STRUCTURAL ESTIMATION OF OUTPUT GAP: THE CASE OF BRAZIL

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June 16, 2013

Abstract

The present paper estimates the output gap for Brazil by applying the major methodologies referenced in the literature and by introducing an alternative method that has not yet been tested for the Brazilian case. This approach estimates the output gap using a dynamic stochastic general equilibrium (DSGE) model. The output gap estimation by the DSGE model managed to identify the recession periods dated by FGV. However, in addition to the episodes reported by FGV, another two critical periods were identified. This type of approach has some advantages such as the possibility to decompose the estimated output gap into the shocks observed in the model. From the decomposition of shocks, it was observed that demand shocks were the ones that most contributed to output gap fluctuations. Still with regard to the decomposition of shocks, it was possible to identify that commodity price shocks could be understood as demand shocks instead of supply shocks. The output gap estimated by a DSGE model is better at forecasting long-term free-price inflation compared to other methods. Thus, output gap estimation by the DSGE model could be an additional tool for monetary policy conduct.

Keywords Output Gap, Potential output, DSGE, Bayesian Estimation.

1 INTRODUCTION

The inflation targeting regime was implemented in Brazil in June 1999, demanding responsibility and operating independence from the Central Bank so that the monetary policy could meet the inflation target.

According to Bogdanski et al. (2000), when a country is under the inflation targeting regime, it is necessary that the monetary authority take a proactive attitude and predict inflation movements given the difference between the actions taken and their effect on economic activity and, therefore, it is paramount that the inflation rate be forecasted. Instead of reacting to current facts, monetary policymakers ought to make decisions based on future inflation forecasts, on interest rate possibilities and on the best estimate of the current economic status. There is then the need to develop models that allow policymakers to make accurate judgments.

The output gap is one among several variables that allow for a better investigation of possible inflationary pressures. The fact that it is not an observable variable led many researchers to try a better way to estimate this variable.

The production function is widely used to estimate the output gap. This method began to be looked into in the 1980s, including several improvements over time. In this method, the productive structure of the economy is assumed to be represented by Cobb-Douglas production technology with its traditional properties: each input has decreasing marginal returns, the function has constant returns to scale and unitary elasticity of substitution, and productivity is Hicks-neutral.

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Also, there are the unobserved components models, which have been widely used to estimate potential output and output gap: univariate approaches such as the one described by Harvey e Jaeger (1993) where output is decomposed into trend and cycle, as well as the Hodrick e Prescott (1997), whose interpretation is also model-based. One of the first examples of a multivariate unobserved components model is provided by Clark (1989), who estimated a bivariate model for the US using real GDP and unemployment based on Okun's law.

Apel e Jansson (1999) obtained estimates of the natural rate of unemployment (NAIRU) and of potential output for the UK, US and Canada, based on an unobserved components model using GDP, inflation, and unemployment. Scott (2003) estimated the output gap for New Zealand using a trivariate system that included unemployment, GDP and installed capacity utilization (ICU). Rünstler e Europeo (2002) work focused on univariate and multivariate unobserved components models for the euro area and provided real-time assessment of the reliability of output gap estimates and of their performance in order to forecast future inflation. Among the multivariate models, the author implements the production function approach based on the capital-output ratio, total factor productivity and installed capacity utilization. Other multivariate approaches are based on extensions of the Hodrick-Prescott (HP) filter. Laxton et al. (1992) extended the penalized least squares model, criterion on which the HP filter is based, so as to add important macroeconomic relationships that are output gap expressions, such as the Phillips curve and Okun's law. In the same vein, Areosa (2004) work estimated the output gap for Brazil by combining a production function with the HP filter, using the Phillips curve as a restriction on the model.

A more recent approach to the estimation of potential output is based on new Keynesian DSGE models. This approach relies on three basic elements. Firstly, it is based on improvements in optimal monetary policy theory, which underscores the role of a consistent model for potential output measures and related output gaps so that appropriate monetary policy decisions are made and also as a source of inflationary pressures. Secondly, it is hinged upon improvements in the estimation of DSGE models, which allow for an internally consistent quantitative interpretation and a complete interpretation of the dynamics of macroeconomic variables (especially of inflation, output, and potential output). Thirdly, DSGE models allow for the use of more traditional concepts of potential output (not only the concepts of consistent models) to better plan monetary policy decisions in an internally consistent model.
These DSGE models are based on the original structure of real business cycle models, which are quantitative macroeconomic models that indicate the optimizing behavior of agents conditional on a set of constraints in an economy that is under permanent equilibrium. The origin of this type of model dates back to the seminal work of Kydland e Prescott (1982). In their work, the authors introduce a small but coherent dynamic economic model built upon optimizing agents, rational expectations and market equilibrium. The current generation of NK DSGE models is based, to a great extent, on a theoretical summary that provides the Keynesian and real business cycle theories with a distinct and complementary role in the analysis of business cycle fluctuations. Some factors in the real business cycle theory (with flexible prices) help explain the behavior of potential output over time. On the other hand, factors emphasized in the Keynesian theory are related to the period needed for the adjustment of nominal wages and prices that lead to transient deviations from the observed output compared to potential output. Both theories belong with the general equilibrium model framework. Unlike the more traditional approaches, where the Keynesian theory explains the short-run dynamics of the economy whereas the classic theory explains it in the long run, the new Keynesian approach stresses that wages, prices and potential output may change in the short run and that this should be taken into account in business cycle analysis. Moreover, in the new Keynesian approach, fluctuations in economic activity are not necessarily desirable and monetary policy is not irrelevant for stabilization (maintaining the observed output close to that of the potential output). Given the distortions associated with the delay in adjusting wages and prices, the consequences of real disturbances might be inefficient and its level of inefficiency might be attenuated in response to the monetary policy.
Back to the discussion on the definition of potential output, we find two different ways to deal with it in the DSGE model literature. Woodford (2003) argues that in the case of DSGE models with physical capital, potential output is the flexible-price equilibrium output, which depends not only on current and future exogenous shocks, but also on the current capital stock of a sticky-price economy. So, this notion of potential output revolves around past monetary policy decisions.

Alternatively, in the work of Neiss e Nelson (2003), the authors define potential output as the equilibrium output whose prices are flexible not only in the present and in the future, but also in the past. This definition differs from the previous one with respect to capital stock since, in this definition, the existing capital stock does not matter; what does matter is the capital stock that there would exist if prices were always flexible.
According to Woodford (2003), its definition is more associated with the determination of equilibrium in a sticky-price economy, as current capital stock (with price rigidity) and its effects on productive capacity are relevant in defining potential output and, therefore, in the conduct of an optimal monetary policy.
The empirical literature on DSGE models for potential output estimation is scarce and the conclusions are still preliminary, due to the fact that this literature is relatively new. Furthermore, some pending questions exist on the robustness of the estimations of flexible-price potential output insofar as the frameworks of alternative models, shock identification systems and data revisions are concerned.
A large strand of the literature compares historical estimates of potential output and their respective output gaps using DSGE models with more traditional measures. There exists some evidence that potential output estimates by DSGE models bear some resemblance to conventional potential output measures. For instance, in the work by Justiniano e Primiceri (2008), the authors estimated an NK DSGE model using data for the US and found that the resulting potential output was similar to that obtained with the HP filter or estimated by the Congressional Budget Office (CBO). However, other studies report that there might be significant differences between the potential output estimated with DSGE models and those obtained from more traditional estimation methods. For example, using the DSGE model for Sweden, Adolfson et al. (2011) obtained a flexible-price potential output that differed from that obtained with the HP filter. The authors reported that, in some periods, the output gap with flexible prices might have a different sign from the output gap estimated with the HP filter.
Another work is that by Hirose e Naganuma (2007), which uses a DSGE model that resembles that of Edge, Kiley e Laforte (2007) for the US, in the present paper, and as also observed by Justiniano e Primiceri (2008), the output gap was close to the estimation obtained with the HP filter.
Along the same line of Hirose e Naganuma (2007), this paper uses a DSGE model and Bayesian estimation techniques to determine the output gap for Brazil. As shown earlier, this is a relatively new approach and no study exists in the literature that assesses output gap for Brazil through this method. The output gap estimated herein is consistent with a fully specified DSGE model, which derives from the optimization problem of firms and households. As the macroeconomic dynamics in this model is ruled by parameters that are not affected by policy changes, this approach is designed to dismiss the critique by Lucas et al. (1976) about reduced-form models, which, according to the author, lack microfoundations.
As argued by Woodford (2003), an optimal monetary policy will replicate the flexible-price equilibrium, resulting in more efficient allocation. Therefore, from the welfare perspective, the corresponding output gap should be a useful indicator of monetary policy.
Another big advantage of this type of estimation over other currently available methods is the possibility of decomposing the output gap into the shocks present in the model. In other words, it is possible to check the contribution of each shock to output gap move-

ments. The use of this output gap estimation method provides some insight into output gap fluctuations.
The paper is organized into three sections. Section 1 describes the major output gap estimation methods and compares them. Section 2 introduces the output gap estimated by the DSGE model and assesses output gap based on the decomposition of shocks obtained from the model. Finally, two exercises are performed: the first one uses a Taylor equation while the second one uses the Phillips curve. The intention of these exercises is to check the robustness of output gap estimation by the DSGE model compared to the other methods described earlier.

2 USUAL METHODS FOR OUTPUT GAP ESTIMATION

There are basically two methods for output gap estimation: estimation by structural relationships and by statistical filters. The first one focuses on the use of the economic theory to determine the structural and cyclical effects of output. The second one seeks to split the time series into permanent and cyclical components using purely statistical methods. This section introduces the different methods for output gap estimation and compares the methods by graphical representation. On the graphs, the gray areas stand for the periods dated by FGV as recessions. The following techniques were used to estimate the output gap:

1. Linear trend
2. Quadratic trend
3. Hodrick-Prescott (HP) filter
4. Beveridge-Nelson decomposition
5. Production function
6. Unobserved components model
7. Production function combined with the Hodrick-Prescott filter

2.1 LINEAR TREND (LT)

This method consists in assuming that output growth trend is well approximated by a deterministic linear trend in relation to time. This is one of the simplest methods described in the literature, which assumes that output can be decomposed into a cyclical component and a linear function over time. That is, the output trend component grows at a constant rate over time. For that to occur, we have a linear regression on the logarithm of the GDP (y_t), a constant (α) and a trend term t:

$$y_t = \alpha + \beta t + \epsilon_t \tag{1}$$

where β is the estimated parameter and ϵ_t is the error term of the regression. Note that the term β denotes the growth rate of the potential output. Potential output growth would then be:

$$\hat{y}_t = \hat{\alpha} + \hat{\beta} t \tag{2}$$

In equation (2), we have the estimated parameters of y, α and β. This gives us the potential output. To estimate the output gap, subtract the effective output from the potential output:

$$h_t = y_t - \hat{y}_t = \alpha + \beta t + \epsilon_t - (\hat{\alpha} + \hat{\beta} t) = \epsilon_t \tag{3}$$

Equation (3) shows that the output gap is simply the error term of the regression equation.
To calculate the output gap for Brazil, we used the logarithm of the seasonally adjusted GDP series at market prices[1] from the first quarter of 2002.

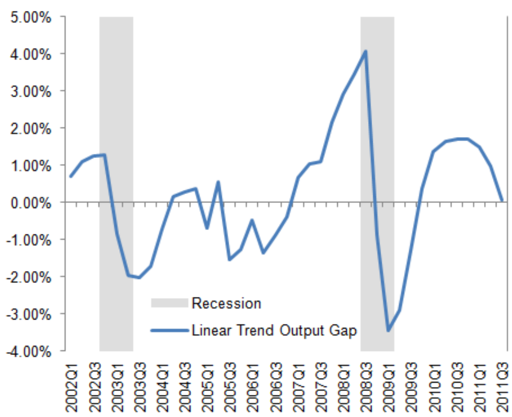

Figure 1: Output Gap: Linear Trend

The gray areas indicate the periods dated by FGV as recessions and help improve the reading of output as it is expected that the beginning and end of recessions occur between the peaks and troughs of the output gap, respectively.
The main criticism about this type of estimation is the presence of several factors that

[1]Source:IBGE

may influence the potential output, such as population growth, increase in productivity and capital stock growth. This runs counter to the assumption that the growth of these variables is such that they yield a constant potential output growth rate over time[2].
Oil price shocks in the 1970s, which caused inflation to rise, economic growth to decrease and total factor productivity (TFP) to fall, led researchers to take into account the "supply side" in addition to the relationship between unemployment, inflation and level of activity, giving rise to the debate about other methods for output gap estimation.

2.2 QUADRATIC TREND (QT)

This method is very similar to the one described in the previous section. The difference between the two methods lies in the definition of trend. Now, potential output follows a quadratic function over time. Hence, we have:

$$y_t = \alpha + \beta_1 t + \beta_2 t^2 + \epsilon_t \tag{4}$$

It is easy to see that equation (4) differs from equation (1) as its trend is squared. The estimation of output gap follows the same procedure used for the estimation of linear trend. In what follows, we show the output gap calculated by quadratic trend and the comparison with the estimation by linear trend.

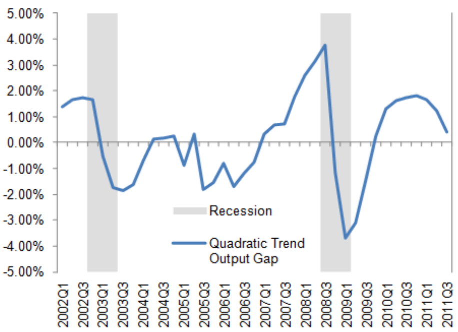

Figure 2: Output Gap: Quadratic Trend

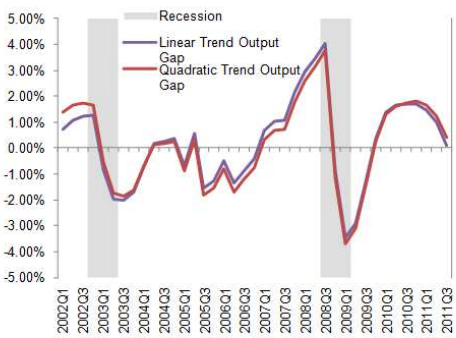

Figure 3: Linear and Quadratic Trends

As shown in Figure (3), the output gap yielded by quadratic trend is very similar to that estimated by linear trend. Despite the "sophisticated" estimation of the trend (potential output), this method receives many of the criticisms leveled at the linear trend model.

[2]A way to come to grips with this problem is to use subsamples of the series for the regression equations; however, one would have to decide about the ideal period for the cutoff.

2.3 HODRICK-PRESCOTT (HP) FILTER

Another method for the decomposition of output into a trend and a stationary component was developed by Hodrick e Prescott (1997). In their work, they proposed to represent a time series by the sum of a trend component that varies smoothly over time and a cyclical component. Thus, we have:

$$y_t = g_t + c_t \text{ para t=1,2,...,T.}$$

The smoothing measure for trend g_t corresponds to the sum of the squared second difference. The cyclical component c_t represents the deviation from g_t, whose mean is close to zero in the long run. These remarks about the determination of growth components lead to the following dynamic optimization problem:

$$\min_{\{g_t\}_{t=0}^{T}} \left\{ \sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\} \quad (5)$$

Where $c_t = y_t - gt$. Parameter λ is a positive number that penalizes the variability of the trend component of the series. That is, the larger the value of λ, the smoother the series. When this value is sufficiently large, the optimization problem can be solved by the use of a linear equation, as outlined in the previous section.
To calculate the potential output for Brazil using the HP filter, we used the seasonally adjusted quarterly GDP series at market prices since 2002[3]. By applying the HP filter to the logarithm of the GDP, we obtain the potential output and, consequently, the output gap.

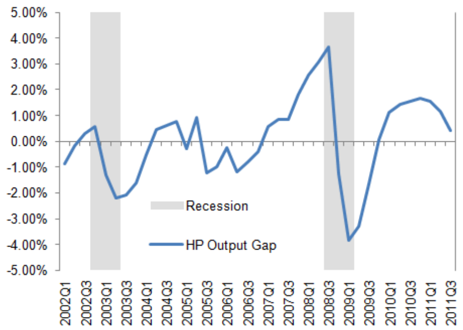

Figure 4: Output Gap: HP Filter

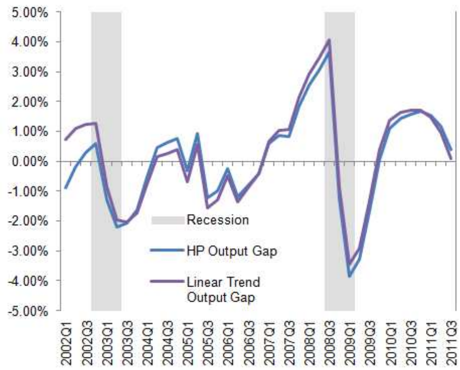

Figure 5: HP, Quadratic Trend and Linear Trend

As indicated in Figure (5), the output gap estimated by linear and quadratic trends were very close to that obtained from the HP filter. An advantage of this estimation method over the linear and quadratic trend models is that it is able to indicate changes

[3]Same series used to calculate the output gap by linear trend.

in potential output. Notwithstanding the improvement in output gap estimation, the HP filter has some shortcomings. To start with, the value of λ is arbitrary. As with the linear trend method, this type of decomposition is based only on statistical criteria, without any economic foundations. The HP filter can lead to a negative growth of the potential GDP if the last observations of the series correspond to a recession period, which gives a non-intuitive result. Also, there is the imposition that the sum of gaps in the whole series should be close to zero[4], which is not in the least justifiable theoretically. To finish with, there exists the end-of-sample bias, also known as "border effect". The values obtained by the HP filter at the end of the sample are strongly influenced by the addition of new data in the series[5].

2.4 BEVERIDGE-NELSON (BN) DECOMPOSITION

The Beveridge e Nelson (1981) decomposition assumes that any nonstationary economic series can be decomposed into two additive parts: a "permanent" one and a "transitory" one. The difference between the trend and the effective value of the series is regarded as cyclical component of the series. This latter component is represented by a stationary autoregressive process that is negatively correlated with the trend.
Consider an ARMA(p,q) model for output changes:

$$\phi(L)\Delta y_t = c + \theta(L)\varepsilon_t \tag{6}$$
$$\phi(L) = 1 - \phi_1(L) - \phi_2(L)^2 - ... - \phi_p(L)^p \tag{7}$$
$$\theta(L) = 1 + \theta_1(L) + \theta_2(L)^2 + ... + \theta_p(L)^p \tag{8}$$

With $\varepsilon_t \sim iid(0, \sigma^2)$, $\mid \phi \mid < 1$ and $\mid \theta \mid < 1$. The ARMA model can be rewritten (according to Wold's representation theorem) as a moving average, taking the following form:

$$\Delta y_t = \mu + \Psi(L)\varepsilon_t \tag{9}$$

where $\Psi(L) = \phi(L)^{-1}\theta(L) = \sum\limits_{j=0}^{\infty} \psi_j L^j$
The Beveridge-Nelson decomposition is given by:

$$y_t = y_0 + \delta t + \Psi(1)\sum_{j=1}^{t} \varepsilon_j + \tilde{\varepsilon_t} \tag{10}$$

Where $\tilde{\varepsilon_t} = \Psi(L)\varepsilon_t$, $\Psi(L) = \sum\limits_{k=0}^{\infty} \psi_k L^k$, $\psi_k = -\sum\limits_{j=k+1}^{\infty} \psi_j$,
$TD_t = y_0 + \delta t =$ deterministic trend,

[4]As defined earlier, $c_t = y_t - gt$ with the sum of c_t tending towards zero in the long run.

[5]A way to minimize the variation in the result on the tail of the series would be to include forecasts some periods ahead in order to reduce the border effect.

$TS_t = \sum_{j=1}^{t} \varepsilon_j =$ stochastic trend,
and $C_t = \tilde{\varepsilon}_t =$ cyclical component

To compute the BN decomposition shown earlier, it is necessary to estimate an ARMA(p,q) model for the output variation. For this estimation, we used the same GDP series described in the previous sections. The only difference was the use of a longer sample[6] for the estimation of the regression equation, starting with the first quarter of 1999 rather than with the first quarter of 2002. The model was selected by the Akaike and Schwarz information criteria.
Among the tested models, the ARMA(2,1) model had the lowest information criterion (both Akaike and Schwarz). With the best ARMA model at hand and according to the criterion used, it is possible to compute the BN decomposition.

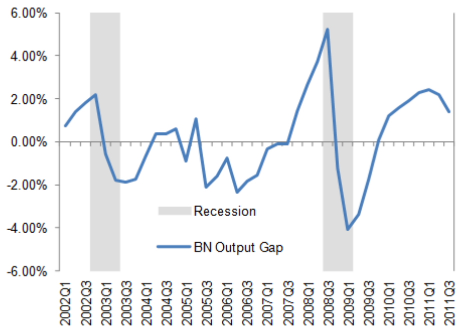

Figure 6: Output Gap - Berevidge-Nelson Decomposition

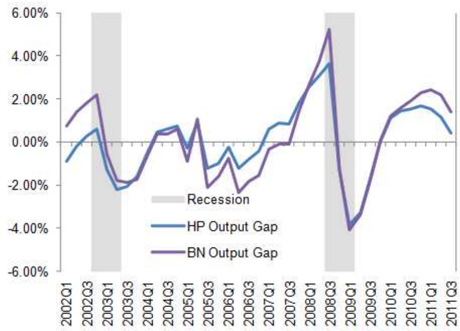

Figure 7: HP Filter and BN Decomposition

Despite the estimation of the output gap since 1999, the graphs were built upon 2002 data for the sake of comparison with the methods mentioned in the previous sections. As displayed in Figure 7, the behavior of the BN decomposition was similar to that of the HP filter, although, unlike the HP filter, the BN method is not submitted to the ”border effect”, as the results depend only on the past values of the series. This decomposition, however, may yield excessively volatile results and cyclical components negatively correlated with the effective GDP, as pointed out by McMorrow, Roeger e Financieros (2001). Nonetheless, Araujo, Areosa e Guillén (2004) assessed the potential output estimation methods based on the forecast of free prices, using a Phillips curve, and concluded that the BN decomposition proved to be the most efficient method.

2.5 PRODUCTION FUNCTION

The production function seeks to draw on the economic theory in order to obtain the potential output. This is an advantage over the methods that are based only on extracting the trend from the series. From the 1980s, this type of estimation gained huge popularity,

[6]Sample size was increased to improve the identification of the model.

with several improvements over time.
In September 1999, the Central Bank of Brazil announced the methods used for output gap estimation, including the production function.
From then on, a supplement to the inflation report was published for each change in the estimation method, giving detailed information about it. To facilitate the demonstration of the different changes in production function, we follow the chronological order of the changes made by the Central Bank of Brazil.
It is assumed that the productive structure of the Brazilian economy can be represented by Cobb-Douglas production technology and its traditional properties: each input has decreasing marginal returns; the function has constant returns to scale; the elasticity of substitution is unitary; and productivity is Hicks-neutral.
Muinhos e Alves (2003) introduce an approach to the estimation of output gap without the need to calculate capital stock and, consequently, without the need to use the depreciation rate, i.e., the major problems observed previously. As shown, while a problem is solved, another one arises. The following equation was used for the estimation:

$$Y_t = A_t(K_t c_t)^{\alpha}(L_t(1-u_t))^{(1-\alpha)} \tag{11}$$

where c_t is the installed capacity utilization and u_t is the rate of unemployment. This is the equation for effective output; the equation for the potential output would be:

$$\bar{Y}_t = A_t(K_t naicu_t)^{\alpha}(L_t(1-nairu_t))^{(1-\alpha)} \tag{12}$$

It is widely known that output gap is merely the difference between effective output and potential output. From the difference in the logarithm of equations 11 and 12, we get:

$$h_t = ln\left(\frac{y_t}{\bar{y}_t}\right) \tag{13}$$

$$h_t = \alpha\left[ln(c_t) - ln(naicu_t)\right] + (1-\alpha)\left[ln(u_t) - ln(nairu_t)\right] \tag{14}$$

Therefore, output gap is merely the weighted average of the capital stock gap and of the labor market gap. As pointed out earlier, although it is not necessary to estimate the capital stock series, the aim is now to find the *nairu* and *naicu*. There is a specific literature on the determination of *nairu* and *naicu* for Brazil. As the present paper focuses on introducing output gap estimation methods, for simplicity, we chose to use the HP filter to calculate the trend of both series and estimate the output gap.
The series used to calculate the output gap by means of the production function were the following:

- Seasonally adjusted quarterly GDP series with market prices since 2002 - Source: IBGE.
- Quarterly average of the seasonally adjusted rate of unemployment since 2002 - Source: IBGE.

- Quarterly average of the seasonally adjusted installed capacity utilization rate (ICUR) since 2002 - Source: Manufacturing Industry Survey by FGV.
- Quarterly average of the seasonally adjusted installed capacity utilization rate (ICUR) since 2002 - Source: Manufacturing Industry Survey by CNI.
- The share of labor in the production function $(1-\alpha)$ was considered to be 0.4. This value is the average wage of employees for 2000 to 2009 as a function of the GDP. Source: IBGE.

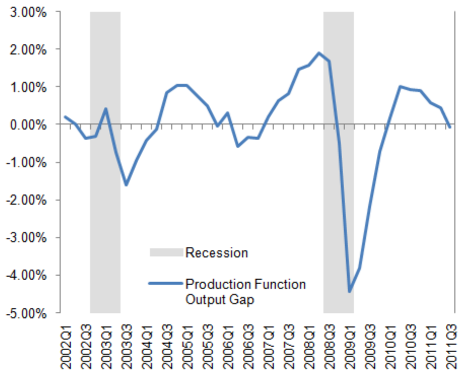

Figure 8: Output Gap: Production Function

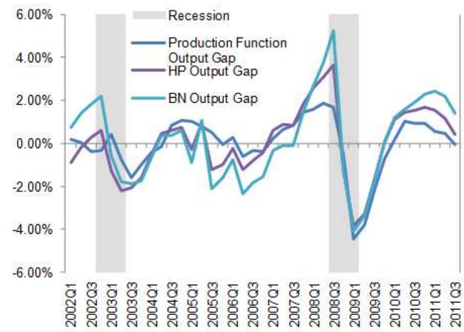

Figure 9: HP filter, BN and production function

Figure (8) shows that, unlike the other methods analyzed so far, the output gap is close to zero. In addition, this estimate does not apparently capture the 2002 recession so well, as when the recession began, the output gap was already negative.

2.6 UNOBSERVED COMPONENTS MODEL

This method is based on structural time series models that seek to decompose the time series into unobservable components. This method belongs with more statistical methods, such as the HP filter. In the present paper, we used a model where the GDP series is decomposed into cycle, trend, and irregular component. To estimate the output gap by this method, it is necessary to use the Kalman filter. This estimation is consistent with the study conducted by Portugal (1993), which made the estimation for Brazil for the period between 1920 and 1980. Another noteworthy study is that of Harvey (1985), which estimated the GDP for the US. Thus, we have:

Essa metodologia se baseia nos modelos estruturais de série de tempo que procuram decompor a série de tempo em componentes não observáveis. Esse método se enquadra no grupo dos métodos mais estatísticos, como o filtro HP. Neste trabalho, foi utilizado um modelo onde a série do PIB é decomposto entre ciclo, tendência e o componente irregular. Para estimar o hiato do produto por esse método será necessário utilizar o filtro de Kalman. Essa estimação segue a mesma linha do estudo realizado por Portugal (1993), que realizou a estimação para o Brasil, cujo período estudado foi de 1920 até

1980. Outro trabalho de destaque neste mesmo sentido é o de Harvey (1985), que fez a estimação para o PIB dos EUA. Sendo assim, temos que:

$$y_t = \mu_t + \psi_t + \varepsilon_t \tag{15}$$

where y_t is the logarithm of the GDP, μ_t is the trend, ψ_t is the cycle and ε_t is the irregular component. The trend is modeled as follows:

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \tag{16}$$

$$\beta_t = \beta_{t-1} + \xi_t \tag{17}$$

where η_t and ξ_t are the white noise error terms uncorrelated with variance σ^2_η and σ^2_ξ. The cyclical component is obtained as follows:

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} cos\lambda & sen\lambda \\ -sen\lambda & cos\lambda \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_t \\ \omega_t^* \end{bmatrix} \tag{18}$$

where ρ is the smoothing factor, λ is the cycle frequency and ω_t ω_t^* stand for the white noise error terms with variance σ^2_ω. Equation (18) can also be written as:

$$\psi_t = \frac{(1 - \rho cos\lambda.L)\omega_t + (\rho sen\lambda.L)\omega_t^*}{1 - 2\rho cos\lambda.L + \rho^2.L^2} \tag{19}$$

The complete model consists of equations (10),(11),(17) and (19) and can be written in state-space form.

$$y_t = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \alpha_t + \varepsilon_t \tag{20}$$

$$\alpha_t = \begin{bmatrix} \mu_t \\ \beta_t \\ \psi_t \\ \psi_t^* \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \rho cos\lambda & \rho sen\lambda \\ 0 & 0 & -\rho sen\lambda & \rho cos\lambda \end{bmatrix} \tag{21}$$

Here, the potential output is represented by the trend of model μ_t. The seasonally adjusted quarterly GDP series with market prices since 1996 was used to estimate the model.

As seen in Figure (11), the output gap estimated by the decomposition of unobservable components was very close to that obtained from the HP filter. As on the other graphs, the peak gap occurs soon after the beginning of recession, as determined by FGV, and its trough occurs before the end of the recession.

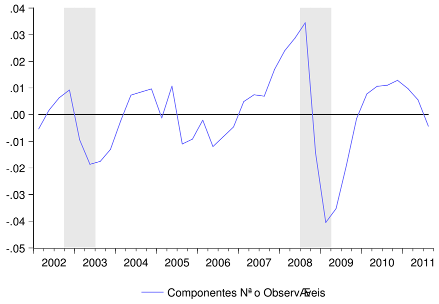

Figure 10: Unobservable components

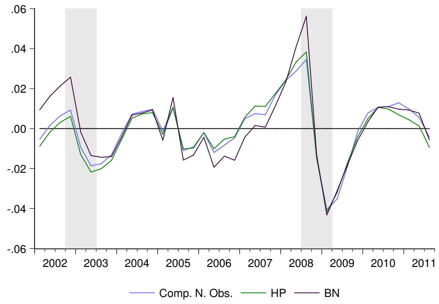

Figure 11: HP, BN, and unobservable components

2.7 PRODUCTION FUNCTION COMBINED WITH THE HP FILTER

This version of the production function was introduced by Areosa (2004) with the aim of making an economic appeal in favor of the use of the HP filter, and estimating the unobservable variables of the model, such as *nairu* and *naicu*. This method then solves the problem found at the end of the previous section. Some of the derivation was performed in the previous section; thus, equation (14) is retrieved and so is equation (5):

$$h_t = \alpha \left[ln(c_t) - ln(naicu_t)\right] + (1-\alpha)\left[ln(u_t) - ln(nairu_t)\right]$$

$$\min_{\{y_t\}_{t=o}^{T}} \left\{ \sum_{t=1}^{N} (y_t^n - y_t)^2 + \lambda_y \sum_{t=1}^{T} [(y_t - y_{t-1}) - (y_{t-1} - y_{t-2})]^2 \right\}$$

As remarked by Areosa (2004), this estimation process consists of two steps. First, equation (14) is used as restriction on the minimization problem observed in equation (5). Second, the objective function is extended so as to estimate the unobservable variables of the production function. Following these steps, we obtain:

$$\min_{\{nairu_t\}_{t=0}^{T}, \{naicu_t\}_{t=0}^{T}} \left\{ \beta_u Op_u + \beta_c Op_c + \beta_y \left[\sum_{t=1}^{N} (y_t^n - y_t)^2 + \lambda_y \sum_{t=1}^{T} [(y_t - y_{t-1}) - (y_{t-1} - y_{t-2})]^2 \right] \right\} \tag{22}$$

s.a.

$$h_t = \alpha \left[ln(c_t) - ln(naicu_t)\right] + (1-\alpha)\left[ln(u_t) - ln(nairu_t)\right]$$

where, $\beta_u Op_u$ e $\beta_c Op_c$ represent the objective function used to estimate the *nairu* and *naicu*, respectively. Using the HP filter to estimate the unobservable series, as proposed in the previous section, we have:

$$\min_{\{nairu_t\}_{t=0}^{T},\{naicu_t\}_{t=0}^{T}} \left\{ \beta_u \left[\sum_{t=1}^{N} (u_t^n - u_t)^2 + \lambda_u \sum_{t=1}^{T} [(u_t - u_{t-1}) - (u_{t-1} - u_{t-2})]^2 \right] + \right.$$
$$+ \beta_c \left[\sum_{t=1}^{N} (c_t^n - c_t)^2 + \lambda_c \sum_{t=1}^{T} [(c_t - c_{t-1}) - (c_{t-1} - c_{t-2})]^2 \right] +$$
$$\left. + \beta_y \left[\sum_{t=1}^{N} (y_t^n - y_t)^2 + \lambda_y \sum_{t=1}^{T} [(y_t - y_{t-1}) - (y_{t-1} - y_{t-2})]^2 \right] \right\} \tag{23}$$

s.a.

$$h_t = \alpha \left[ln(c_t) - ln(naicu_t) \right] + (1 - \alpha) \left[ln(u_t) - ln(nairu_t) \right]$$

This procedure yields a multivariate filter whereby the potential output, *nairu* and *naicu* are estimated simultaneously. Note that if we did not have the restriction, the solution would be to use the HP filter in each series, which leads us to the exercise conducted in the previous section. However, Areosa (2004) warns that using the filter in the employment series and installed capacity is not the right procedure, as we should take into account the imposed restriction; otherwise, the model loses its economic appeal with the simple use of a statistical filter in the series.
Areosa (2004) shows that this model results in an output gap with very high volatility. To improve the estimation, a Phillips curve is added to equation (xx) so that a more accurate estimation of the NAIRU is obtained. As pointed out by Boone (2000), an estimation process that does not contemplate its effect on inflation can give an inefficient NAIRU estimation, and biased parameters with inefficient NAIRU forecasts. To circumvent this optimization problem with the restriction, it is necessary to use the Kalman filter.
For the estimation, we used data from the first quarters of 2002 of the following series:

- Quarterly average of the open unemployment rate[7] - Source: Seade and Dieese.

- Quarterly average of the seasonally adjusted installed capacity utilization - Source: FGV.

- Seasonally adjusted quarterly GDP with market prices - Source: IBGE.

- Quarterly inflation rate (IPCA) - Source: IBGE.

As mentioned by Cerra e Saxena (2000), the Kalman filter is very sensitive to the initially defined parameters. In order to obtain a similar result to that of Areosa (2004), we used the values shown in his paper as initial guess.

[7]The unemployment series was seasonally adjusted with an X12 process.

And that enabled us to calculate the output gap. Figure 12 shows the output gap estimated by the production function combined with the HP filter. Note that it differs from the other gap estimates, principally in the most recent period, in which the gap is positive while the other methods show a negative gap.

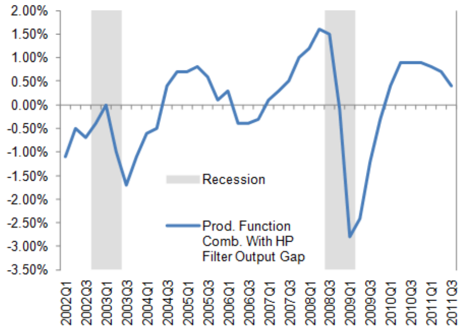

Figure 12: Output Gap: Production Function Combined with the HP Filter

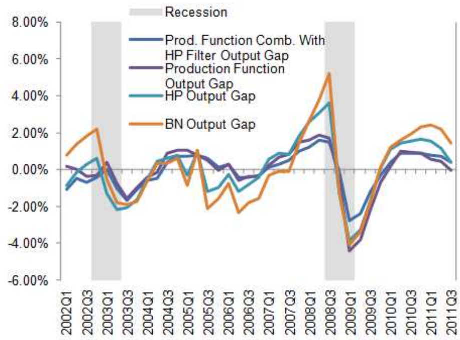

Figure 13: Production Functions, Production Function Combined with HP and HP plus BN.

3 STRUCTURAL ESTIMATION

3.1 THE MODEL

The structural model used herein is a variant of the standard new Keynesian DSGE model - the same one applied by Hirose e Naganuma (2007).
The model is consistent with the optimization of household and firm behaviors in a monopolistically competitive market with price rigidity, where households own the firms and monetary policy follows an interest rate rule. In this case, a standard Taylor rule is used. First of all, we develop the model. After solving the model, we obtain the gap derived from this method and analyze it.

3.1.1 Households

The representative household lives for an infinite number of periods and its utility derives from three components: goods consumption C_t, real monetary balances $\dfrac{M_t}{P_t}$ and leisure $1-N_t$. We introduced the multiplicative habit formation in consumption, as investigated by Fuhrer (2000). Households maximize the following expected utility function:

$$E_t \sum_{i=0}^{\infty} \beta^i D_{t+i} \left[\frac{1}{1-\tau} \left(\frac{C_{t+i}}{C^h_{t+i-1}} \right)^{1-\tau} + \frac{\mu}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right] \tag{24}$$

Where D_t is construed as an IS shock or as a real demand shock and C^h_{t-1} represents the habit of the representative family with a persistence parameter $0 < h < 1$. The parameter $0 < \beta < 1$ represents the discount factor, $\frac{1}{\tau} > 0$ indicates the intertemporal elasticity of substitution, $b > 0$ and $\eta > 0$ are associated with elasticities of substitution in consumption, and finally $\mu > 0$ and $\chi > 0$ are the scale factors. The representative household uses its earnings in goods consumption and purchase of government bonds, saving some of it as real money balance. The resources of period t are allocated as a function of wage, of the amount of monetary balance from the previous period, of the profits of the bonds purchased in the previous period and, finally, of the firm's profits, since the family owns the firm. Given the aggregate price index, the budgetary constraint is as follows:

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t}\right) N_t + \frac{M_{t-1}}{P_t} + R_{t-1}\left(\frac{B_{t-1}}{P_t}\right) + \Pi_t \tag{25}$$

Where B_t are the nominal treasury bonds, paid at a nominal interest rate R_t, $\frac{W_t}{P_t}$ is the real wage and Π_t is the firm's real profit.
The first-order conditions of the optimization problem of the representative household are:

$$\frac{U^*_{C,t}}{C_t} = \beta R_t E_t \left(\frac{U^*_{C,t+1}}{C_{t+1}} \frac{P_t}{P_{t+1}}\right), \tag{26}$$

$$\frac{D_t \mu (M_t/P_t)^{-b}}{U^*_{C,t}/C_t} h = \frac{R_{t-1}}{R_t}, \tag{27}$$

$$\frac{D_t \chi N_t^{\eta}}{U^*_{C,t}/C_t} = \frac{W_t}{P_t}. \tag{28}$$

where

$$U^*_{C,t} = D_t \left(\frac{C_t}{C^h_{t-1}}\right)^{1-\tau} - \beta h E_t \left[D_{t+1}\left(\frac{C_{t+1}}{C^h_t}\right)^{1-\tau}\right] \tag{29}$$

Performing the log-linear approximation of equations (26) and (29) around their steady state and taking into account the equilibrium condition, where $C_t = Y_t$, we obtain the following Euler equation:

$$u^*_{c,t} - y_t = E_t u^*_{c,t+1} - E_t y_{t+1} + r_t - E_t \pi_{t+1} \tag{30}$$

with

$$u^*_{c,t} = \frac{(1-\tau)}{(1-\beta h)}\left[(1+\beta h^2)y_t - hy_{t-1} - \beta h E_t y_{t+1}\right] + \frac{1}{1-\beta h}d_t - \frac{\beta h}{1-\beta h}E_t d_{t+1}, \quad (31)$$

Where lower-case letters represent the percentage deviation from the steady state. With the approximation of equation (28), we get:

$$d_t + \eta n_t - u^*_{c,t} + c_t = w_t - p_t. \quad (32)$$

3.1.2 The Firms

The firm produces the final good Y_t using intermediate goods $Yt(j)$, $j \in [0,1]$ manufactured by firms in a monopolist competitive environment, with the following technology:

$$Y_t = \left[\int_0^1 Y_{t(j)}^{\frac{\lambda_t - 1}{\lambda_t}} d_j\right]^{\frac{\lambda_t}{\lambda_t - 1}} \quad (33)$$

Where λ_t is the elasticity of demand for each intermediate good (note that this term varies over time). The profit maximization of firms is as follows:
$\max \Pi_{t(j)} = p_t y_y - \int_0^1 p_{t(j)} y_{t(j)} d_j$
s.a $Y_t = \left[\int_0^1 Y_{t(j)}^{\frac{\lambda_t - 1}{\lambda_t}} d_j\right]^{\frac{\lambda_t}{\lambda_t - 1}}$
The solution to this problem results in a standard demand for intermediate goods and an aggregate price:

$$Y_{t(j)} = \left(\frac{P_{t(j)}}{P_t}\right)^{\lambda_t} Y_t \quad (34)$$

$$P_t = \left[\int_0^1 P_{t(j)}^{1-\lambda_t} d_j\right]^{\frac{1}{1-\lambda_t}} \quad (35)$$

Each monopolistically competitive firm has a demand curve with a negative slope (equação (34)) for output Y_j. The production function is linear in relation to the labor input $N_{t(j)}$:

$$Y_{t(j)} = A_t N_{t(j)} \tag{36}$$

Where A_t represents an exogenous productivity shock.
The minimization problem of the firm subjected to the production function in equation (36) is given by:
$\min \dfrac{W_t}{P_t} N_t + \Phi_t (Y_{t(j)} - A_t N_{t(j)})$,
Where Φ_t represents the real marginal cost of the firm. The first-order condition yields:

$$\Phi_t = \frac{W_t/P_t}{A_t} \tag{37}$$

According to Calvo (1983), it is assumed that firms can change the prices charged for the produced goods in a given period with probability $1-\omega$. Each firm j chooses the price $P_{t(j)}$ so as to maximize its expected profit:

$$E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+1} \left[\frac{P_{t(j)}}{P_{t+1}} Y_{t+i(j)} - \Phi_{t+i} Y_{t+i(j)} \right], \tag{38}$$

Where $Q_{t,t+i} = \beta^i \dfrac{U^*_{C,t+i}/C_{t+i}}{U^*_{C,t}/C_t}$ is the stochastic discount factor. Subjected to demand curve (34) with equilibrium condition $Y_{t(j)} = C_{t(j)}$, the first-order condition for each firm implies optimal price P^*_t chosen by all firms that adjusted their prices at t:

$$\frac{P^*_t}{P_t} = Z_t \frac{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} Y_{t+i} \Phi_{t+i} \left(\dfrac{P_{t+i}}{Pt} \right)^{\lambda_t}}{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} Y_{t+i} \left(\dfrac{P_{t+i}}{Pt} \right)^{\lambda_t - 1}} \tag{39}$$

Where $Z_t = \dfrac{\lambda_t}{\lambda_t - 1}$ represents the markup. From equation (35), the aggregate price is given by:

$$P_t = \left[\omega P_{t-1}^{1-\lambda_t} + (1-\omega) P_t^{*1-\lambda_t} \right]^{\dfrac{1}{1-\lambda_t}}. \tag{40}$$

Performing a linear approximation around the steady state of P_t and P^*_t, we obtain the new Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\omega)(1-\omega)}{\omega} \varphi_t + \frac{1-\omega}{\omega} (z_t - \beta\omega E_t z_{t+1}), \tag{41}$$

Where π_t corresponds to the inflation rate, $\varphi_t = w_t - p_t - a_t$ is the real marginal cost, and markup z_t is interpreted as a cost shock faced by price-setting firms. All variables are expressed as percentage deviation from the steady state.

3.1.3 Equilibrium with flexible prices and the output gap

The output gap is defined as the deviation of the current output from some potential output measure. Here, the result of the flexible-price equilibrium in the absence of cost shocks is regarded as potential output. As argued by Woodford (2003), an optimal monetary policy replicates a flexible-price steady state economy, which can be the result if we assume that the government rewards monopolistic distortions with appropriate subsidies. Thus, the output gap defined herein ought to be a useful measure of welfare for monetary policymakers.

Let us consider the case where all firms adjust prices every period, disregarding the problem with cost shock z_t. A situation in which prices are flexible occurs when $\omega = 0$, $P_t^* = P_t$ and $Z_t = \overline{Z}$ in equation (39). So, given the definition of the real marginal cost according to equation (37), we have:

$$\frac{W_t}{Pt} = \frac{At}{\overline{Z}}. \tag{42}$$

Using this relationship along with equation (28), which relates real wage to the marginal rate of substitution between consumption and leisure, the flexible-price equilibrium satisfies the following equation:

$$\frac{D_t \chi N_t^{\eta}}{U_{C,t}^{*}/C_t} = \frac{At}{\overline{Z}}. \tag{43}$$

Performing the log-linear approximation around the steady state, we have:

$$d_t + \eta n_t^f - u_{c,t}^{*f} + c^f = a_t \tag{44}$$

Where f means flexible-price equilibrium. Likewise, the production function corresponding to equation (36) can be linearized, taking the following form:

$$y_t^f = n_t^f + a_t \tag{45}$$

Using equations (44) and (45), together with the equilibrium condition where $y_t^f = c_t^f$, we have the following output equation in the flexible-price equilibrium:

$$y_t^f = a_t + \frac{1}{1+\eta} u_{c,t}^{*f} - \frac{1}{1+\eta} d_t, \tag{46}$$

with

$$u_{c,t}^{*f} = \frac{(1-\tau)}{(1-\beta h)} \left[(1+\beta h^2) y_t^f - h y_{t-1}^f - \beta h E_t y_{t+1}^f \right] + \frac{1}{1-\beta h} d_t - \frac{\beta h}{1-\beta h} E_t d_{t+1}. \tag{47}$$

Then, output fluctuations in the flexible-price equilibrium depend not only on changes in productivity, but also on demand shocks (specifically, preference shocks). Hence, the output gap is defined as:

$$hiato_t = y_t - y_t^f, \tag{48}$$

Measuring the percentage deviation from the potential output.

3.1.4 Monetary Policy

To close the model, it is still necessary to specify a monetary policy rule. This will be a standard Taylor rule, i.e., the central bank adjusts the nominal interest rate in response to inflation movement and to the output gap deviation from its target. The log-linearized version of the monetary policy rule is as follows:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)\left[\psi_\pi \pi_t + \psi_y (y_t - y_t^f)\right] + \varepsilon_{r,t}, \varepsilon_{r,t} \sim N(0, \sigma_r^2) \tag{49}$$

Where, $0 <= \rho_r > 1$ determines the degree of interest rate smoothing, $\psi_\phi > 0$, and $\psi_y > 0$. The term $\varepsilon_{r,t}$ represents an exogenous policy shock that could be interpreted as an unsystematic component of monetary policy.

3.1.5 Exogenous Shock Processes and Equilibrium System

As a source of dynamic equilibrium, it is assumed that demand shocks d_t, cost shocks z_t and productivity shocks follow the stationary first-order autoregressive processes below:

$$d_t = \rho_d d_{t-1} + \varepsilon_{d,t}, 0 <= \rho_d < 1, \rho_d \sim N(0, \sigma_d^2) \tag{50}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, 0 <= \rho_z < 1, \rho_z \sim N(0, \sigma_z^2) \tag{51}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, 0 <= \rho_a < 1, \rho_a \sim N(0, \sigma_a^2) \tag{52}$$

Now, we have a general equilibrium system that consists of equations (30), (31), (32), (56), (46) and (47) together with shocks (50), (51) and (52).

3.2 ESTIMATION STRATEGY

In this paper, we used Bayesian estimation techniques for the structural parameters in a fully specified DSGE model. After estimating the parameters, the system is represented in state-space form for the use of the Kalman filter algorithm.

3.2.1 Data and Estimation

The quarterly GDP variation rate (GDP_t), the inflation rate ($IPCA_t$) and the benchmark interest rate ($Selic_t$) were used for the estimation. To calculate the output growth rate, we used the seasonally adjusted GDP series with market prices provided by IBGE. The quarterly inflation rate was calculated from the seasonally adjusted IPCA series. Finally, we used the effective Selic rate accumulated during the quarter. All the series include quarterly data, from the first quarter of 2002 to the third quarter of 2012. The measurement equation that relates the model's variables to the series used takes the following form:

$$\begin{bmatrix} GDP_t \\ IPCA_t \\ Selic_t \end{bmatrix} = \begin{bmatrix} \gamma^* \\ \pi^* \\ r^* + \pi^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} s_t$$

Where γ^* is the steady state growth rate, r^* and π^* are the steady state real interest and inflation rates, respectively.
In the present paper, we decided to estimate all parameters. The decision about prior distributions followed the work of Smets e Wouters (2003), who used the beta distribution for the parameters that could assume values on the interval $[0, 1]$, the gamma distribution for the parameters that take strictly positive values and the inverse gamma distribution for the standard deviations of structural shocks. The mean values of prior parameters were decided based on the works of Hirose e Naganuma (2007), Ornellas e Portugal (2011) and Santos (2011). The table below displays the parameters:

The discount factor β was parameterized based on the steady state real interest rate r^* and is expressed as $\beta = [exp(r^*/100)]^{-1}$.

3.3 RESULTS

Now, we analyze the results obtained from the estimation of the model described previously. First, we analyzed the posterior distributions of the structural parameters. After that, we assessed the output gap derived from the estimation.

3.3.1 Posterior distribution of the structural parameters

Table (1) shows the posterior distributions of the estimated parameters. Note that the steady state real interest rate r^* and the steady state inflation rate π^* were quite close to their respective prior means. In annualized terms, the posterior means represent an inflation rate of $6,36\%$ and a real interest rate of $6,43\%$ and, consequently, a nominal interest rate of $13,19\%$. These results were not so different from the mean of these rates in the analyzed period. From 2002 to September 2012, the average annual inflation rate was $6,5\%$, whereas the mean Selic rate was $14,3\%$ for the period. The parameter ω, known as the Calvo parameter, yielded a value that was higher than that of the prior mean. The mean value of 0.79 indicates that firms adjust prices nearly every four quarters.

		Priori		Posteriori		
Parameter	Density	Mean	Stand. Dev.	Mean	Conf. Int. (90%)	Range
τ	Gamma	1.86	0.15	1.19	[1.09; 1.29]	$\Re^+$
h	Beta	0.50	0.10	0.84	[0.77; 0.91]	$[0, 1)$
ω	Beta	0.66	0.05	0.79	[0.76; 0.81]	$[0, 1)$
r^*	Gamma	1.5	0.25	1.57	[1.11; 2.01]	$\Re^+$
η	Gamma	2.00	0.25	1.64	[1.29; 1.99]	$\Re^+$
ψ_π	Gamma	1.50	0.15	1.54	[1.29; 1.78]	$\Re^+$
ψ_y	Gamma	0.50	0.05	0.52	[0.36; 0.68]	$\Re^+$
ρ_r	Beta	0.50	0.10	0.64	[0.53; 0.76]	$[0, 1)$
ρ_d	Beta	0.50	0.15	0.30	[0.16; 0.44]	$[0, 1)$
ρ_z	Beta	0.50	0.15	0.61	[0.40; 0.82]	$[0, 1)$
ρ_a	Beta	0.50	0.20	0.84	[0.74; 0.94]	$[0, 1)$
γ^*	Gamma	0.75	0.51	0.93	[0.88; 0.99]	$\Re^+$
π^*	Gamma	1.50	0.50	1.55	[1.28; 1.82]	$\Re^+$
σ_r	Inv. Gamma	0.50	0.14	0.49	[0.29; 0.68]	$\Re^+$
σ_d	Inv. Gamma	0.50	0.14	0.51	[0.29; 0.69]	$\Re^+$
σ_z	Inv. Gamma	0.50	0.15	0.49	[0.29; 0.70]	$\Re^+$
σ_a	Inv. Gamma	0.50	0.14	0.49	[0.29; 0.68]	$\Re^+$
$ln\rho(Y^T)$	-212.48					

Table 1: Prior and posterior distribution of the parameters

The Taylor equation parameters are also noteworthy. The table shows that both the posterior mean of the output gap parameter ψ_y and that of inflation ψ_π were very close to their prior means, which were defined based on Taylor (1993). Finally, the posterior mean of parameter ρ_r, which shows the weight of the interest rate from the previous period in the determination of the current interest rate was above the value that had been initially estimated.

3.3.2 Shock Propagation

Before analyzing the output gap estimated by the model, it is necessary to assess the behavior of key variables in response to the shocks observed in the model. Figure (14) shows the response of output, potential output, inflation and interests to technological shocks, monetary policy shocks, marginal cost and demand shocks.

In Figure (14), each line corresponds to a type of shock whereas the columns stand for the observed variables. Each graph represents the posterior mean and its respective confidence interval associated with the impulse response to a shock corresponding to a standard deviation in percentage terms from the steady state. It was observed that the model's variables respond properly to the shocks.

Table (2) shows the variance decompositions for output growth, inflation, nominal in-

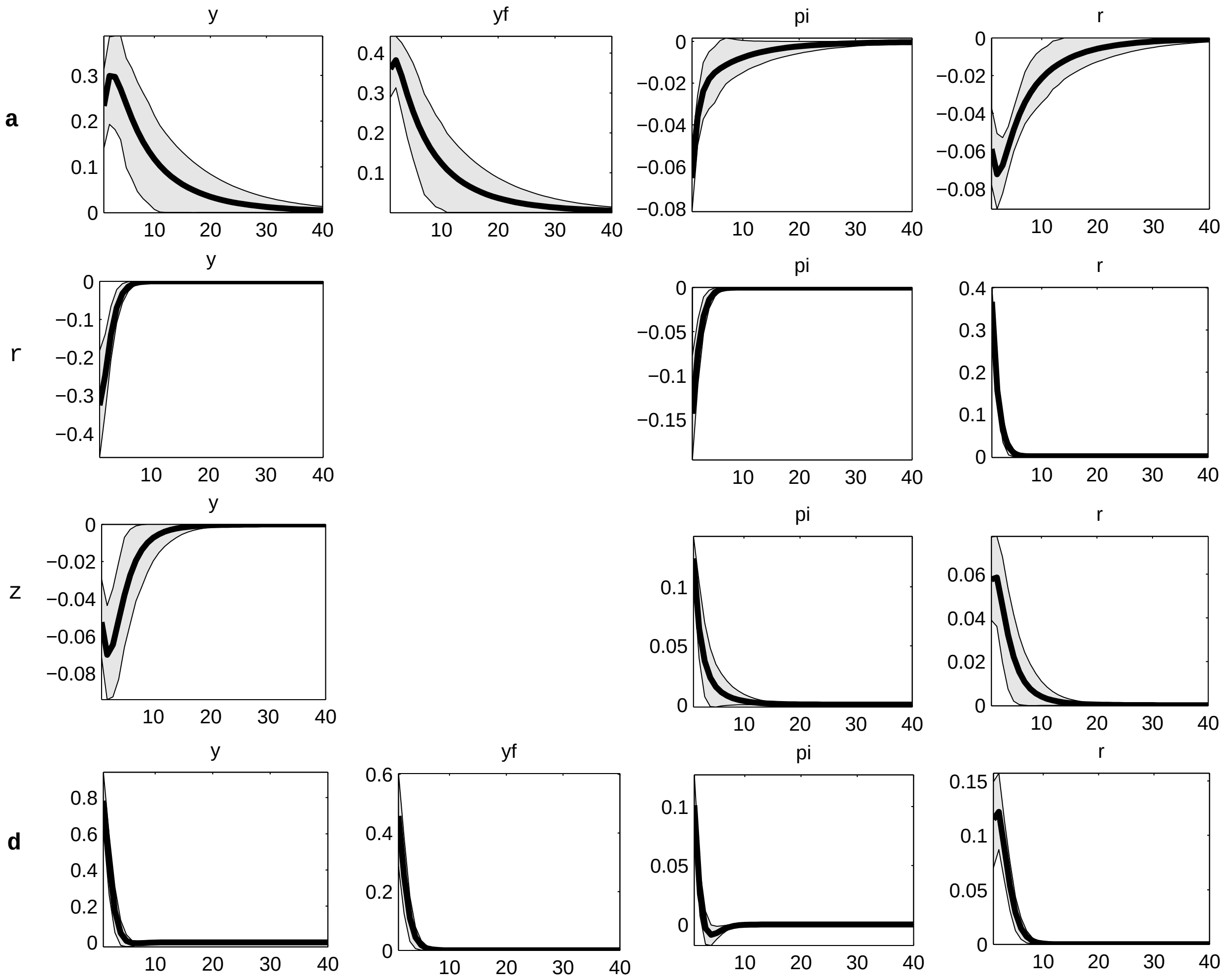

Figure 14: Impulse response function

Shock	Average	Shock	Average
GDP (y)		Inflation (π)	
Demand	0,50	Demand	0,15
Productivity	0,36	Productivity	0,12
Marginal Cost	0,01	Marginal Cost	0,32
Monetary Policy	0,13	Monetary Policy	0,41
Interest Rate (r)		Potential GDP (y^f)	
Demand	0,14	Demand	0,24
Productivity	0,11	Productivity	0,76
Marginal Cost	0,05	Marginal Cost	0
Monetary Policy	0,70	Monetary Policy	0

Table 2: Variance decomposition

terest rate and potential output growth. In order to understand the output gap variability, the decompositions of output and potential output are of great importance. Output growth is stimulated by demand and productivity shocks while potential output movements are due mainly to productivity shocks.
For both series, the contributions of cost and policy shocks are negligible. Based on these findings and on the observation of the impulse response, the analysis of output gap is demonstrated in detail in the next subsection.

3.3.3 Output Gap

Figure (15) shows the output gap estimated from the DSGE model described earlier. The gray areas on the graph represent the recessions dated by FGV[8]. Note that the output gap estimated herein includes both periods dated by FGV as recessions.
Recall that NBER classifies the duration of a recession as the period that goes from the peak of the cycle to its trough. Bearing this in mind, it is possible to notice that, according to FGV, the recession shown in Figure (15) begins in the same period when output gap is at its maximum in both moments. However, there is some controversy over the end of the recession experienced in 2002 and 2003.
FGV reports that the crisis ends two periods before the output gap. Also, there are two periods in which output gap is negative (from 2005 to 2007 and from the third quarter of 2011 to the end of the series). The analysis of what happened in these two periods will be made later when we present the output gap decomposed by the shocks.

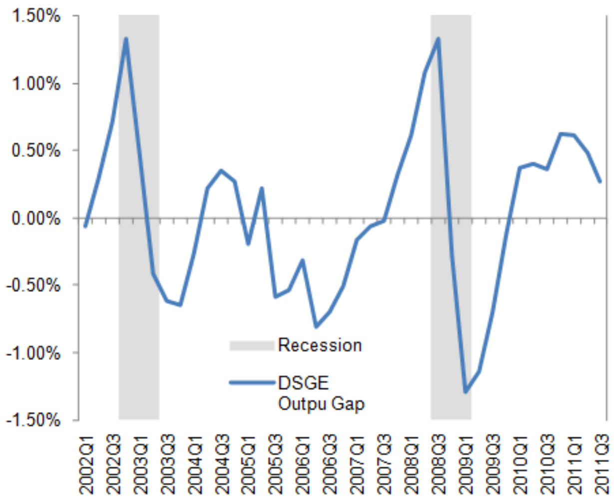

Figure 15: Output gap derived from the DSGE model

Justiniano e Primiceri (2008) and Coenen, Smets e Vetlov (2009) also estimated the output gap using a DSGE model for the US and the European Union and found a highly

[8]Assessment made by the Brazilian Business Cycle Dating Committee (CODACE), whose last update occurred in March 2012.

volatile output gap, but in the period investigated by Coenen, Smets e Vetlov (2009) the gap was negative less frequently, with a mean different from zero. Nevertheless, in the output gap estimated in the present paper, we observed that it is not affected by the problems described previously.

One of the advantages of output gap estimation by a DSGE model is the possibility to infer the forces that act upon the output gap and upon the other structural variables. Figure (16) shows the contribution of each shock to output gap movements, where eps_a, eps_z, eps_r and eps_d, are the technological shock, cost shock, monetary policy shock and demand shock, respectively. The graph indicates that the demand shock has a larger weight on output gap movements. In both recessions dated by FGV, the demand shock yields a negative output gap. As commented earlier, there exist two moments in which the output gap is negative, but the FGV does not date them as recession periods. In the first period, the output gap is negative from the third quarter of 2005 to the third quarter of 2007. This is the time interval on which the output gap is always less than zero; however, the classification of recession by NBER indicates that it goes from the third quarter of 2004 to the second quarter of 2006.

Figure (16) shows that, in such period, one of the major factors for output gap reduction was the negative monetary policy shock. In fact, in September 2004, the Central Bank adopted a tight monetary policy that ended only in May 2005. During this period, the benchmark Selic rate rose from 16.00% to 19.75%, an increase of 3.75 percentage points. As asserted several times by the Central Bank[9], monetary policy changes have a lagged effect on economic activity and this can be seen in Figure figura (16). It is widely known that interest rates were increased at the end of the third quarter of 2004, that the monetary policy shock assumed negative values only in the first quarter of 2005 (a lag of approximately two quarters) and that this lasted up to the third quarter of 2007. This increase in the Selic rate was motivated by the rise of the price index. In September 2004, month in which the Central Bank adopted a tight monetary policy, the inflation accumulated over the past 12 months hovered around 7%, and the target for the same year was pushed up twice. This attempt to take inflation to a lower level (the inflation target for 2005 was 4.5%) had an impact on aggregate demand and the GDP, which had grown 5.7% in 2004, dropped to 3.2% in the subsequent year.

Another period that is not contemplated by FGV goes from the fourth quarter of 2010 to the end of the available series. This period is characterized by crisis in some European Union countries such as Greece, Portugal, Spain, and Italy. The crisis led these countries to lower their global demand for products. This reduction produced some effects on Brazil's economic activity, as demonstrated in Figure figura (16). Since the fourth quarter of 2011, the negative demand shock was fully accountable for the decrease of output gap to negative levels. Note that from September 2011 to October 2012, the Central Bank reduced the benchmark Selic rate by 5.25 percentage points, with a positive effect of monetary policy shock in the period. By taking into account the third quarter of 2012 alone, it is possible to observe that despite the strong negative effect of demand shock, positive productivity and monetary policy shocks contributed to reducing the output gap in that quarter. It is important to recall that a positive productivity shock leads to an increase in the observed output and in the potential output as well; however, as outlined

[9]See, for instance, the Inflation Report Box of June 2009 entitled "Lags in Monetary Policy Transmission to Prices."

in Figure (14), the effect on potential output is greater than that on the observed output; hence, a positive productivity shock has a negative effect on the output gap. Therefore, in the last quarter of the year, the decrease in productivity caused the potential output to fall more sharply than did the observed output, contributing to minimizing the difference between output and its potential.

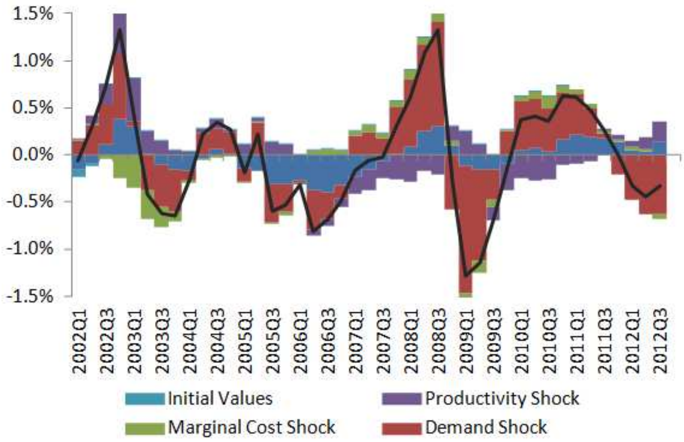

Figure 16: Decomposition of shocks

Whether the Central Bank ought to act in the case of a commodity price shock has always been a matter for debate. In general, the commodity price shock is classified as a supply shock; thus, the Central Bank should not change the benchmark interest rate as the Centrtal Bank must use the monetary policy only to control demand shocks in order to keep the inflation rate within the established target. Nonetheless, in the case of Brazil, which is a big exporter of commodities, a positive shock to the price of these goods is actually a demand shock given that the increase in commodity price produces a positive wealth effect owing to the improvement of the terms of trade.
After the decomposition of the shocks, the behavior of the output gap during the periods with commodity price shocks was analyzed and so were the decisions taken by the Central Bank at the time. First, the shocks used in the model were categorized into two groups according to Galí, Smets e Wouters (2011), being then classified as either "demand" or "supply" shocks. The "demand" shock refers to monetary policy shocks and to demand shocks themselves. The "supply" shock consists of cost and productivity shocks. Figure (17) shows the contributions of these shocks to output gap movements. It is perceived that the demand shock is the main cause for output gap fluctuations.

The following step is to identify the periods in which commodity price was submitted to important shocks. Based on Brazil's Commodity Price Index (IC-Br) data published by the Central Bank, it is possible to notice three commodity price shocks from early 2002 to the third quarter of 2012. The first shock occurred between April 2002 and February 2003, in which period the index increased 93.6%. The second shock took place between June 2010 and February 2011, with a 48.5% increase in the index. And the last shock began in June 2012 and went up to September 2012, with an 8.0% rise in the index.

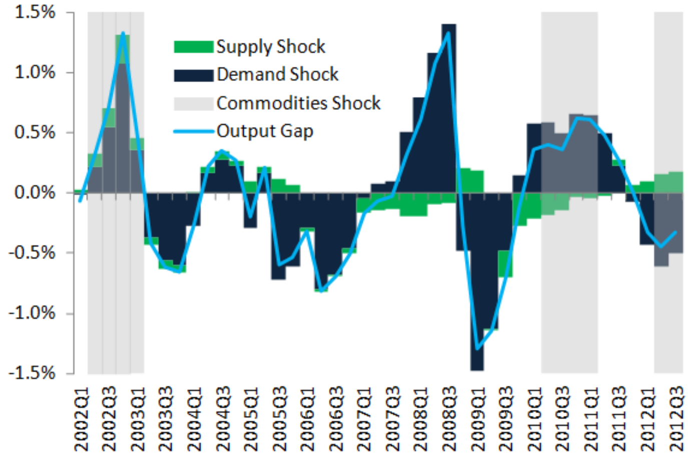

Figure 17: Decomposition of shocks

Graph (17) shows that the output gap was negative only at the time of the last commodity price shock, having remained positive during the other two shocks. In the three periods, the demand shock was the major contributing factor to output gap. It is now necessary to check the monetary authority's behavior in these three episodes.

Between April 2002 and February 2003, the IC-Br increased the most. As shown in Figure (17), the output gap was positive in the second quarter of 2002 and the Central Bank kept the Selic rate at 18.5% p.a. In the third quarter of 2002, the monetary authority decided to reduce the Selic rate by 0.5 p.p, and the output gap continued to rise. Only in the last quarter of 2002 did the Central Bank increase the Selic rate, which went from 18.0% to 25.0%, and the output gap continued to escalate. The increase in interest rates only ended in February 2003. Note that the output gap in the first quarter of 2003 showed a slight decrease.

In the second commodity price shock identified herein, the output gap remained positive throughout the period in which commodity price rose more remarkably. In this case, the shocks showed a distinct behavior, with predominance of the "demand" shock relative to the "supply" shock; as a matter of fact, the "supply" shock was negative throughout the analyzed period. Differently from the findings obtained for the period analyzed earlier, the Central Bank had adopted a tight monetary policy in April of that year. The beginning of these increases in the Selic rate was not related to the commodity price shock because, by definition, it is not possible to predict a shock with any certainty. Notwithstanding, this interest rise may have contributed to stopping the output gap from deteriorating further. Finally, in June 2012, the commodity price went up markedly. However, as a result of the effects of the European crisis on domestic activity, the Central Bank maintained an expansionary behavior in order to revive the economy. Therefore, it is difficult to assess the impact of the increase in commodity price on the output gap during this period. By and large, except for the last shock, the remaining episodes of sudden increase in commodity price showed a positive effect of the "demand" shock. This is possibly some evidence that, in Brazil, positive commodity price shocks should be construed as demand rather than supply shocks.

4 PREDICTIVE TESTS

4.1 CENTRAL BANK REACTION FUNCTION

After estimating the output gaps by the methods described earlier, the next step is to qualify these gaps. A possibility is to use the Central Bank reaction function to try to determine which output gap estimate makes the estimated interest rate mirror best what actually happened. In fact, this exercise seeks to identify which output gap is more consistent with the Central Bank preferences regarding the benchmark interest rate instead of determining which output gap is the best. Anyway, this is a valid exercise as it may add to the vast extant literature on Central Bank preferences.
The reaction function or Taylor rule used for the estimation was as follows:

$$i_t = \beta_1 i_{t-1} + (1-\beta_1)(\beta_3 h_t + \beta_2(E\pi_t - \pi_t^*)) + \epsilon_t \tag{53}$$

Where i_t is the effective annualized monthly Selic interest rate, h_t is the output gap, $E\pi_t$ is the inflation expectation for the year and π_t^* is the inflation target for the year. Since in the Brazilian inflation targeting regime, the inflation target for years t and $t+1$ is known by the monetary authority at the beginning of year t, it is understandable that the monetary policy is based on the inflation target for the current and subsequent years. According to Minella et al. (2003), the variable denoted as Dj_t was used, which represents a weighted average of the deviation of the expected inflation from its target for years t and $t+1$, respectively, i.e.:

$$Dj_t = \frac{12-j}{12}(E_j\pi_t - \pi_t^*) + \frac{j}{12}(E_j\pi_{t+1} - \pi_{t+1}^*) \tag{54}$$

Where j is a monthly index, $E_j\pi_t$ is an inflation expectation for year t in month j, $E_j\pi_{t+1}$ is the inflation expectation for year $t+1$ in month j, π_t^* is the inflation target for year t and π_{t+1}^* is the inflation target for year $t+1$. Therefore, the Central Bank reaction function is as follows:

$$i_t = \beta_1 i_{t-1} + (1-\beta_1)(\beta_3 h_t + \beta_2 Dj_t) + \epsilon_t \tag{55}$$

The following data were used for the estimation:

- Effective annualized Selic interest rate - Source: Bacen.
- Inflation target published by the Brazilian National Monetary Council - Source: Bacen.

- Inflation expectation series (IPCA) for the year - Source: Focus - Bacen.

Quarterly data were used. All series begin in the first quarter of 2002 and end in the third quarter of 2012. To assess with which output gap the Central Bank reaction function is more consistent in the effective Selic interest rate series, the mean squared error (MSE) was used as selection criterion. A Taylor equation was estimated for each output gap series by modifying the sample from 2002T1-2010T1 to 2002T1-2012T3, with a 4-step-ahead forecast for each estimate.

Mean Squared Error				
	step 1	step 2	step 3	step 4
HP Filter	0,38	0,42	0,52	0,46
DSGE	0,42	0,41	0,49	0,48
Production Function	0,29	0,41	0,69	0,87
Linear Trend	0,27	0,26	0,30	0,23
Production Function Combined With The HP Filter	0,62	1,01	1,63	1,91
Quadratic Trend	0,33	0,35	0,44	0,41
Beveridge-Nelson Decomposition	0,52	0,60	0,76	0,76
Unobserved Components Model	0,30	0,29	0,35	0,32

Table 3: Mean Squared Error of the Central Bank's Reaction Equation.

As shown in Table (3), the model that used the linear trend had the lowest MSE, followed by the gap in the unobserved components model. For the one-step-ahead forecast, the MSE of the output gap estimated with the production function was similar to that obtained from the linear trend; however, when estimated for additional periods, the predictive power deteriorated more quickly.
Regarding the output gap estimated by the DSGE model, it is possible to note that it had a moderate performance, similarly to the one observed in the output gap calculated by the HP filter. We expected the MSE of the estimation with the DSGE model not to be the lowest as the Central Bank, in principle[10], does not follow this output gap measure.

4.2 PHILLIPS CURVE

In Araujo, Areosa e Guillén (2004), as well as in the present study, the output gap was estimated by several methods and the estimates were used to forecast the free-price inflation based on the Phillips curve. The result found in the referenced study was that the models in which output gap was estimated through univariate models were better than those based on unobserved components models when the MSEs were compared. The Phillips curve was also used to qualify the several output gap measures in the work of Proietti, Musso e Westermann (2002). Since the output gap should serve as a measure of inflationary pressure, it should improve the predictive power of models that seek to

[10]Unlike the other output gaps presented herein, there is no official record by the Central Bank that asserts that the monetary authority observes the output gap estimated by this method.

forecast inflation.
In the present study, we also use the Phillips curve to forecast inflation with output gaps derived from the proposed methods. The purpose is to check whether the structural gap is a better predictor of future inflation. The following Phillips curve was used for the proposed exercise:

$$\pi_t^L = \beta_1 \pi_{t-1} + \beta_2 E_t\left[\pi_{t+1}\right[+ \beta_3 h_{t-1} + \epsilon_t \tag{56}$$

Where π_t^L represent the free-price inflation accumulated over the past 12 months, π_t is the full inflation accumulated in the past 12 months, $E_t\left[\pi_{t+1}\right[$ is the inflation expected for the next 12 months and h_t represents the output gap.

The following data were used for the estimation:

- Free IPCA accumulated in the past 12 months - Source: IBGE.
- IPCA accumulated in the past 12 months - Source: IBGE.
- Smoothed series of the expectation for IPCA for the next 12 months -Source: Focus-Bacen.

Free IPCA was chosen because it is more sensitive to monetary policy changes than the full index that aggregates administered prices. Only quarterly data were used. Just as in the previous section, we use the MSE as our measure. Likewise, for each output gap series, we estimated a Phillips curve modifying the sample from 2002T1-2010T1 to 2002T1-2012T3, and a four-step-ahead forecast was utilized for each estimate.

Mean Squared Error				
	step 1	step 2	step 3	step 4
HP Filter	0,43	0,44	0,34	0,35
DSGE	0,66	0,48	0,22	0,23
Production Function	0,56	0,67	0,66	0,43
Linear Trend	0,54	0,56	0,26	0,39
Production Function Combined With The HP Filter	0,47	0,64	0,65	0,47
Quadratic Trend	0,45	0,74	0,49	0,57
Beveridge-Nelson Decomposition	0,39	0,49	0,39	0,39
Unobserved Components Model	0,43	0,41	0,34	0,45

Table 4: Mean Squared Error of the Phillips Curve

As shown in Table (4), here we do not have an output gap estimate that stands out on its own as the best predictor of free-price inflation. This is the opposite of what was

obtained from the Taylor rule when the output gap estimation via linear trend yielded the lowest MSE for both the short-run and long-run forecasts. In the case of the one-step-ahead forecast, the best results were those obtained by the Phillips curve using the output gap derived from the BN decomposition.
For long-run forecasts, the lowest MSE was that of the estimation by the DSGE model. In other words, the output gap estimation by the DSGE model is the one that gathers more information so that free-price inflation can be forecasted three and four steps ahead. These findings are not supported by Araujo, Areosa e Guillén (2004), given that the methods for the output gap estimation were not the same and that the sample is not the same used in the present paper.

5 CONCLUSION

This paper introduces several estimation methods for output gap in Brazil. However, the main goal of this paper was to present an output gap estimation for Brazil using a dynamic stochastic general equilibrium (DSGE) model. This means that the output gap is obtained from a fully specified DSGE model, which is derived from the optimization problem of firms and households. As the macroeconomic dynamics in this model is ruled by parameters that are not affected by policy changes, this approach is designed to dismiss the critique by Lucas et al. (1976) about reduced-form models, but according to the author, it lacks microfoundations. An advantage of output gap estimation by this method is the possibility to decompose the estimated output gap into shocks observed in the model.
The output gap estimated here managed to identify the periods dated by FGV as recessions. However, in addition to the episodes reported by FGV, we identified another two critical periods. The first one, from the third quarter of 2004 to the second quarter of 2006, according to NBER, was submitted to two types of shocks: monetary policy and demand shocks. The negative monetary policy shock was due to a cycle of increase in the benchmark interest rate aimed at reducing inflation to a lower level than that observed at the end of 2004 (an accumulated rate of 7.6%). This increase in the Selic rate had an impact on aggregate demand, causing GDP growth to drop by 2.6 percentage points between late 2004 and early 2005. The second one began immediately after the European crisis, which led to the reduction in global demand and ended up affecting the economic activity in Brazil. The decomposition of shocks showed that the negative demand shock was the main determinant of output gap reduction, as expected. Nevertheless, the cycle of loose monetary policy, combined with a slump in productivity, resulted in a smaller difference between the current output and the potential observed in the third quarter of 2012.
The present study also contributed to the discussion on how commodity price shocks ought to be interpreted. In general, the commodity price shock is classified as a supply shock; therefore, the Central Bank should not change the benchmark interest rate as it should use the monetary policy only to control demand shocks in order to keep the inflation rate within its target. However, in the case of Brazil, which is a big exporter of commodities, a positive shock to the price of these goods actually translates into a

demand shock since an increase in commodity price produces a positive wealth effect owing to the improvement of the terms of trade.

From the decomposition of shocks, it was also possible to assess how the commodity price shock affects the output gap. It is widely debated in Brazil whether the commodity price shock should be viewed as a demand or a supply shock. It is commonly known that if it is understood as a supply shock, the monetary authority should not change its monetary policy; otherwise, the Central Bank must act in order to keep tabs on the inflation rate. Based on the IC-Br, we identified three commodity price shocks. It was difficult, though, to analyze the last remarkable increase in commodity price in 2012 due to the effect of the European crisis on the Brazilian economy. So, except for the last study period, the other spells of sudden increase in commodity price were positively affected by the "demand" shock. This is possibly evidence that, in Brazil, positive commodity price shocks should be construed as demand shock rather than supply shock.

In addition to the output gap estimated by the DSGE model, other more traditional methods were also used so as to check for the validity of this new method. To achieve that, we ran two exercises: the first one consisted of a Taylor equation and the second one used the Phillips rule. The first exercise seeks to demonstrate which output gap best fits the Selic rate movements, i.e., an attempt to determine which output gap estimation is preferred by the Central Bank. In this exercise, the estimation obtained from a linear trend model yielded the best results while the DSGE model was not appropriate for this type of forecast. This result was expected because the Central Bank, in principle, does not follow this output gap measure. The second exercise seeks to check which output gap estimation method best forecasts free-price inflation using the Phillips curve. In this case, no estimation method stood out on its own. It was observed, though, that the output gap derived from the BN decomposition was the best at forecasting inflation one step ahead. For long-run forecasts, the lowest MSE was obtained from the DSGE model. That is, output gap estimation by the DSGE model provides more information for the forecast of free-price inflation via DSGE three and four steps ahead. These findings are not supported by Araujo, Areosa e Guillén (2004).

This paper showed that output gap estimation by the DSGE model can be an additional tool for monetary policy conduct. The method used for output gap estimation allows learning more about the role of shocks in output gap movements and identifying some shocks, such as that to commodity price. Furthermore, the output gap proved to be a good predictor of future inflation, indicating that it can better identify future inflationary pressures. For future studies, it would be interesting to test other DSGE model configurations and compare them with the results found here. For instance, using an open-economy model might improve the output gap estimation. Another suggestion would be to carry out a real-time analysis of output gap to check for its consistency. In the present paper, it was not possible to run this test due to the sample size used for output gap estimation, which would yield a too short real-time series, thus compromising the analysis.

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