkeley Working Papers. UC Berkeley.

Avellaneda, M., & Lee, J. H. 2010. Statistical arbitrage in the US equities market. *Quantitative Finance*, **10**, 1–22.

Caldeira, João F., & Portugal, Marcelo S. 2010. Estratégia Long-Short, Neutra ao Mercado, e Index Tracking Baseadas em Portfólios Cointegrados. *Revista Brasileira de Finanças*, **8**(4), 469–504.

Chan, E. P. 2009. *Quantitative Trading: How to Build Your Own Algorithmic Trading Business.* New Jersey: Wiley trading series.

Dunis, C. L., & Ho, R. 2005. Cointegration portfolios of european equities for index tracking and market neutral strategies. *Journal of Asset Management*, **1**, 33–52.

Dunis, C. L, Giorgioni, G., Laws, J., & Rudy, J. 2010. *Statistical Arbitrage and High-Frequency Data with an Application to Eurostoxx 50 Equities.* CIBEF Working Papers. CIBEF.

Elliot, R. J., Hoek, J. V. D., & Malcolm, W. P. 2005. Pairs trading. *Quantitative Finance*, **5**(3), 271–276.

Fung, W., & Hsieh, D. A. 1999. Is mean-variance analysis applicable to hedge funds? *Economics Letters*, **62**(1), 53–58.

Gatev, E., Goetzmann, G. W., & Rouwenhorst, K. 2006. Pairs Trading: Performance of a Relative Value Arbitrage Rule. *The Review of Financial Studies*, **19**, 797–827.

Hamilton, James D. 1994. *Times Series Analysis.* New Jersey: IE-Princeton.

Hendry, D., & Juselius, K. 2001. Explaining cointegration analysis: part II. *Energy Journal*, **22**, 75–120.

Jacobs, B., Levy, K., & Starer, D. 1993. Long-Short Equity Investing. *Journal of Portfolio Management*, **1**, 52–64.

Johansen, S. 1995. *Likelihood-based inference in cointegrated vector autoregressive models.* Advanced texts in econometrics. Oxford University Press.

Khandani, A. E., & Lo, A. W. 2007. What happened to the quants in August 2007? *Journal of Investment Management*, **5**, 5–54.

Litterman, B. 2003. *Modern Investment Management - An Equilibrium Approach.* New York: Wiley.

Lo, A. W., & MacKinlay, A. C. 1990. When Are Contrarian Profits Due to Stock Market Overreaction? *Review of Financial Studies*, **3**(2), 175–205.

Lucas, A. 1997 (Jan.). *Strategic and tactical asset allocation and the effect of long-run equilibrium relations.* Serie Research Memoranda 0042. VU University Amsterdam, Faculty of Economics, Business Administration and Econometrics.

Pole, A. 2007. *Statistical Arbitrage: Algorithmic Trading Insights and Techniques.* USA: Wiley Finance.

Poterba, J. M., & Summers, L. H. 1988. Mean reversion in stock prices : Evidence and Implications. *Journal of Financial Economics*, **22**(1), 27–59.

Soerensen, Jan Tang. 2006. *Hedge Fund Portfolio Construction.* M.Phil. thesis, The Fletcher School - Tufts University.

Stock, J.H., & Watson, M.W. 1988. Testing for Common Trends. *Journal of the American Statistical Association*, **83**, 1097–1107.

Vidyamurthy, G. 2004. *Pairs Trading, Quantitative Methods and Analysis.* Canada: John Wiley & Sons.

Zebedee, A. A., & Kasch-Haroutounian, M. 2009. A closer look at co-movements among stock returns. *Journal of Economics and Business*, **61**(4), 279–294.