Measuring inflation persistence in Brazil using a multivariate model[♦]

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Abstract

In this article, we estimate inflation persistence in Brazil in a multivariate framework of unobserved components, accounting for the following sources affecting inflation persistence: Deviations of expectations from the actual policy target; persistence of the factors driving inflation, such as the behaviour of interest rate and output may also affect the dynamics of inflation; and the usual intrinsic measure of persistence is evaluated through lagged inflation terms. Data on inflation, output and interest rates are decomposed into unobserved components, which are identified in a linear Gaussian state-space model. To simplify the estimation of a great number of unknown variables, we employ Bayesian analysis. Our results indicate that expectations-based persistence matters considerably for inflation persistence in Brazil, which has experienced an overall decrease in the last few years. These findings imply that traditional price-setting frictions used in macroeconomic models may be misleading in terms of representing real inflation persistence.

Keywords: Inflation persistence, inflation expectations, Kalman filter, Bayesian analysis

JEL classification: C11; C22; C32; E31

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1 Introduction

Vestiges of inflationary memory, particularly in developing countries, which experienced decades of high inflation levels, may still be an important obstacle in the process of price stabilization. In the 1980s and early 1990s, Brazil underwent a period of high inflation rates. The inertial component of inflation was admittedly strong in that period. Although the Real Plan in 1994 managed to reduce inflation rates, it is not yet clear whether a substantial decrease in inflation persistence has followed, according to the Central Bank of Brazil's Inflation Report of December 2008, see BCB (2008).

Most of the literature on measures of inflation persistence is based on inflation data only, usually building on univariate autoregressive equations. Examples include Pivetta and Reis (2007), Cogley and Sargent (2005) and Levin and Piger (2004), as well as Petrassi and Oliveira (2010) in the Brazilian case. These measures are said to represent **unconditional** inflation persistence, since they do not consider the underlying inflation generating process.

In this article, we instead use a model based on Dossche and Everaert (2005),[1] recognizing that there are specific effects on the process of inflation other than past inflation values, which have a considerable impact on inflation persistence. We first estimate univariate inflation persistence for Brazil with a slightly different approach, by introducing an expectations-based source of persistence, in the sense of Angeloni et al. (2004). Then, we provide estimates of inflation persistence in a multivariate framework of unobserved components, dealing with the following sources of inflation persistence: First, the fact that it may derive from deviations of expectations from the actual policy target. This source is known as expectations-based persistence, and it may be understood as similar to the "sticky information measure", as in Mankiw and Reis (2002). According to these authors, firms gather information on prices slowly because of the costs incurred in acquiring and processing new data.[2] If this is really the case, then substantial differences between private agents' expected targets and central bank policy targets have a potential influence on inflation persistence that should not be overlooked.[3] Second, persistence of the factors driving

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[1] Their work reflects research conducted in the context of the Eurosystem Inflation Persistence Network (IPN) of the European Central Bank, which aimed to explain price setting and inflation dynamics in order to address patterns, causes and policy implications of inflation persistence in the euro area.

[2] A related argument is found in Sims (2003). According to his theory of "rational inattention", it may happen that people simply have limited ability to obtain and process information. Other possibilities are the assumptions that the central bank has imperfect credibility, as in Kozicki and Tinsley (2005) or that agents are uncertain about central bank preferences, as in Cukierman and Meltzer (1986), or also that agents are learning about the true model of the economy, as argued by Milani (2007).

[3] In fact, both Caetano and Moura (2009) and Guillen (2008) found evidence that agents update information in Brazil in a similar frequency as found in studies for the USA and Europe, such as Mankiw et al. (2003).

inflation, such as the stance of interest rate and of potential output may also affect the persistence of inflation. According to Dossche and Everaert (2005), persistence of output gaps in response to business cycle shocks add to the persistence of inflation, referred to by them as "extrinsic persistence." Third, the usual "intrinsic persistence" related to the nature of the price-setting mechanism is also evaluated as we account for lags in the inflation equation.

Data on inflation, output and interest rates are decomposed into unobserved components, forming a linear Gaussian state-space model. Due to the large number of unknown components, to simplify the estimation, we use data from other studies of the Brazilian economy (or from other countries whenever Brazilian data are not available) as priors in a Bayesian analysis.

The main outcomes are the time distributions of state variables such as the perceived and actual inflation target and coefficients that represent the sources of persistence, as well as an estimate of the evolution of natural interest rates. Besides playing a role in inflation dynamics, this variable is useful to check whether interest rates have departed from the estimated natural stance.

This article is organized into five sections. In Section 2, we present the related literature, concerning both empirical and theoretical contributions to inflation persistence. In Section 3, we present the model and discuss the inputs and estimation strategies adopted. Section 4 contains the main results and section 5 concludes.

2 Inflation persistence literature

Modern literature on inflation persistence follows two major paths. The first one deals with macroeconomic approaches as an alternative to the new Keynesian model described in Calvo (1983), whose dynamics does not include the inflation persistence observed in the real world.

The second path, which is also the focus of the present study, seeks to empirically measure inflation persistence. A common practice is to adopt univariate time series approaches, in which persistence is represented by the sum of autoregressive coefficients of an AR model of inflation. Examples include Pivetta and Reis (2007) and Cecchetti and Debelle (2006). Pincheira (2009) estimates inflation persistence for Chile, and concludes that it has decreased in the past few years. This is, however, a simpler form of analysing the

variable, which does not contemplate the full dynamics of inflation, since it only captures the intrinsic persistence derived from price and wage inflation.[4]

The paper by Dossche and Everaert (2005) is the main reference used in the present study. By decomposing the inflation generating process into unobserved variables, using the Kalman filter, the authors measure inflation persistence in the Euro area and in the USA and identify types of persistence at different levels. According to them, measures of persistence have often been overvalued by emphasising on intrinsic persistence.

2.1 The Brazilian Context

In the 1980s and in the early 1990s, Brazil underwent a period of high inflation rates. In some episodes, inflation persistence was a key component that further fuelled inflation rates, as demonstrated by constant and deliberate price indexation and packages of sweeping economic reforms. Although the implementation of the Real Plan back in 1994 brought down inflation rates, episodes of high levels of persistence still occurred, as described in the Open Letter of the Brazilian Central Bank (BCB, 2003). In recent years, inflation persistence in Brazil has apparently decreased, but it is still a cause for concern. BCB (2008), for example, argued that "[...] it is not clear whether the decrease in inflation rates in these countries was followed by a reduction in the persistence of inflation [...]", when referring to emerging countries, such as Brazil, Argentina and Turkey. Specifically in regard to Brazil, it reads: "[...] given the extremely high rates of inflation the country had experienced for decades, inflationary memory might still be of relevance."

Consequently, attempts to accurately measure inflation persistence seem to be at least as important in Brazil as they were in the Euro area by the time the Inflation Persistence Network was established. As a matter of fact, some authors have provided recent persistence estimates for Brazil. For example, Petrassi and Oliveira (2010) and Rebelo et al. (2009), compare Brazilian estimates to those obtained for other countries.[5] However, in none of these models, a proper account of the persistence of expectations and output deviations is taken. Despite different conclusions, existing models in the Brazilian literature elaborate on **unconditional** estimates of inflation persistence.

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[4] Controversy still exists over whether persistence has decreased or not in the past few years, as in these approaches, this strongly depends on how inflationary trend is modelled, as pointed out by Marques (2004). When structural breaks or Markov shifts are allowed, autoregressive coefficients naturally tend to decrease, indicating that inflation persistence becomes more constant, which does not necessarily apply to the type of modelling we propose here.

[5] More recently, both Figueiredo and Marques (2011) and Silva and Leme (2011) focused on ARFIMA models accounting respectively for regime switching and structural breaks.

3 Methodology

3.1 Macroeconomic model

As previously argued, a considerable part of the literature measures inflation persistence by focusing on one of the following equivalent values: Either the coefficient for the sum of lagged terms φ_i in a simple univariate decomposition of inflation such as:

$$\pi_t = \mu + \sum_{i=1}^{k} \varphi_i \pi_{t-i} + v_t \tag{1}$$

$$v_t \sim N(0, \sigma_v^2)$$

or the largest autoregressive root (LAR), on the other hand, as in Pivetta and Reis (2007) and Petrassi and Oliveira (2010), represented by ρ in an equation such as:

$$\pi_t = \mu + \rho \pi_{t-1} + \sum_{i=1}^{k} \phi_i \Delta \pi_{t-i} + v_t \tag{2}$$

In both cases, the dynamics of inflation relies on a measure of its average, μ, and its autoregressive components. Therefore, any estimate of persistence based on these equations is an unconditional measure.

Our model begins with a slight modification of this univariate setting. As in Dossche and Everaert (2005) and Kozicki and Tinsley (2005), inflation is allowed to follow a stationary AR process around the perceived inflation target π_t^P:

$$\pi_t = (1 - \sum_{i=1}^{k} \varphi_i)\pi_t^P + \sum_{i=1}^{k} \varphi_i \pi_{t-i} + v_{1t} \tag{3}$$

$$v_{1t} \sim N(0, \sigma_{v_1}^2)$$

where π_t^P is treated as an unobserved component that represents what agents expect inflation target to be. We assume π_t^P is related to the actual inflation target π_t^T, which is also an unobserved component here[6], in the following way:

$$\pi_{t+1}^P = (1 - \delta)\pi_t^P + \delta \pi_{t+1}^T + \eta_{1t} \tag{4}$$

$$\eta_{1t} \sim N(0, \sigma_{\eta_1}^2)$$

The so-called intrinsic persistence is represented in (3) by $\sum_{i=1}^{k} \varphi_i$, since it indirectly measures the speed at which shifts on π_t^P have an impact on observed inflation. On the other hand, $(1 - \delta)$ is a good approximation of the expectations-based source of persistence. Clearly, if δ is close to 1, private agents perfectly predict the actual inflation target, so there is no effect on persistence derived from erroneous expectations. The error term η_{1t} represents

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[6] While in an inflation targeting system, the target is itself an essential component, usually made available publicly, the variable π_t^T represents here the effectively pursued target, which is obviously not observable in the economy.

shocks to the perceived inflation target, and only has a short-run impact on π_t^P. We assume the actual inflation rate target follows a random walk,

$$\pi_{t+1}^T = \pi_t^T + \eta_{2t} \tag{5}$$
$$\eta_{2t} \sim N(0, \sigma_{\eta_2}{}^2)$$

Shifts in the inflation target are meant to represent changes in central bank preferences, for instance, due to changes in the composition of the Monetary Policy Committee or as a consequence of modifications in the economic outlook. Therefore, η_{2t} represents shocks to the Central Bank's inflation target that have a long-run impact on π_t^P. Using the simplification that $\eta_{1t} = 0$, equation (4) thus becomes:

$$\pi_{t+1}^P = (2 - \delta)\pi_t^P + (\delta - 1)\pi_{t-1}^P + \delta\eta_{2t} \tag{6}$$

Consequently, adding to the direct impact of lagged coefficients on inflation, there is an indirect effect derived from imperfect information about the actual target, which also translates into inflation persistence. This differentiates our model from the usual univariate approaches.

However, in our univariate setting, it is still not possible to disentangle these types of persistence from extrinsic persistence, since the level of output and interest rates do not play any role. In order to do so, we further consider a structural macroeconomic model based on Rudebusch and Svensson (1999) and in line with an inflation targeting regime.[7] Differently from Dossche and Everaert (2005), we further consider alternative observation periods, in order to capture shifts in inflation persistence. The basic model consists of three observation equations, using the terminology of state-space models. The first one is a conversion of equation (3) into a new Keynesian Phillips curve, by adding a lagged output gap term h_{t-1}, that is:

$$\pi_t = (1 - \sum_{i=1}^k \varphi_i)\pi_t^P + \sum_{i=1}^k \varphi_i \pi_{t-i} + \phi_1 h_{t-1} + \nu_{1t} \tag{7}$$
$$\nu_{1t} \sim N(0, \sigma_{\nu_1}{}^2)$$

The second observation equation is a Central Bank's reaction rule, under which interest rates respond to an inertial component i_{t-1}, to a neutral position of the interest rate $(\pi_t^P + r_t^*)$ and to deviations of inflation from its target $(\pi_{t-1} - \pi_t^T)$:

$$i_t = \rho_2 i_{t-1} + (1 - \rho_2)(\pi_t^P + r_t^*) + \rho_1(\pi_{t-1} - \pi_t^T) + \nu_{2t} \tag{8}$$
$$\nu_{2t} \sim N(0, \sigma_{\nu_2}{}^2)$$

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[7] Bogdanski et al. (2000) describe the main features of the model used for the Brazilian economy, which are similar to Rudebusch and Svensson (1999).

where $(\pi_t^P + r_t^*)$ can be understood as the nominal natural rate of interest. This rule meets general theoretical principles for economies that adopt inflation targets and allows extracting information about shifts in inflation targets.

Finally, the following equation contemplates the aggregate demand side. As usual, we assume real output is decomposed into potential output and output gap, $y_t^r = y_t^P + h_t$, while the latter is explained by lagged terms and by a monetary policy transmission mechanism $(i_t - \pi_{t-1}^P - r_{t-1}^*)$, corresponding to an IS relationship:

$$h_t = \phi_2 h_{t-1} + \phi_3 h_{t-2} + \phi_4(i_t - \pi_{t-1}^P - r_{t-1}^*) + v_{3t} \tag{9}$$
$$v_{3t} \sim N(0, \sigma_{v_3}^2)$$

Following Dossche and Everaert (2005), extrinsic persistence can be represented by the sum of terms ϕ_2 and ϕ_3, which clearly include the persistence of output deviations from their potential level. The unobserved variables in equations (7) through (9) have the following behaviour over time:

$$y_{t+1}^P = y_t^P + \lambda_{t+1} + \eta_{3t} \tag{10}$$
$$\lambda_{t+1} = \lambda_t + \eta_{4t} \tag{11}$$
$$r_{t+1}^* = \gamma \lambda_{t+1} + \tau_{t+1} \tag{12}$$
$$\tau_{t+1} = \theta \tau_t + \eta_{5t} \tag{13}$$

in addition to equation (6) for the perceived inflation target. As usual, η_{3t}, η_{4t} and η_{5t} are normally distributed with zero mean and corresponding variances $\sigma_{\eta_3}^2$, $\sigma_{\eta_4}^2$ and $\sigma_{\eta_5}^2$.

Note that potential output follows a random walk with drift. The drift term λ_t represents the trend growth of potential output, which changes according to technological improvements.

Equations (11) through (13) are based on Laubach and Williams (2003), who focus on measuring the natural rate of interest. Such rate (r_t^*) depends on the trend growth of potential output and other determinants, such as households' rate of time preference (τ_t). The central argument of Laubach and Williams (2003) is that the natural rate of interest in the USA has varied significantly and thus should be taken into consideration in monetary policy decisions. Since these variables are jointly estimated in the present model, and the natural rate of interest is of paramount importance to the conduct of monetary policy, we provide estimates for the Brazilian economy in the analysed period.

3.2 Estimation strategy

The Kalman filter was used due to its superiority in estimating models with time-varying parameters. Instead of taking for granted that economic agents instantly recognize the true model, it is assumed that they learn about it (and, especially, about the changes), using new information efficiently. The Kalman filter algorithm is implemented with the appropriate state-space representation of the time series model.

First, a measurement (or observation) equation relates a vector y_t of N known variables with a state vector α_t, of dimension $M \times 1$, as follows:

$$y_t = Z\alpha_t + Ad_t + \varepsilon_t \tag{14}$$

where Z and A are coefficient matrices, d_t is a vector of exogenous variables $K \times 1$, and ε_t is an error vector, such that $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = H$.

The vector of unobserved components, α_t, behaves dynamically following the state equation[8]

$$\alpha_t = T\alpha_{t-1} + R\eta_t \tag{15}$$

where T and R are coefficient matrices, and η_t is an error vector with zero mean and $Var(\eta_t) = Q$. Here, an analogous state equation is used, with a slight change in the timing of the state variable, which does not alter the Kalman filter result, provided that ε_t and η_t are uncorrelated:

$$\alpha_{t+1} = T\alpha_t + R\eta_t. \tag{16}$$

The algorithm also requires an initial state vector α_0, for which $E(\alpha_0) = a_0$, such that $E(\varepsilon_t \alpha_0') = 0$ e $E(\eta_t \alpha_0') = 0$ and $Var(\alpha_0) = P_0$, where P_0 is positive semidefinite.

In practice, as observations of y_t become available, the algorithm updates the mean and variance of the state vector up to period t. It is assumed that the information about t-1 is known, that is, α_{t-1} is normally distributed with known mean a_{t-1} and variance P_{t-1}. According to this procedure, the mean of the state vector at t is:

$$a_{t|t-1} = T\alpha_{t-1} \tag{17}$$

which is commonly referred to as a forecasting equation. The distribution predicted by the algorithm for the next observation, y_t, is normally distributed with mean

$$\hat{y}_{t|t-1} = Z\alpha_{t|t-1} + Ad_t \tag{18}$$

and covariance matrix

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[8] Note that, in their general forms, state equations should neither contain lagged variables in more than one period nor other state variables. However, these restrictions can be overcome in the state-space specification, as shown further ahead.

$$F_t = ZP_{t|t-1}Z' + H_t. \tag{19}$$

When the observation of y_t is known, the mean and variance of the updated conditional distribution of the state variable are obtained, which correspond, respectively, to:

$$a_t = a_{t|t-1} + P_{t|t-1}Z'F_t^{-1}(y_t - Z\alpha_{t|t-1}Ad_t) \tag{20}$$

and

$$P_t = P_{t|t-1} - P_{t|t-1}Z'F_t^{-1}ZP_{t|t-1} \tag{21}$$

This process is repeated up to the last period $t = T$ for which information on y_T is available. These general specifications allow us to express the univariate model (equations 3 through 6) in state space:

$$y_t = [\pi_t] \qquad\qquad H = [\sigma_{v_1}^2]$$

$$Z = \left[\left(1 - \sum_{i=1}^{4} \varphi_i\right) \; 0\right] \qquad T = \begin{bmatrix} 2 - \delta & \delta - 1 \\ 1 & 0 \end{bmatrix}$$

$$\alpha_t = [\pi_t^P \; \pi_{t-1}^P]' \qquad\qquad R = \begin{bmatrix} \delta \\ 0 \end{bmatrix}$$

$$A = [\varphi_1 \; \varphi_2 \; \varphi_3 \; \varphi_4] \qquad \eta_t = [\eta_{2t}]$$

$$d_t = [\pi_{t-1} \; \pi_{t-2} \; \pi_{t-3} \; \pi_{t-4}]' \qquad Q = [\sigma_{\eta_2}^2]$$

$$\varepsilon_t = [v_{1t}]$$

Analogously, the equations of the multivariate model can be represented in state space form, as follows:

$$y_t = [\pi_t \; i_t \; y_t^T]'$$

$$Z = \begin{bmatrix} 0 & (1-\sum\varphi_i) & 0 & 0 & -\phi_1 & 0 & 0 & 0 & 0 & 0 \\ \rho_1 & (1-\rho_2) & 0 & 0 & 0 & 0 & (1-\rho_2)\gamma & 0 & (1-\rho_2) & 0 \\ 0 & 0 & \phi_4 & 1 & -\phi_2 & -\phi_3 & 0 & \phi_4\gamma & 0 & \phi_4 \end{bmatrix}$$

$$\alpha_t = [\pi_t^T \; \pi_t^P \; \pi_{t-1}^P \; y_t^P \; y_{t-1}^P \; y_{t-2}^P \; \lambda_t \; \lambda_{t-1} \; \tau_t \; \tau_{t-1}]'$$

$$A = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 & \phi_1 & 0 & 0 \\ \rho_1 & 0 & 0 & 0 & 0 & 0 & \rho_2 \\ 0 & 0 & 0 & 0 & \phi_2 & \phi_3 & -\phi_4 \end{bmatrix}$$

$$d_t = [\pi_{t-1} \; \pi_{t-2} \; \pi_{t-3} \; \pi_{t-4} \; y_{t-1} \; y_{t-2} \; i_{t-1}]'$$

$$\varepsilon_t = [v_{1t} \; v_{2t} \; v_{3t}]'$$

$$H = \begin{bmatrix} \sigma_{v_1}^2 & 0 & 0 \\ 0 & \sigma_{v_2}^2 & 0 \\ 0 & 0 & \sigma_{v_3}^2 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta & (1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & \delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\eta_t = [\eta_{1t} \ \ \eta_{2t} \ \ \eta_{3t} \ \ \eta_{4t} \ \ \eta_{5t}]'$$

$$Q = \begin{bmatrix} \eta_{1t} & 0 & 0 & 0 & 0 \\ 0 & \eta_{2t} & 0 & 0 & 0 \\ 0 & 0 & \eta_{3t} & 0 & 0 \\ 0 & 0 & 0 & \eta_{4t} & 0 \\ 0 & 0 & 0 & 0 & \eta_{5t} \end{bmatrix}$$

Operationalization of the usual algorithm of the Kalman filter requires that matrices Z, A, H, T, R and Q be known. Since they depend on a vector of unknown parameters ψ, it is necessary to additionally make an inference about this vector. Including data obtained from other studies, it is possible to treat ψ as a vector of random parameters with prior density $p(\psi)$ and to estimate the posterior density $p(\psi|y)$ using Bayesian inference[9]. Alternatively, the goal is to find the posterior mean $\bar{g}$ given by:

$$\bar{g} = E[g(\psi|y)] = \int g(\psi)p(\psi|y)d\psi. \tag{22}$$

Following Dossche and Everaert (2005) and using Bayes' theorem, this turns into:

$$\bar{g} = \frac{\int g(\psi)z^g(\psi,y)g(\psi|y)d\psi}{\int z^g(\psi,y)g(\psi|y)d\psi} \tag{23}$$

where

$$z^g(\psi,y) = \frac{p(\psi)p(y|\psi)}{g(\psi|y)}. \tag{24}$$

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[9] An alternative approach is the optimization of the diffuse likelihood function from the exact Kalman filter. However, in a framework containing so many unobserved components, this turns out to be unfeasible, as Dossche and Everaert (2005) pointed out.

Departing from a sample of n independent choices of ψ, called $\psi^{(i)}$, we obtain a $\bar{g}_n$ estimator of $\bar{g}$:

$$\bar{g}_n = \frac{\sum_{i=1}^n g(\psi^{(i)}) z^g(\psi^{(i)}, y)}{\sum_{i=1}^n z^g(\psi^{(i)}, y)}. \tag{25}$$

In practice, we introduce estimators of the posterior mean of $\bar{\psi} = E(\psi|y)$. This is possible by admitting $g(\psi^{(i)}) = \psi^{(i)}$ and $\tilde{\psi} = \bar{g}_n$ in the previous equation, where $\tilde{\psi}$ is the estimator of $\bar{\psi}$. The 5th and 95th percentiles are also computed. Briefly, an estimate of the 5th percentile, $\tilde{\psi}^{5\%}$, derives from $F(\tilde{\psi}^{5\%}|y) = 0.05$, where $F(\tilde{\psi}|y) = Prob(\psi_j^{(i)} \leq \psi_j)$ denotes the jth element in ψ. The same applies to the 95th percentile.

3.3 Data and prior information

In this study, we used observed Brazilian data on inflation, output and interest rate. For inflation, a quarterly series of seasonally adjusted consumer price index (IPCA) was utilized as reference for the targeting regime. For the output, we employed seasonally adjusted quarterly GDP, in constant prices, obtained from the Brazilian Institute of Geography and Statistics (IBGE). Finally, for the interest rate, we used for each quarter the average of daily frequencies of the SELIC rate in the corresponding quarter. The sampling period for the univariate and multivariate cases stretches from the first quarter of 1995 to the first quarter of 2011, totalling 65 observations. Additionally, we estimate two models that are identical with the multivariate one with respect to the equations, but include 49 observations, since they consider the alternative periods 1995-2007 and 1999-2011. In such way, we are able to assess the dynamic behaviour of persistence measures. On the final graphs, all values are annualized for the sake of illustration.

In order to simplify the Bayesian procedure in the presence of several unobserved series, we decided to use the values of some studies carried out for the Brazilian economy and those of some international studies, as prior information. Tables 1 and 2 summarize the values of the distributions used respectively for the univariate and multivariate cases, along with the estimated posteriors.

In the case of the univariate estimation, more specifically with regard to intrinsic persistence, the major reference was Petrassi and Oliveira (2010), as previously commented. In their study, the mean of ρ values for the different models was $\rho = 0.46$, which, in our

case,[10] corresponds to $\sum_{i=1}^{4} \varphi_i$, as in equation 3. This value was distributed among φ_i, so as to provide more recent lags with a heavier weight. As prior information for the parameter that measures expectations-based persistence, δ, we used the figures proposed by Guillen (2008) and Caetano and Moura (2009), which quantify the presence of sticky information in Brazil, following Mankiw and Reis (2002). The value of $\delta = 0.16$ means that, on average, 84% of the inflation target perceived by agents, results from the target perceived in the previous quarter and only 16% originates from the actual target used by the Central Bank. This is easily seen from equation (4). The remaining coefficients required for the univariate estimation were obtained from foreign studies. The variances of errors v_1 and η_2 derive respectively from Dossche and Everaert (2005) and from a combination of Kozicki and Tinsley (2005) and Smets and Wouters (2005). The variances for the construction of prior and posterior distributions of the parameters followed Dossche and Everaert (2005).

To initialize the multivariate estimation, we first used some posterior means obtained for the variables estimated in the univariate model $(\varphi_i, \delta, \sigma_{v_1}^2, \sigma_{\eta_2}^2)$. Thus, the univariate estimation was also useful for adjusting and refining the values to be used in the main model. For the coefficient of the output gap in the Phillips curve (equation 7), we used the value of the linear curve introduced by Muinhos (2004), $\phi_1 = 0.28$. With respect to the IS curve parameters, the results obtained by Aragón and Portugal (2009), which take into account changes in Central Bank's policy preferences, were used as reference. The relevant values for equation (9) are: $\phi_2 = 0.5$, $\phi_3 = -0.18$ and $\phi_4 = 0.008$. For the interest rule parameters (equation 8), our reference was Sin and Gaglianone (2006), who estimated a DSGE model for Brazil, based on Smets and Wouters (2005). The prior means $\rho_1 = 0.3$ and $\rho_2 = 0.7$ originate from their work. From Dossche and Everaert (2005) we obtain $\sigma_{v_2}^2 = 0.3$. Finally, Laubach and Williams (2003) was our source of information for the coefficients related to equations (10) through (13). Their figures for parameters γ, θ, $\sigma_{v_3}^2$, $\sigma_{\eta_3}^2$, $\sigma_{\eta_4}^2$ and $\sigma_{\eta_5}^2$, as well as the other parameters of the multivariate estimation are shown in Table 2.

For the estimation of alternative periods (1995/1-2007/1 and 1999/1-2011/1), the prior distributions were exactly the same as those of the multivariate estimations, in order not to interfere with the coefficient results.

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[10] As shown by Levin and Piger (2004), one can compute intrinsic persistence either in terms of the sum of the AR coefficients or by the largest AR root, since both measures are equivalent. Therefore, our prior mean for $\sum_{i=1}^{k} \varphi_i$ is considered to equal an average ρ from Petrassi and Oliveira (2010).

4 Results

For both the univariate and multivariate estimations, we show two sets of results. First, in Tables 1 and 2, the prior means and distributions of the unobserved coefficients and then their respective posteriors are shown. Second, in Figures 1 and 2 we illustrate key state variables relating to the estimated and perceived targets in comparison to the Brazilian inflation rate.

As previously highlighted, the univariate case was first estimated, with the decomposition of inflation persistence into intrinsic persistence, derived from the sum of AR coefficients, and into expectations-based persistence, represented by 1-δ.

The posteriors obtained for the unobserved components were relatively similar to the values from Brazilian studies used as priors. However, the variances $\sigma^2_{v_1}, \sigma^2_{\eta_2}$ proved to be quite different from Dossche and Everaert (2005) figures. This is probably due to the different nature of shocks hitting inflation in Brazil and in the euro area.

Table 1
Prior and posterior distributions - Univariate model

Variable	Prior Mean	5%-95%	Posterior Mean	5%-95%
φ_1	0.23	[0.15, 0.31]	0.262	[0.18, 0.34]
φ_2	0.10	[0.02, 0.18]	0.079	[0.00, 0.16]
φ_3	0.08	[0.00, 0.16]	0.082	[0.01, 0.16]
φ_4	0.05	[-0.03, 0.13]	0.051	[-0.02, 0.13]
δ	0.16	[0.08, 0.24]	0.169	[0.09, 0.25]
$\sigma^2_{v_1}$	1.3	[0.36, 2.73]	0.859	[0.64, 1.17]
$\sigma^2_{\eta_2}$	0.12	[0.03, 0.25]	0.053	[0.01, 0.17]

Figure 1 shows the behaviour of state variables in the univariate case compared to IPCA observed inflation. In the first observations, the values are erratic, as a consequence of the filter algorithm. After that, the estimated inflation target had its highest rates around 2001, reaching a peak of yearly 8.25% in the third quarter of 2001, and then dropped vigorously until the 1st quarter of 2006 to 5.15%. After that it has rather been slowly increasing again. As for the target perceived by agents, there was a mean-reverting behaviour and, in more recent years, a greater stability and adherence to the target we estimated to be the actual target followed by the Central Bank.
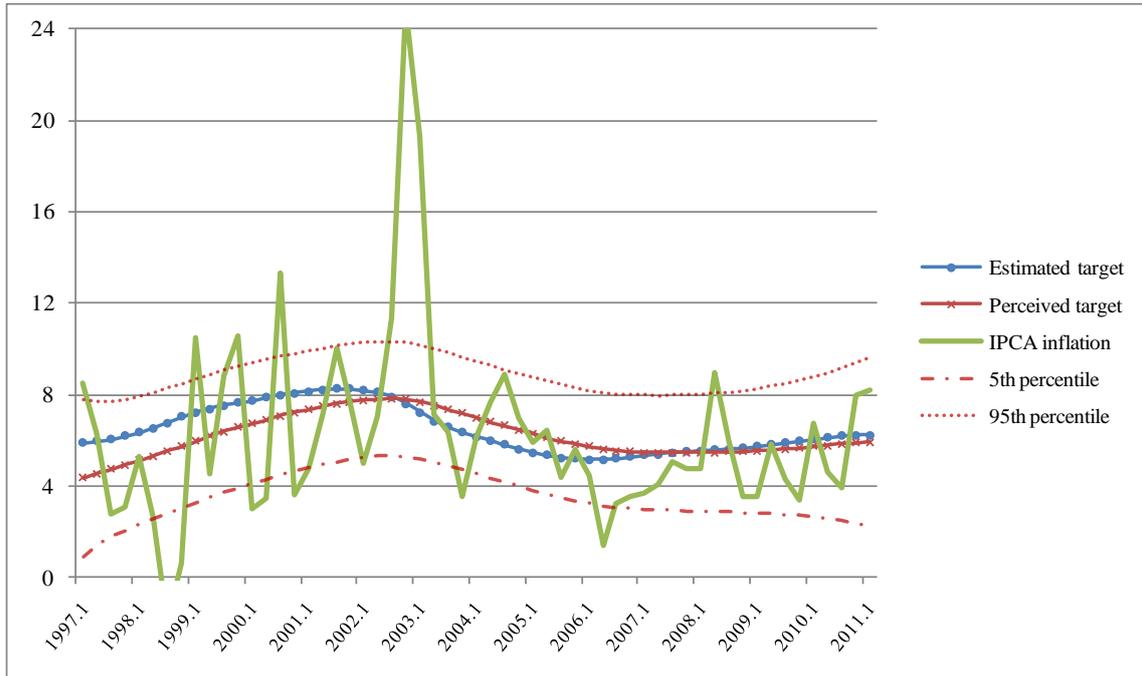
Figure 1: Unobserved targets and inflation (yearly, %) - Univariate model

In regard to the results of the multivariate estimation (see Table 2), we have a broader scenario that now includes extrinsic persistence, as mentioned before. The priors generally proved to result in good approximations for most coefficients. The initial profile of the autoregressive coefficients slightly changed, since the 2nd inflation lag (φ_2) is oddly lower than the 3rd and 4th lags (φ_3, φ_4). Dossche and Everaert (2005) found a lower 3rd lag among the results of the euro area. Both their result and ours reflect the importance of the choice of several lags as a measure of intrinsic persistence, as opposed to simply choosing the first lag. More importantly, the sum of autoregressive components resulted in greater intrinsic persistence (0.62 instead of 0.47). On the other hand, lower expectations-based persistence was found (0.77 instead of 0.83). Extrinsic persistence, which is specific to the multivariate setting, summed 0.44.

Together these figures suggest that, on the one hand, extrinsic persistence matters, since its introduction impacts the results of other sources of persistence, but on the other hand, it has a lower share on aggregate persistence than the other two drivers.

Table 2

Prior and posterior distributions - Multivariate model

| Variable	Prior Mean	5%-95%	Posterior Mean	5%-95%
$* \varphi_1$	0.26	[0.10, 0.43]	0.369	[0.30, 0.46]
$* \varphi_2$	0.08	[-0.09, 0.24]	0.030	[-0.01, 0.07]
$* \varphi_3$	0.08	[-0.08, 0.25]	0.108	[0.06, 0.16]
$* \varphi_4$	0.05	[-0.11, 0.22]	0.110	[0.10, 0.12]
$** \phi_1$	0.28	[-0.12, 0.44]	0.264	[0.20, 0.33]
$** \phi_2$	0.5	[0.34, 0.66]	0.461	[0.43, 0.50]
$** \phi_3$	-0.18	[-0.34, -0.02]	-0.018	[-0.03, -0.01]
$** \phi_4$	0.008	[-0.40, 0.42]	0.0004	[0, 0.001]
$*** \gamma$	1	[0.92, 1.08]	0.994	[0.93, 1.06]
$** \rho_1$	0.3	[0.26, 0.34]	0.26	[0.26, 0.26]
$** \rho_2$	0.7	[0.66, 0.74]	0.685	[0.68, 0.69]
$* \delta$	0.17	[0.09, 0.25]	0.227	[0.21, 0.24]
$*** \theta$	0.97	[0.95, 0.99]	0.970	[0.97, 0.97]
$* \sigma^2_{v_1}$	0.86	[0.23, 1.78]	0.827	[0.78, 0.86]
$*** \sigma^2_{v_2}$	0.3	[0.21, 0.41]	0.377	[0.34, 0.41]
$*** \sigma^2_{v_3}$	0.16	[0.11, 0.22]	0.168	[0.16, 0.18]
$*** \sigma^2_{\eta_1}$	0.0001	[0.00, 0.0001]	0.0001	[0.00, 0.0001]
$* \sigma^2_{\eta_2}$	0.05	[0.04, 0.07]	0.048	[0.04, 0.05]
$*** \sigma^2_{\eta_3}$	5.86	[4.10, 7.91]	7.058	[5.81, 8.39]
$*** \sigma^2_{\eta_5}$	0.10	[0.07, 0.14]	0.111	[0.10, 0.12]
$*** \boldsymbol{\sigma^2_{\eta_4}}$	0.01	[0.00, 0.01]	0.006	[0.00, 0.01]

Notes:

*Denotes variables, for which univariate posterior means were used as prior means.

**Variables, for which figures from other Brazilian studies were used.

***Variables, for which figures from foreign studies were used.

Likewise, the behaviour of the main state variables (Figure 2) confirms that inflation persistence is partially explained by the difference between the target perceived by agents and the actual inflation target. Despite the similar mean-reverting tendency of the target perceived by agents, expectations distortions decreased in the multivariate case, i.e., the expected inflation target followed the estimated target more closely. This suggests that expectations-based persistence is less present than in the univariate case, but this is probably because in the multivariate model we can disentangle extrinsic persistence from the other two sources. Therefore, this result actually means that persistence of expectations plays a major role on aggregate persistence.
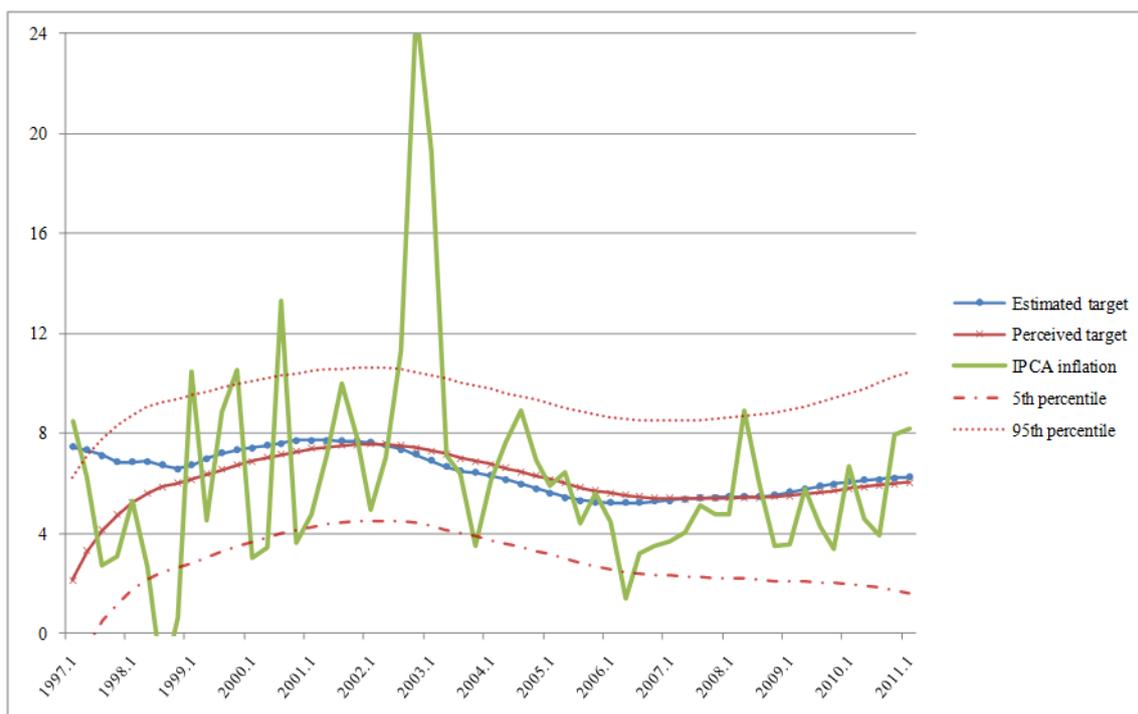
Figure 2: Unobserved targets and inflation (yearly %) - Multivariate model

It is interesting to compare the estimated central bank targets with actual policy followed by the Central Bank of Brazil. Figure 3 shows the estimated target π_t^T and the lower and upper tolerance bounds defined by the Brazilian National Monetary Council (CMN) during the inflation targeting regime. It should be underscored that the years in which estimated targets were above the tolerance region were also those years in which headline inflation did not manage the predicted target. From the second half of 2000 until the beginning of 2002, for example, although the preset tolerance bound showed a falling trend, our estimated targets suggest lenient policy towards price stabilization has been carried out. After that, following the election of President Lula da Silva, the Central Bank needed to tighten the monetary policy to curb inflation and its inertia or persistence as well.[11] Not until 2004 could the estimated targets return to the tolerance region, where it remained comfortably until the beginning of 2009. Clearly, after the improvement of the inflation targeting regime, with credibility gains and anchoring of expectations, the Central Bank could put into practice an inflation target that is more centred inside the tolerance region. However, the recent financial crisis pushed up estimated targets, and our result for the beginning of 2011 (6.24%)

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[11] In BCB (2003) the Central Bank of Brazil announced adjusted targets for 2003 and 2004 headline inflation. The respective revised target limits can be seen in Figure 3. Moreover, in BCB (2004), persistence concerns are evident, being based on the finding that the inertial component had contributed 63.7% to inflation in 2003.

is close to the upper announced bound (6.5%). Such recent evolution has prompted the Central Bank to repeatedly raise interest rates during the first half of 2011.
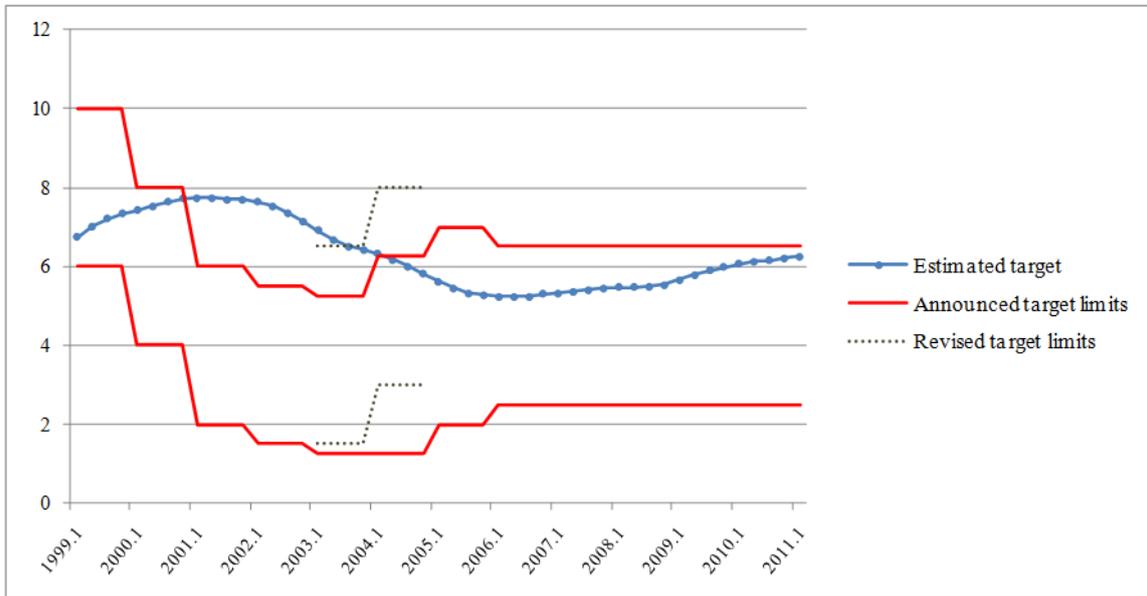
Figure 3: Estimated Central Bank targets, announced tolerance limits (yearly, %)

Consolidated estimates of the relevant parameters for the sources of persistence are shown in the first columns of Table 3. Departing from the univariate to the multivariate approach, there is a decrease in expectations-based persistence and an increase in intrinsic persistence. The reduction in expectations-based persistence is probably an effect of the introduction of extrinsic persistence, as already mentioned. The result $\delta = 0.23$ demonstrates that the speed of expectations updating is higher than what would be expected from "sticky information" estimates produced by Guillén (2008) for Brazil.

We also estimate the same model, considering two alternative periods. The periods were formed removing 16 quarters from each extremity. Thus, the first period starts in 1995 and ends in 2007, while the second term consists of 1999-2011. In the choice of the number of observations we had in mind the accuracy of estimations; obviously the division into two separate periods would generate inaccurate results, due to the number of unobserved variables. Moreover, the second period coincides with the inflation-targeting era in Brazil. It should be noted that both intrinsic and expectations-based persistence decreased over time, as expected. With the adoption of the inflation targeting regime in Brazil in the second period, the Central Bank could anchor inflation expectations more strongly to its actual targets, as a result of credibility improvements. Nevertheless, as already pointed out, expectation

distortions have clearly been a key source of persistence over all periods, as opposed to the traditional view of intrinsic persistence. This finding stays in line with the Euro area and US results from Dossche and Everaert (2005).

Table 3

Summarized measures of persistence

| Variable	Univariate (1995-2011)	Multivariate (1995-2011)	Multivariate 1st period (1995-2007)	Multivariate 2nd period (1999-2011)
$\sum \varphi_i$	0.47	0.62	0.60	0.38
$(1 - \delta)$	0.83	0.77	0.81	0.78
$\phi_2 + \phi_3$	-	0.44	0.33	0.35

Extrinsic persistence, on the other hand, has not quite changed over the periods considered and was considerably lower than the one found in the Euro area and in the US. Therefore, persistent deviations of output from its natural level may be more present in these developed economies than in Brazil.

Most of the Brazilian literature on persistence measures (for example, Campelo and Cribari-Neto (2003) and Durevall (1999)) focuses on periods of higher inflation levels, that is, before 1994, so that a direct comparison to our results is unfruitful. Our univariate intrinsic persistence measures match closely Petrassi and Oliveira (2010) in a similar period.[12] Dossche and Everaert (2005) also reported similar findings when comparing their intrinsic persistence results with the ones from univariate frameworks in the USA and in the euro zone. However they do not consider alternative periods.

Finally, the natural rate of interest is another estimation result with an important economic value. It informs the monetary authority about the natural stance of the interest rate over time. In our model, in particular, its introduction prevented shifts in the actual inflation target from being affected by changes in the benchmark that represents the natural rate of interest. Equations (11) through (13) determine the evolution of the equilibrium interest rate over time, together with the trend growth rate and the parameter that measures time preferences. The average natural rate of interest for the period between the third quarters of 1999 and 2005 was 10.5%, slightly higher than figures found by Portugal and Neto (2009) for the same period, 9.6%. The extreme values found in the estimations are 19.8% for the 4th

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[12] Their intrinsic persistence estimates for the 1995 to 2009 period range from 0.416 to 0.509, whereas our main result for the univariate model was 0.47.

quarter of 1997 and 5.9% for 2009/3. The declining trend is in line with recent estimations from the Central Bank of Brazil (BCB, 2010).[13] As for its determinants, shifts in the trend growth rate did not seem to significantly affect the recent behaviour of the natural rate of interest, as did shifts in time preferences. The same pattern was observed in the data obtained by Dossche and Everaert (2005) for the Euro zone and the USA.
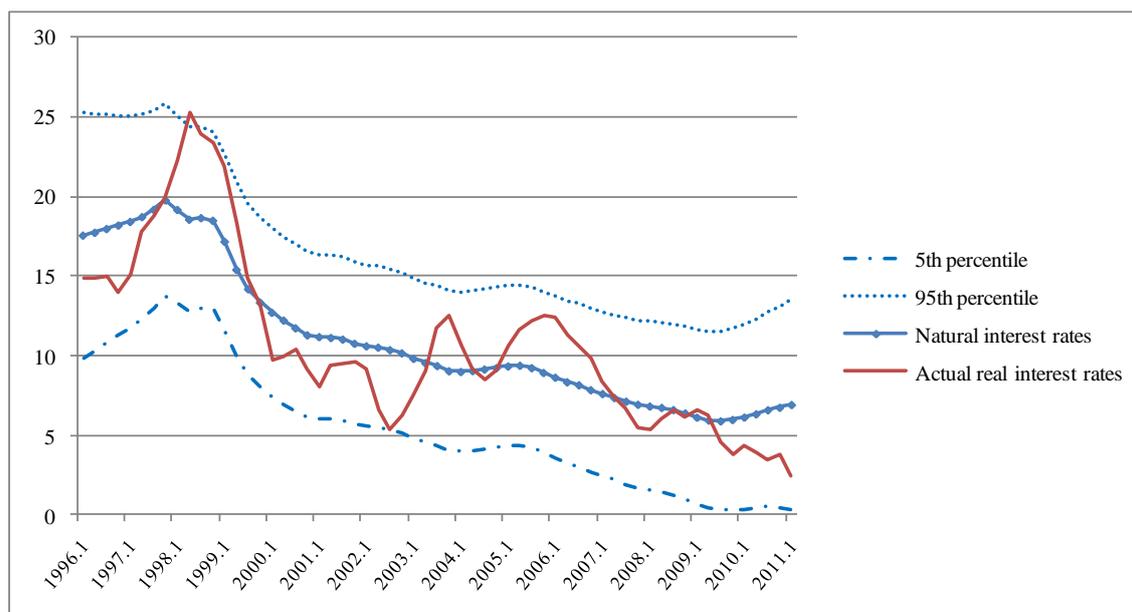
Figure 4: Natural vs. actual real interest rates (yearly, %)

We also compared the estimated natural rates of interest with actual real interest rates, computed by simply subtracting the observed IPCA inflation from nominal interest rates for each quarter, that is, $r_t = \pi_t - i_t$. If the monetary authority follows a neutral stance regarding policy rate adjustment, both rates should ideally evolve similarly. Considering the whole period, this is certainly not the case, but if we take into account only the period under the inflation targeting regime, interest rate gaps are relatively smaller considering historical interest rate figures.

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[13] The Central Bank of Brazil reports the recent reduction in sovereign debt risk, as a consequence of positive developments in national accounts, as the main driver for this decline in the natural rate of interest. For a review on alternative approaches to the measurement of natural rates of interest, see Giammarioli and Valla (2004).

5 Conclusions

In spite of the fundamental importance of measuring inflation persistence and including it in general equilibrium models for the formulation of monetary policy and of empirical regularities, there is no consensus agreement on how to do that. The aim of this chapter was to measure inflation persistence for a recent time period in Brazil, and also to identify the types of inflation persistence using three major sources: intrinsic, extrinsic, and expectations-based.

Aside from the usual intrinsic persistence, the impact of macroeconomic shocks, such as output deviations, on persistence, which lead to extrinsic persistence, was also determined. It was also assumed that shifts in the inflation target (which are not always known and perceived by all agents) may result in permanent changes in mean inflation. Private agents' perception of the policy target sometimes differs considerably from the actual target followed by the central bank, which also induces inflation persistence. These facts are often not considered by conventional empirical models when measuring persistence.

Following Dossche and Everaert (2005), the inflation generating process was decomposed[14] into unobserved variables using the Kalman filter. The main contributions were the following: first, this type of estimation is novel for Brazil, because the dynamics of inflation and of its determinants are jointly assessed in a model that contemplates not only the sticky price behaviour, but also the idea of sticky information. Second, by clarifying inflation persistence in Brazil and simultaneously adding more information on the natural rate of interest, this study provides important data that may enrich the framework of monetary policy in the country.

The following major results were obtained. First, the role of inflation expectations, of shifts in inflation targets, and of significant deviations of output and of the natural rate of interests in inflation persistence should not be neglected in any representation of inflation dynamics. Not surprisingly, the impact of accounting for such factors in measures of inflation persistence is that the usual intrinsic persistence is quite different from traditional figures from univariate approaches. These factors are especially important for Brazil, which has gone through the implementation of an inflation targeting regime and episodes of inflation during foreign crises and the domestic crisis in 2002/2003. Second, although intrinsic inflation persistence has significantly decreased in the past few years, the other two sources did not

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[14] In the present study, we only consider the IPCA (broad consumer price index). However, a possible extension could deal with administered prices separately in order to quantify persistence in these prices and its importance to monetary policy.

experience such a trend. Hence, one should cautiously interpret literature findings that point to an undisputable decrease in inflation persistence, especially if they are based on intrinsic measures of persistence. As we particularly showed, expectations-based persistence proved to be both high and almost unchanged over recent years. Also, according to Portugal and Neto (2009), estimates suggest that interest rates have fluctuated around the estimated natural stance, contradicting arguments that the Central Bank of Brazil has been too conservative in its interest rate policy. Of course one should be cautious since we deal with a restricted period and the natural rate of interest entails a long-run definition. Finally, this study indicates that persistence is less distressing for stable inflationary settings (such as the period between 2006-2009 in Brazil), but that in unstable ones, the cost of disinflation in terms of output loss, or the sacrifice ratio, tends to be even higher. This is due to agents' perceptions not being easily anchored, when the central bank inflation targets change more often.

References

Angeloni, I., Aucremanne, L., Ehrmann, M., Gali, J., Levin, A., Smets, F., 2004. Inflation Persistence in the Euro Area: Preliminary Summary of Findings. Paper presented at the ECB conference on inflation persistence in the euro area.

Aragón, E., Portugal, M., 2009. Central Bank preferences and monetary rules under the inflation targeting regime in Brazil. Brazilian Review of Econometrics 29 (1), 79-109.

BCB, 2003. Open Letter to the Finance Minister. Banco Central do Brasil.

_____, 2004. Open Letter to the Finance Minister. Banco Central do Brasil.

_____, 2008. Inflation Report, Banco Central do Brasil, 151-155.

_____, 2010. Inflation Report, Banco Central do Brasil, 94-103.

Bogdanski, J., Tombini, A. , Werlang, S. Implementing inflation targeting in Brazil. BCB Working Paper No. 1, Banco Central do Brasil.

Caetano, S., Moura, G., 2009. Reajuste informacional no Brasil: Uma aplicação da Curva de Phillips sob rigidez de informação. Anais do XXXVII Encontro Nacional de Economia.

Campelo, A. K., Cribari-Neto, F., 2003. Inflation inertia and inliers: The case of Brazil. Revista Brasileira de Economia 57 (4), 713-739.

Cecchetti, S. G., Debelle, G., 2006. Has the inflation process changed? Economic Policy 21 (46), 311-352.

Cogley, T., Sargent, T. J., 2005. Drifts and volatilities: Monetary policies and outcomes in the post WWII US. Review of Economic Dynamics 8 (2), 262-302.

Cukierman, A., Meltzer, A. H., 1986. A theory of ambiguity, credibility, and inflation under discretion and asymmetric information. Econometrica 54 (5), 1099-1128.

Dossche, M., Everaert, G., 2005. Measuring inflation persistence: A structural time series approach. ECB Working Paper No. 495.

Durevall, D., 1999. Inertial inflation, indexation and price stickiness: evidence from Brazil. Journal of Development Economics 60 (2), 407-421.

Figueiredo, E. A., Marques, A. M., 2011. Inflação inercial sob mudanças de regime: Análise a partir de um modelo MS-ARFIMA. Economia Aplicada 15 (3), 443-457.

Giammarioli, T., Valla, N., 2004. The natural real interest rate and monetary policy: A review. Journal of Policy Modeling 26, 641-660.

Guillen, D., 2008. Ensaios sobre a formação de expectativas de inflação. Master's Dissertation, Faculdade de Economia, PUC-RJ, Brazil.

Kozicki, S., Tinsley, P. A., 2005. Permanent and transitory policy shocks in an empirical macro model with asymmetric information, Journal of Economic Dynamics and Control 29 (11), 1985-2015.

Laubach, T., Williams, J. C., 2003. Measuring the natural rate of interest. The Review of Economics and Statistics 85 (4), 218-231.

Levin, A. T., Piger, J. M., 2004. Is inflation persistence intrinsic in industrial economies? ECB Working Paper No. 334.

Mankiw, N. G., Reis, R., 2002. Sticky information versus sticky prices: A proposal to replace the new Keynesian Phillips curve. The Quarterly Journal of Economics 117 (4), 1295-1328.

Mankiw, N. G., Reis, R., Wolfers, J., 2003. Disagreement about inflation expectations. NBER Macroeconomics Annual Vol. 18, 209-270.

Marques, C. R., 2004. Inflation persistence: Facts or artefacts? ECB Working Paper No. 371.

Milani, F., 2007. Expectations, learning and macroeconomic persistence. Journal of Monetary Economics 54 (7), 2065-2082.

Petrassi, M., Oliveira, F. N., 2010. Is inflation persistence over? BCB Working Paper No. 230. Banco Central do Brasil.

Pincheira, P., 2009. La dinámica de la persistencia inflacionaria en Chile. Notas de Investigación de Banco Central de Chile 12 (1).

Pivetta, F., Reis, R., 2007. The persistence of inflation in the United States. Journal of Economic Dynamics and Control 31 (4), 1326-1358.

Portugal, M. S., Neto, P.C., 2009. The natural rate of interest in Brazil between 1999 and 2005. Revista Brasileira de Economia 63 (2), 103-118.

Rebelo, A., Silva, C., Lopes, D., 2009. Persistência inflacionária: Comparações entre o caso brasileiro e outros países emergentes. Anais do XXXVII Encontro Nacional de Economia.

Silva, C. G., Leme, M. C., 2011. An analysis of the degree of persistence of inflation, inflation expectations and real interest rate in Brazil. Revista Brasileira de Economia 65 (3), 289-302.

Sims, C. A., 2003. Implications of rational inattention. Journal of Monetary Economics 50 (3), 665-690.

Sin, H. L., Gaglianone, W. P., 2006. Stochastic simulation of a DSGE model for Brazil. Unpublished.