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ENDOGENEITY AND NONLINEARITIES IN CENTRAL BANK OF BRAZIL'S REACTION FUNCTIONS: AN INVERSE QUANTILE REGRESSION APPROACH

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Abstract: In this work, we seek to investigate nonlinearities in the reaction function of the Central Bank of Brazil by estimating quantile regressions. As the monetary policy rule has endogenous regressors, we followed the procedures suggested by Wolters (J Macroecon 34:342-361, 2012) and the method of inverse quantile regression, proposed by Chernozhukov and Hansen (Econometrica 73:245–261, 2005). This method enabled us to detect nonlinearities in the Central Bank of Brazil's reaction function without the need to make specific assumptions about the factors that determine these nonlinearities. In particular, we observed that: i) the response of the interest rate to the current and expected inflation was, in general, stronger in the upper tail of the conditional interest rate distribution; ii) the response to the output gap showed a growing and significant trend in the lower tail of the conditional Selic rate distribution; iii) the response of the Central Bank of Brazil to the real exchange rate was positive and higher in the upper tail of the conditional Selic rate distribution.

Keywords Monetary policy rules · Quantile regression · Endogenous regressors · Central Bank of Brazil.

JEL Classification C32 · E52 · E58

1 Introduction

The inflation-targeting regime was adopted by the Central Bank of Brazil (CBB) in July 1999. This decision was taken six months after the transition from an exchange rate band system to a floating system. Owing to exchange rate overshooting and to the rise in inflation and in inflation expectations, the Brazilian government aimed to implement a policy regime that was institutionally committed to maintaining price stability and providing a new nominal exchange rate anchor for inflation.

For the analysis of the CBB's monetary policy decisions in the inflation-targeting regime, several papers have estimated the Taylor (1993) rule or the forward-

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looking reaction function proposed by Clarida et al. (2000).¹ For instance, Minella et al. (2003) and Minella & Souza-Sobrinho (2013) estimated a forward-looking reaction function and observed that the CBB strongly reacted to inflation expectations. Mello & Moccero (2009) utilized cointegration analysis and M-GARCH model estimations to check for the presence of long-term relationships between the monetary policy interest rate (Selic rate), inflation expectations, and inflation target, and to verify the presence of volatility spillovers between inflation expectations and monetary policy. For Brazil, the results gathered by these authors revealed there exist long-term relationships between the interest rate, expected inflation, and inflation target, and that higher volatility in monetary policy increases the volatility of the expected inflation. Sanches-Fung (2011) estimated reaction functions for the CBB in a data-rich environment. Sanches-Fung's (2011) evidence points out that the CBB adjusted the Selic interest rate according to the Taylor principle, but that it did not react systematically to the exchange rate behavior.

An important assumption of the papers mentioned above is that interest rate rules are linear functions relative to the variables describing economic conditions. By contrast, the economic literature has come up with numerous reasons why the monetary authority responds nonlinearly to inflation and/or to the output gap. Nobay & Peel (2000), Schaling (2004) and Dolado et al. (2005) demonstrate that an optimal nonlinear monetary rule emerges when the central bank has a quadratic loss function and the Phillips curve is nonlinear. Bec et al. (2002), Nobay & Peel (2003), Dolado et al. (2004), Surico (2007) and Cukierman & Muscatelli (2008) show nonlinearities in the optimal monetary rule may arise if the monetary authority's preferences are asymmetric in relation to inflation and/or to the output gap. By assessing an optimal monetary policy in an economy where the central bank is uncertain over the Phillips curve slope, Tillmann (2011) evidences that the interest rate adjustment is nonlinear. Lastly, the zero lower bound for the nominal interest rate can prompt the central bank to respond nonlinearly to the inflation rate (Kato & Nishiyama, 2005; Adam & Billi, 2006).²

For Brazil, studies on nonlinearities in the monetary policy rule assess specific features of the CBB's asymmetric reaction. For example, Aragón & Portugal (2010), Sá & Portugal (2011) and Aragón & Medeiros (2013) reveal that the Brazilian monetary authority had an asymmetric preference for an above-target inflation in the inflation-targeting regime. Moura & Carvalho (2010) find empirical evidence of nonlinearities in the reaction function that corroborates the CBB's asymmetric preference concerning inflation. Lopes & Aragón (2014) describe that the nonlinearity in the interest rate rule stems from time-varying asymmetric preferences rather than from possible nonlinearities in the Phillips curve. Schiffino et al. (2013) show that the nonnegativity constraint on the Selic interest rate may affect the calibration of the CBB's preferences, implying nonlinearities in the optimal monetary rule. Aragón and Medeiros (2014) estimate a reaction function whose parameters vary over time and conclude that the reaction of the Selic rate to inflation varies remarkably throughout the period, showing a downtrend during the inflation-targeting regime.

Unlike the afore-mentioned studies, the present paper seeks to verify nonlinearities in the CBB's reaction function by quantile regression estimation. An important advantage of this approach over conventional methods (e.g., least ordinary

¹ According to the monetary rule proposed by Taylor (1993), the central bank changes the nominal interest rate in response to deviations of the current inflation from the inflation target and to the current output gap. In turn, the policy rule formulated by Clarida et al. (2000) assumes the monetary authority adjusts the interest rate based on expected future inflation rates and on the output gap.

² Kato & Nishiyama (2005) and Adam & Billi (2006) argue that, close to the zero bound, the central bank responds more strongly to a decrease in inflation rate in order to minimize the likelihood of deflation.

squares (OLS) and instrumental variables (IV)) is that it allows estimating the Selic interest rate rule across different quantiles of the conditional interest rate distribution and not only in the conditional mean of this variable. This permits detecting nonlinearities in the CBB's reaction function without having to make specific inferences about the causal factors of these nonlinearities. Thus, as nonlinearity is determined by the data, the quantile regression method allows comparing the estimates of the monetary rule parameters obtained for the quantiles of the conditional interest rate distribution with the mean from the linear reaction function.

Empirically, we used inverse quantile regression (IVQR), proposed by Chernozhukov & Hansen (2005, 2006), to estimate the CBB's quantile reaction function parameters during the inflation-targeting regime. This method was chosen because of the presence of endogenous regressors (inflation rate and output gap) in the interest rate rule. Some authors, such as Chevapatrakul et al. (2009), Wolters (2012), and Chevapatrakul & Paez-Farrell (2014), estimate the reaction function by quantile regression. To add the presence of endogeneity, Chevapatrakul et al. (2009) and Chevapatrakul & Paez-Farrell (2014) apply the two-stage quantile regression (2SQR) method, while Wolters (2012) uses IVQR.³ Note that IVQR is a good alternative to the 2SQR method because: i) it yields consistent and unbiased estimates of all parameters in the model; and ii) the estimates are consistent even when endogenous regressors change the distribution of the dependent variable (Wolters, 2012).⁴

The IVQR estimation results for the CBB's reaction function can be summarized as follows. While conditional mean estimations showed an insignificant response of the Selic rate to the current inflation gap, quantile regression results indicated that the CBB's short-term response to this variable was significant and increasing between quantiles 0.5 and 0.9. Conversely, the short-term response of the Selic rate to the output gap increased from quantile 0.2 to quantile 0.7 and was not statistically different from zero at the extreme quantiles of the conditional interest rate distribution. We also perceived that the short-term response of the Selic rate to expected inflation was significant from quantile 0.4, exhibiting an uptrend. Regarding the long-term response, results suggest the Selic rate responded strongly to current and expected inflation when the interest rate was above the median. On the other hand, the long-term response to the output gap was significant only at some quantiles on the [0.05, 0.7] interval. This suggests that the CBB does not react to demand pressures when the interest rate is too high. When we included the real exchange rate as an interest rate rule regressor, we noticed the CBB responded positively to the real exchange rate both in the conditional mean and across the interest rate distribution. Moreover, results show that the reaction to the real exchange rate was, in general, stronger in the upper tail of the conditional Selic rate distribution.

Aside from this introduction, this paper is organized into four sections. Section 2 introduces the theoretical model used to derive the interest rate rule adopted by the monetary authority. Section 3 describes the empirical specifications of the CBB's reaction function and its estimation method across different quantiles of the conditional interest rate distribution. Section 4 interprets the results. Section 5 concludes.

2 Theoretical Model

³ Chevapatrakul et al. (2009) assess monetary policy conduct in the United States and in Japan, whereas Chevapatrakul & Paez-Farrell (2014) focus their analysis on Australia, Canada, and New Zealand. In turn, Wolters (2012) estimates the Federal Reserve's reaction function.

⁴ For further details on the 2SQR method, see Amemiya (1982), Powell (1983) and Kim & Muller (2004, 2008).

The theoretical model used in this paper to analyze monetary policy optimal decisions is based on Clarida et al. (1999). The model employs the new Keynesian framework introduced by these authors and consists of three components. The first component is a system of equations that restrict the monetary authority's dynamic control problem. This system of equations comprises: i) an IS curve, which governs output dynamics; and ii) a Phillips curve, which describes inflation dynamics. The second component concerns the central bank's quadratic loss function, which describes monetary policy goals. Finally, the third component is the monetary policy optimal rule, which shows how the central bank traces the optimal path for the nominal interest rate.

2.1 Monetary authority's optimization problem

To assess monetary policy conduct, suppose that the monetary authority's decisions are made before demand shocks, u_t^d , and before cost shocks, u_t^s . Therefore, conditional on the information available at the end of the previous period, the central bank tries to choose the current nominal interest rate, i_t , and a sequence of future interest rates so as to minimize:

$$E_{t-1} \sum_{j=0}^{\infty} \beta^j L_{t+j} \quad (1)$$

where $\beta \in (0,1)$ is the fixed discount factor and the loss function at t is denoted by:

$$L_t = \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda_y y_t^2 + \lambda_i (i_t - i^*)^2 + \lambda_{\Delta i} (i_t - i_{t-1})^2 \right] \quad (2)$$

where π_t is the inflation rate, π^* is the inflation target, y_t is the output gap (i.e., the difference between actual output and potential output), λ_y is the relative weight of the deviation of output from potential output, and λ_i and $\lambda_{\Delta i}$ are the relative weights of interest rate stabilization around an implicit target, i^* , and the interest rate at $t-1$, i_{t-1} .⁵ The monetary authority presumably stabilizes inflation around the inflation target, keeps the output gap closed at zero, and stabilizes the nominal interest rate around target i^* and the nominal interest rate at $t-1$.

The monetary authority's goal is to minimize (1) conditional on the following system of equations describing the economic structure:

$$y_t = E_t y_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}) + u_t^d \quad (3)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t^s \quad (4)$$

where $E_t y_{t+1}$ and $E_t \pi_{t+1}$ are the expected values for output gap and inflation rate given the information set available at t , and demand shocks (u_t^d) and cost shocks (u_t^s). These

⁵ The literature lists several reasons for interest rate smoothing. Among them, we may cite the following: i) presence of uncertainties about the values of economic data and of the coefficients of the macroeconomic model; ii) remarkable changes in interest rates could destabilize the exchange rate and financial markets; and iii) constant fluctuations in the short-term interest rate, albeit small, would strongly impact the aggregate demand and inflation rate. For details about the smoothing of the monetary policy instrument, see Clarida et al. (1998), Sack (2000), Woodford (1999, 2003) and Sack & Wieland (2000).

shocks follow first-order autoregressive processes. Parameters σ and κ are positive constants.⁶

The IS curve, given by equation (3), is a log-linearized version of Euler equation for consumption derived from household optimal decision on consumption and savings after imposition of the market clearing condition. The value expected for the output gap shows that, as households prefer to smooth consumption over time, the expectation for a higher consumption level eventually increases current consumption, also boosting the current demand for output.

The Phillips curve, given by equation (4), describes the characteristics of overlapping nominal prices, where companies show a constant probability of maintaining the output price fixed in any time period (Calvo, 1983). The discrete nature of price adjustment encourages every company to set a higher optimal price the higher the expectation of future inflation. In addition, the presence of output gap in Phillips curve captures the movements in marginal costs associated with excess demand.

2.2 Optimal monetary rule

The central bank's optimization problem (1) is solved discretionally.⁷ This implies that the central bank takes the expectations of future variables as given and chooses the current interest rate in each time period. Since there is no endogenous persistence in inflation and in output gap, the intertemporal optimization problem can be reduced to a sequence of static optimization problems. Combining the first-order conditions and solving i_t , we get:

$$i_t = (1 - \theta_1) \left[\beta_0 + \beta_1 E_{t-1}(\pi_t - \pi^*) + \beta_2 E_{t-1}(y_t) \right] + \theta_1 i_{t-1} \quad (5)$$

where $\beta_0 = i^*$; $\beta_1 = \kappa \sigma^{-1} / \lambda_i$; $\beta_2 = \lambda_y \sigma^{-1} / \lambda_i$; $\theta_1 = \lambda_{\Delta i} / (\lambda_i + \lambda_{\Delta i})$. This equation shows that the optimal nominal interest rate at t responds linearly to deviations of the expected inflation rate from the inflation target, and to the output gap expected for time t .

3 Empirical specifications

In this section, we initially introduce the CBB's reaction function to be estimated in the conditional mean of the interest rate. This linear function is based on the theoretical model described in the previous section. Thereafter, we describe the monetary policy rule to be estimated by quantile regression and the estimation method for this function. Finally, we take into consideration an alternative specification of the CBB's reaction function.

3.1 The monetary policy rule in the conditional mean

For estimation purposes, we made four amendments in the reduced form of reaction function (5). Firstly, we included an exogenous random shock for the interest rate, m_t , in this equation. This shock is assumed to be i.i.d and can be interpreted as the monetary policy's purely random component. Secondly, we consider a variable inflation target (π_t^*). This change is necessary because the inflation targets established by the Brazilian

⁶ Equations (1) and (2) are obtained explicitly from the optimizing behavior of firms and households in an economy with currency and nominal price stickiness. For further details, see Clarida et al. (1999).

⁷ Palma & Portugal (2011) provide evidence in favor of a discretionary monetary policy in Brazil for the period 2000-2010.

National Monetary Council varied annually in the period 1999-2004. Thirdly, the nominal interest rate at $t-2$ is inserted in the policy rule to avoid possible serial autocorrelation problems.⁸ Fourthly, the expected values for inflation and output gap in (5) are replaced with their observed values. By making these amendments, the specification of the policy rule to be estimated is given by:

$$i_t = \beta'_0 + \beta'_1(\pi_t - \pi_t^*) + \beta'_2 y_t + \theta_1 i_{t-1} + \theta_2 i_{t-2} + \varepsilon_t \quad (6)$$

where $\beta'_i = (1 - \theta_1 - \theta_2)\beta_i$, $i=0,1,2$, and $\varepsilon_t = -[\beta'_{1,t}(\pi_t - E_{t-1}(\pi_t)) + \beta'_{2,t}(y_t - E_{t-1}(y_t))]$ + m_t . The coefficients β'_1 and β'_2 (β_1 and β_2) measure the short-term (long-term) response of the interest rate to inflation and to output gap.

Once inflation and output gap forecast errors are an integral part of term ε_t , π_t and y_t are correlated with this error term. In view of that, (6) in the conditional mean of the monetary policy's interest rate will be estimated by IV and by the generalized method of moments (GMM).

3.2 The monetary policy rule across different conditional quantiles

Quantile regression models manage to determine the heterogeneous impacts of variables at different points along a distribution. Quantile regression was first proposed by Koenker & Bassett (1978) and has rather attractive features, namely: i) it can be used to assess the response of the dependent variable to explanatory variables at different points of the dependent variable distribution; ii) quantile regression estimators are more efficient than OLS estimators when the error term is non-Gaussian; and iii) quantile regression estimators are less sensitive to the presence of outliers in the dependent variable (Koenker, 2005).

Quartiles split observations into four segments with equal proportions of benchmark observations in each segment. Quintiles and deciles, similarly to quartiles, split observations into 5 and 10 segments, respectively. Quantiles or percentiles refer to the general case (Koenker & Hallock, 2001). For our monetary policy problem, the τ th conditional quantile is defined as $q_\tau(i_t | i_{t-1}, i_{t-2}, \pi_t - \pi_t^*, y_t)$ such that the likelihood of the nominal interest rate being smaller than $q_\tau(i_t | i_{t-1}, i_{t-2}, \pi_t - \pi_t^*, y_t)$ is equal to τ , i.e.:

$$\int_{-\infty}^{q_\tau(i_t | \pi_t - \pi_t^*, y_t, i_{t-1}, i_{t-2})} f_{i_t | \pi_t - \pi_t^*, y_t, i_{t-1}, i_{t-2}}(i_t | \pi_t - \pi_t^*, y_t, i_{t-1}, i_{t-2}) di = \tau, \quad \tau \in (0, 1) \quad (7)$$

where $f_{i_t | i_{t-1}, i_{t-2}, \pi_t - \pi_t^*, y_t}(i_t | i_{t-1}, i_{t-2}, \pi_t - \pi_t^*, y_t)$ is the conditional density of i_t given i_{t-1} , i_{t-2} , $\pi_t - \pi_t^*$ and y_t . This is a nonparametric specification in which τ can vary continually between zero and one; hence, there are an infinite number of possible parameter vectors.⁹ For $\tau = 1/2$, equation (7) shows the conditional median function of i_t given i_{t-1} , i_{t-2} , $\pi_t - \pi_t^*$ and y_t .

Taking (7), the CBB's reaction function at quantile τ can be expressed as:

$$q_\tau(i_t | \pi_t - \pi_t^*, y_t, i_{t-1}, i_{t-2}) = \beta'_0(\tau) + \beta'_1(\tau)(\pi_t - \pi_t^*) + \beta'_2(\tau)y_t + \theta_1(\tau)i_{t-1} + \theta_2(\tau)i_{t-2} \quad (8)$$

According to equation (8), the parameters of the CBB's reaction function can be estimated at different quantiles, thereby allowing for a complete description of the conditional distribution of the monetary policy interest rate.

Unfortunately, by virtue of the presence of endogenous regressors π_t and y_t , the estimation of reaction function (8) by the quantile regression method proposed by

⁸ This procedure was also adopted by Aragón & Portugal (2010) and Minella & Souza-Sobrinho (2013).

⁹ This requires fewer details about the specification of the distribution of $y|x$ (Greene, 2012).

Koenker & Bassett (1978) yields biased estimates (Kim & Muller, 2012). To circumvent this problem, an alternative would be to use two-stage quantile regression (2SQR). This method is based on the two-stage least absolute deviation estimator developed by Amemiya (1982) and Powell (1983), and extended to quantile regression by Chen & Portnoy (1996) and Kim & Muller (2004, 2012). For our problem, the two stages of the 2SQR method consist in: i) estimating regressions on endogenous regressors π_t and y_t as a function of a set of selected instruments and calculating the adjusted values of these regressors; ii) estimating monetary rule (8) by quantile regression replacing π_t and y_t with their adjusted (or predicted) values obtained in step (i).

Although the 2SQR method yields consistent estimators for slope parameters, the intercept estimator is biased (Kim & Muller, 2012). Because of that, we utilize the inverse quantile regression (IVQR) method, proposed by Chernozhukov and Hansen (2005, 2006).¹⁰ The advantage of this procedure is that it yields unbiased estimates even when changes in endogenous regressors alter the conditional distribution of the dependent variable. As pointed out by Wolters (2012), this appears to be the case of the estimation of the monetary authority's reaction function in which the nominal interest rate exhibits a zero bound. Given such constraint, it is reasonable to assume that a decrease in inflation followed by a reduction in nominal interest rates alters the conditional distribution of this policy instrument. In what follows, we briefly describe the IVQR method.

3.2.1 Inverse quantile regression

The IVQR method derives from the following moment condition regarded as the major identification constraint:

$$P(Y \leq q_\tau(D, X) | X, Z) = \tau \quad (9)$$

where $P(\cdot|\cdot)$ stands for the conditional probability, Y is the dependent variable, D is a vector of endogenous variables, X is a vector of exogenous variables including the constant, and Z is a vector of additional instrumental variables. In the case of interest rate rule (8), Y is the policy instrument i_t , D is made of inflation output ($\pi_t - \pi_t^*$) and output gap (y_t), X is the vector that includes the intercept, i_{t-1} and i_{t-2} , and Z is the vector of additional instruments that may include lagged values of inflation gap and output gap.

In IVQR, the moment condition is equivalent to stating that 0 is the τ th quantile of the random variable $Y - q_\tau(D, X)$ conditional on (X, Z) . Thus, equation (9) is the transform within an analogous sample. For that reason, we have to find the parameters for function $q_\tau(D, X)$ such that zero is the solution to the quantile regression problem, in which the error term regressor is $Y - q_\tau(D, X)$ in any function of (X, Z) . Let $\delta_D = [\beta_{\pi-\pi^*} \beta_y]'$ be the vector of parameters of endogenous variables, $\delta_X = [\beta_0 \theta_1 \theta_2]'$ the vector of parameters of exogenous variables and Λ a set of possible values for δ_D . Therefore, the conditional quantile as a linear function is $q_\tau(Y|D, X) = D'\delta_D(\tau) + X'\delta_X(\tau)$.

According to Wolters (2012), the algorithm that implements the IVQR estimator can be summarized in three steps. The first step consists in estimating regressions by least squares, relating endogenous regressors (D) to the vectors of exogenous variables (X) and instruments (Z), and obtaining the vector of predicted values (\hat{D}). In the

¹⁰ This method is also known as instrumental variable quantile regression (Chernozhukov & Hansen, 2006).

second step, for all $\delta_D \in \Lambda$, we obtain the estimates for vectors δ_X and δ_Z as the solution to the following minimization problem:

$$\left[\tilde{\delta}_X(\delta_D) \quad \tilde{\delta}_Z(\delta_D) \right]' = \arg \min_{\{\delta_X, \delta_D\}} \frac{1}{T} \sum_{t=1}^T \varphi_\tau \left(Y_t - D_t' \delta_D - X_t' \delta_X - \hat{D}_t' \delta_Z \right) \quad (10)$$

where $\varphi_\tau(u) = (\tau - 1(u < 0))u$ is the asymmetric loss function of the least absolute deviation from the standard quantile regression and δ_Z is the vector of parameters related to additional instruments in the regressions shown in the previous step. In the third step, the estimate of δ_D is obtained as the solution to the problem:

$$\tilde{\delta}_D = \arg \min_{\{\delta_D \in \Lambda\}} \sqrt{\tilde{\delta}_Z(\delta_D)' \tilde{\delta}_Z(\delta_D)} \quad (11)$$

This minimization ensures that $Y - q_\tau(D, X)$ no longer depends on \hat{D} , i.e., on (X, Z) . As noted in (10) and (11), the estimates of the parameters of the model are obtained by the estimation of an array of standard quantile regressions (in which convex optimization problems are solved in order to estimate δ_X and δ_Z , in combination with a grid search only for the values of the vector of parameters δ_D).¹¹

3.2.2 Moving blocks bootstrap

To obtain the standard errors of the coefficients of the reaction function estimated by IVQR, we used moving blocks bootstrap (MBB), proposed by Fitzenberger (1997). This author demonstrates that MBB yields standard errors that are robust to unknown forms of heteroskedasticity and autocorrelation, both in linear regressions estimated by OLS and in quantile regressions. As in Clarida et al. (1998) and Wolters (2012), we restricted the autocorrelation to the time horizon of 1 year, which is reasonable for monthly data. Note that in MBB each bootstrap block of the variables (including the dependent variable, the endogenous variables, the exogenous variables, and the instruments) is obtained randomly from the whole sample. After that, the estimates of the parameters by IVQR are obtained for each of the 1000 bootstraps, and the standard errors are calculated as the standard deviation of the 100 estimates obtained for each parameter.¹²

3.3 An alternative specification for the CBB's reaction function

Consonant with Minella et al. (2003), Aragón & Portugal (2010) and Minella & Souza-Sobrinho (2013), we also estimate a specification of the reaction function that includes the deviation of inflation expectations from the inflation target (or from the expected inflation gap). In this case, the reaction function with constant parameters is given by:

$$i_t = \beta_0' + \beta_1' Dj_t + \beta_2' y_t + \theta_1 i_{t-1} + \theta_2 i_{t-2} + \varepsilon_t \quad (12)$$

Whereas the reaction function at quantile τ can be expressed as

$$q_\tau(i_t | Dj_t, y_t, i_{t-1}, i_{t-2}) = \beta_0'(\tau) + \beta_1'(\tau) Dj_t + \beta_2'(\tau) y_t + \theta_1(\tau) i_{t-1} + \theta_2(\tau) i_{t-2} \quad (13)$$

With variable Dj_t denoted as

¹¹ For further details, see Koenker (2005) and Chernozhukov & Hansen (2006).

¹² For more details about MBB, see Fitzenberger (1997).

$$Dj_t = \frac{(12-j)}{12}(E_j\pi_T - \pi_T^*) + \frac{j}{12}(E_j\pi_{T+1} - \pi_{T+1}^*) \quad (14)$$

where j is the monthly index, $E_j\pi_T$ is the inflation expectation in month j for year T , $E_j\pi_{T+1}$ is the inflation expectation in month j for year $T+1$, π_T^* is the inflation target for year T and π_{T+1}^* is the inflation target for year $T+1$. As inflation expectations and output gap are potentially endogenous variables, the IVQR method will be used to estimate the coefficients of monetary rule (13).¹³

4 Results

4.1 Data and unit root tests

To estimate the CBB's reaction functions, we utilized monthly series for the period between January 2000 and December 2013. The series were obtained from the websites of the Applied Economics Research Institute (IPEA) and CBB.

The dependent variable, i_t , is the annualized Selic rate accumulated on a monthly basis. This variable has been used as the major monetary policy instrument in the inflation-targeting regime.

The inflation rate, π_t , is the inflation accumulated over the past 12 months, measured by the broad consumer price index (IPCA).¹⁴ Since inflation targets are considered to be time-varying, we interpolated the annual rates to obtain the monthly series of the target for the inflation accumulated over the next 12 months.

The variable Dj_t is built from the inflation targets set for years T and $T+1$, and from the inflation expectations series obtained from the survey conducted by the CBB with financing and consulting firms. In this survey, firms indicate the inflation rate they expect for years T ($E_j\pi_T$) and $T+1$ ($E_j\pi_{T+1}$).

The output gap (y_t) is measured by the percentage difference between the seasonally adjusted industrial production index and potential output. Potential output is an unobservable variable and, for that reason, it should be estimated. We obtained the proxy for potential output using the Hodrick-Prescott (HP) filter.

The histogram for the Selic rate and the behavior of this variable and of the deviation of inflation from its target are depicted in Figure 1. By comparing the behavior of inflation gap with that of the Selic rate, we note that the CBB has increased (decreased) the use of this policy instrument in response to rises (reductions) in inflation rate. The correlation coefficient between i_t and $\pi_t - \pi_t^*$ was 0.72, suggesting a close relationship between these series. The histogram for the Selic rate indicates that the distribution of this series is asymmetric and skewed to the right and platykurtic.¹⁵ So, the Jarque-Bera statistic (6.66) indicates the null hypothesis of normality of the Selic rate is rejected at 5%. Additionally, it should be noted that the Selic rate is way above zero at the lower quantiles. This suggests that the fear of a lower bound with value zero cannot explain possible asymmetric reactions of the CBB in the lower tail of the conditional distribution of i_t .

¹³ For the determinants of inflation expectations in Brazil, see Bevilaqua et al. (2008) and Carvalho & Minella (2012).

¹⁴ IPCA is calculated by the Brazilian Institute of Geography and Statistics (IBGE) and is the price index used by the National Monetary Council as benchmark for the inflation-targeting regime.

¹⁵ The coefficient of asymmetry was 0.45 and the coefficient of kurtosis was 2.62.

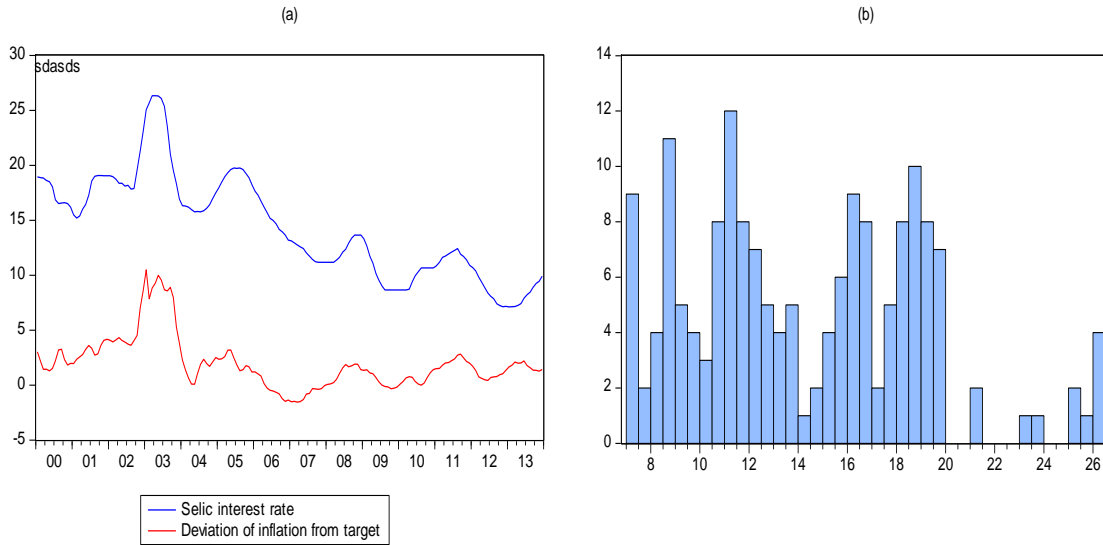


Figure 1 Selic rate and deviation of inflation from its target (panel a) and histogram for the Selic rate (panel b)

Before resuming the estimations, we checked whether the variables used in this study are stationary. Initially, we investigated the order of integration of the variables by the application of three tests: ADF (Augmented Dickey-Fuller), and MZ_{α}^{GLS} and MZ_t^{GLS} tests, suggested by Perron & Ng (1996) and Ng & Perron (2001).¹⁶ As pointed out by Ng & Perron (2001), the selection of the number of lags (k) was based on the modified Akaike information criterion (MAIC) regarded as the maximum number of lags of $k_{max} = \text{int}(12(T/100)^{1/4}) = 13$. Constant (c) and a linear trend (t) were included as deterministic components for the cases in which these components were statistically significant.

Table 1 – Unit root tests

| Variable | Exogenous regressors | ADF(k) | $MZ_{\alpha}^{GLS}(k)$ | $MZ_t^{GLS}(k)$ |
|-----------|----------------------|---------------|------------------------|-----------------|
| i_t | c,t | -3.309* (4) | -11.471 (9) | -2.386 (9) |
| π_t | C | -1.909(13) | -13.77** (1) | -2.599*** (1) |
| π_t^* | C | -3.225** (0) | -6.142* (0) | -1.698* (0) |
| Dj_t | C | -2.088 (10) | -11.75** (10) | -2.410** (10) |
| y_t | C | -3.508*** (0) | -18.95*** (0) | -3.053*** (0) |

Note: *** Significant at 1%. ** Significant at 5%. * Significant at 10%.

The results in Table 1 show that, in general, it is possible to reject the unit root hypothesis in inflation, inflation target, output gap, and Dj_t series. For the Selic rate, the MZ_{α}^{GLS} and MZ_t^{GLS} test results indicate this variable is nonstationary in the level.

Since the failure to reject the unit root null hypothesis in the Selic rate may be related to the existence of a structural break in the trend function, two procedures were performed.¹⁷ First, we used the Exp- W_{FS} statistic, proposed by Perron & Yabu (2009), to test the null hypothesis of no structural break in the trend function of the Selic rate against the alternative hypothesis of a break in intercept and slope of the trend function

¹⁶ The null hypothesis of the tests is that the series is nonstationary (or unit root).

¹⁷ See, for instance, Perron (1989).

at an unknown date.¹⁸ The value of this statistic (9.42) implies rejection of the hypothesis of no structural break at a 1% significance level. Therefore, two unit root tests with structural breaks were run. Following Carrion-i-Silvestre et al. (2009), the $MZ_{\alpha}^{GLS}(\lambda^0)$ and $MZ_t^{GLS}(\lambda^0)$ statistics were used to test the unit root null hypothesis, allowing for three breaks in the trend function at an unknown date under the null and alternative hypotheses. The values obtained for MZ_{α}^{GLS} (-113.4) and MZ_t^{GLS} (-7.52) allow rejecting the unit root hypothesis in the Selic rate at 1%.

4.2 The CBB's reaction function in the conditional mean

First, we estimated reaction functions (6) and (12) in the conditional mean using IV and GMM with the optimal weighting matrix, taking into account possible heteroskedasticity and serial autocorrelation in residuals. Specifically, we applied the method proposed by Newey & West (1987) with the Bartlett kernel and fixed bandwidth to estimate the covariance matrix. The following instruments were used: a constant term, lags 1-2 of the Selic rate and deviation of (current or expected) inflation from the target, lags 2-3 of the output gap, and nominal exchange rate movement at $t-1$ (ΔE_{t-1}).¹⁹

The set of instruments implies three overidentification constraints. We tested the validity of these constraints with Hansen's (1982) J test. Additionally, another two tests were employed: i) Durbin-Wu-Hausman' test to verify the null hypothesis of exogeneity of regressors $\pi_t - \pi_t^*$ and y_t in equation (6), and Dj_t and y_t in equation (12); and ii) Cragg-Donald's F test, proposed by Stock & Yogo (2005), to test the null hypothesis that the instruments are weak.^{20,21} The results of these tests, shown in Table 2, indicate we may reject the hypotheses that (current or expected) inflation gap and output gap are exogenous and that the instruments used in the regressions are weak. Also, the J test shows we cannot reject the hypothesis that the overidentification constraints are met.

The estimates of the CBB's reaction function parameters obtained by IV and GMM are quite similar. For specification (6), the values of the coefficients that measure short-term (β'_1) and long-term (β_1) responses of the Selic rate to inflation were not statistically different from zero in the conditional interest rate mean. This suggests that the CBB has not adopted a stabilization policy for the current inflation around the inflation target, as the increase in inflation has not been followed by a significant increase in the Selic rate. On the other hand, the Selic rate responded to the changes in output gap. The long-term coefficients of this variable were equal to 2.2 and 2.4 for rule (6) estimated by IV and GMM, respectively, and were significant at 1%. Finally, the Selic rate smoothing ($\theta_1 + \theta_2$) yielded approximately 0.98. This result is consistent with

¹⁸ Perron & Yabu (2009) present some tests for the structural break in the trend function that do not require knowing *a priori* whether the noise component of the series is stationary or has a unit root. These authors also demonstrate that, in the case in which the structural break is unknown, the Exp- W_{FS} functional of Wald's test provides a test with almost identical limit values for a noise component $I(0)$ or $I(1)$. Therefore, test procedures with similar sizes can be performed for those two cases.

¹⁹ Exchange rate movement is the percentage variation of the Real/Dollar nominal exchange rate (mean for the period).

²⁰ As underscored by Stock & Yogo (2005), the presence of weak instruments may yield biased IV estimators. Thus, following these authors, we considered instruments to be weak when the bias of the IV or GMM estimator relative to the bias of the OLS estimator was greater than any value b (for example, $b = 5\%$).

²¹ The critical values of this test are described in Stock & Yogo (2005).

the literature on short-term interest rate smoothing and indicates the adjustment of this policy instrument at discrete intervals and in discrete amounts.²²

Table 2 – Estimates of the CBB's reaction functions

| Parameters | Eqn. (6) | | Eqn. (12) | |
|-------------------------------|----------------------|----------------------|----------------------|----------------------|
| | IV | GMM | IV | GMM |
| β_0 | 0.179*** (0.067) | 0.171*** (0.061) | 0.120 (0.088) | 0.134* (0.076) |
| β_1 | 0.025 (0.017) | 0.016 (0.014) | 0.115*** (0.029) | 0.111*** (0.030) |
| β_2 | 0.037*** (0.009) | 0.039*** (0.008) | 0.042*** (0.010) | 0.043*** (0.010) |
| θ_1 | 1.753*** (0.062) | 1.716*** (0.058) | 1.627*** (0.068) | 1.627*** (0.063) |
| θ_2 | -0.770*** (0.062) | -0.732*** (0.057) | -0.644*** (0.070) | -0.645*** (0.063) |
| β_1 | 1.452 (1.005) | 0.995 (0.798) | 7.067** (3.104) | 6.377*** (2.190) |
| β_2 | 2.196*** (0.771) | 2.407*** (0.865) | 2.602** (1.125) | 2.469*** (0.931) |
| <i>J-statistic (p-value)</i> | 0.213 | 0.486 | 0.803 | 0.664 |
| <i>Hausman test (p-value)</i> | 0.008 | 0.036 | 0.000 | 0.017 |
| <i>Cragg-Donald F-stat</i> | 26.61 [†] | 26.61 [†] | 24.00 [†] | 24.00 [†] |
| <i>R²-adjusted</i> | 0.996 | 0.996 | 0.996 | 0.996 |

Note: *** Significant at 1%. ** Significant at 5%. * Significant at 10%. Standard deviation (in brackets). [†] Indicates that the relative bias of the IV (or GMM) in relation to the OLS estimator corresponds to at most 5%.

With respect to monetary rule (12), the estimates of coefficient β_1 indicate that, in the conditional Selic rate mean, the CBB has reacted strongly to the deviation of expected inflation from the inflation target. Specifically, the values obtained for this parameter show the monetary policy rule fulfills the Taylor principle (1993), i.e., the CBB has increased the Selic rate just enough to rise the real interest rate in response to an increase in expected inflation. This result is in line with those encountered by Minella et al. (2003), Moura & Carvalho (2010), Sanches-Fung (2011), Aragón & Medeiros (2013) and Minella & Souza-Sobrinho (2013). Compared to the estimates of β_1 for reaction function (6), the CBB has responded more strongly to expected inflation than to current inflation. This procedure is consistent with a forward-looking policy rule and indicates the CBB has been concerned mainly with anchoring inflation expectations to the inflation target set by the National Monetary Council. In regard to coefficient β_2 , the results were analogous to those obtained for monetary rule (6) and show the Brazilian monetary authority has also reacted to the demand pressure.

4.3 Quantile regression results

Now, we present the results for the CBB's reaction function estimated by IVQR. Table 3 contains the coefficients estimated by quantile regressions and their respective standard errors (in brackets) for specification (8). The estimates for each quantile $\tau \in \{0.05, 0.1, 0.2, \dots, 0.9, 0.95\}$ are shown. Unlike IV and GMM results, the short-term Selic interest rate response to inflation gap, $\beta_1'(\tau)$, is statistically different from zero from quantile 0.5 to quantile 0.9. In contrast, the response to inflation is not significant for the

²² For short-term interest rate smoothing, see Goodfriend (1991) and Rudebusch (1995).

lower quantiles of the conditional Selic rate distribution. Hence, results reveal that the CBB's response to inflation gap is stronger when the Selic rate is adjusted to a higher level than its conditional median. In addition, the response to inflation is more intense between quantiles 0.5 and 0.9. This suggests the CBB has reacted more aggressively to inflation for higher levels of the Selic rate (and of the inflation gap). This result is also observed by Chevapatrakul et al. (2009) and Wolters (2012) for the Federal Reserve, and by Chevapatrakul & Paez-Farrell (2014) for the Central Bank of Australia.

Table 3 also shows that the short-term response of the Selic rate to output gap (β_1) is significant from quantile 0.1 to quantile 0.9 and is not statistically different from zero at the extreme quantiles of the conditional interest rate distribution. In comparison with the IV results, the response to the output gap in the conditional mean is, in general, stronger than the estimates obtained for the quantiles. However, this difference is subtle as the confidence interval for the IV estimate includes those estimates obtained by IVQR.

Table 3 – IVQR estimates for reaction function (8)

| Quantile | β_1 | β_2 | θ_1 | θ_2 |
|----------|---------------------|---------------------|---------------------|----------------------|
| 0.05 | -0.020 (0.042) | 0.035 (0.022) | 1.796*** (0.107) | -0.838*** (0.108) |
| 0.1 | -0.043 (0.032) | 0.032** (0.015) | 1.784*** (0.110) | -0.813*** (0.110) |
| 0.2 | 0.002 (0.025) | 0.027** (0.011) | 1.764*** (0.125) | -0.784*** (0.124) |
| 0.3 | -0.001 (0.018) | 0.027*** (0.010) | 1.758*** (0.101) | -0.777*** (0.100) |
| 0.4 | 0.013 (0.018) | 0.029*** (0.011) | 1.734*** (0.090) | -0.749*** (0.088) |
| 0.5 | 0.030* (0.017) | 0.032*** (0.011) | 1.683*** (0.082) | -0.696*** (0.080) |
| 0.6 | 0.048*** (0.018) | 0.034*** (0.011) | 1.678*** (0.075) | -0.691*** (0.074) |
| 0.7 | 0.062*** (0.023) | 0.041*** (0.013) | 1.656*** (0.081) | -0.667*** (0.080) |
| 0.8 | 0.083*** (0.029) | 0.034** (0.017) | 1.626*** (0.087) | -0.639*** (0.089) |
| 0.9 | 0.087** (0.037) | 0.023 (0.021) | 1.635*** (0.114) | -0.646*** (0.113) |
| 0.95 | 0.091 (0.060) | 0.034 (0.027) | 1.655*** (0.189) | -0.674*** (0.183) |

Note: *** Significant at 1%. ** Significant at 5%. * Significant at 10%.

The results regarding the interest rate smoothing coefficients are significantly different from zero. Between quantiles 0.05 and 0.8, there was a reduction in coefficient $\theta_1(\tau)$, whereas $\theta_2(\tau)$ increased. By adding up $\theta_1(\tau) + \theta_2(\tau)$, we verify that the Selic rate smoothing went up from 0.959 at quantile 0.05 to 0.981 at quantile 0.95. This demonstrates that the CBB's monetary policy is characterized by large smoothing of the Selic rate and that this smoothing increases at the higher quantiles along the distribution.

Figure 2 depicts the long-term responses of the Selic rate to deviations of inflation from the target and to output gap for specification (8). The solid line shows the coefficients obtained by IVQR and the horizontal lines show the IV estimates with a 90%CI (dashed lines). Consonant with Wolters (2012), we do not provide the confidence interval for the coefficients at the quantiles because, in general, we had high

standard errors which, consequently, implied rather broad confidence intervals.²³ A possible explanation for that is that the sum of the smoothing parameters is very close to 1, yielding very high estimates for the standard errors obtained by the Delta method.²⁴

That being said, we may note that, when the Selic rate is in the lower tail of the conditional distribution, the reaction to inflation and to output gap is more passive and becomes more active as we move towards the right side of the distribution. In addition, we verified that, in the upper tail of the distribution, the reaction of the interest rate to inflation was stronger than that obtained by IV. Note that the estimates of $\beta_1(\tau)$ were significant at quantiles 0.6 (3.81 with a standard error of 1.89) and 0.7 (5.59 with a standard error of 3.31), whereas the IV estimate was not statistically different from zero. This suggests that the response of the Selic rate to inflation is stronger when this interest rate is above its conditional median. The upper tail of the distribution exhibits a weaker response to inflation than in the IV estimation, although the coefficients are insignificant in both cases. Compared with the coefficient of inflation, the long-term response to output gap is more stable along the whole distribution, as the point estimates obtained by quantile regression usually fall within the confidence interval of the IV estimate.

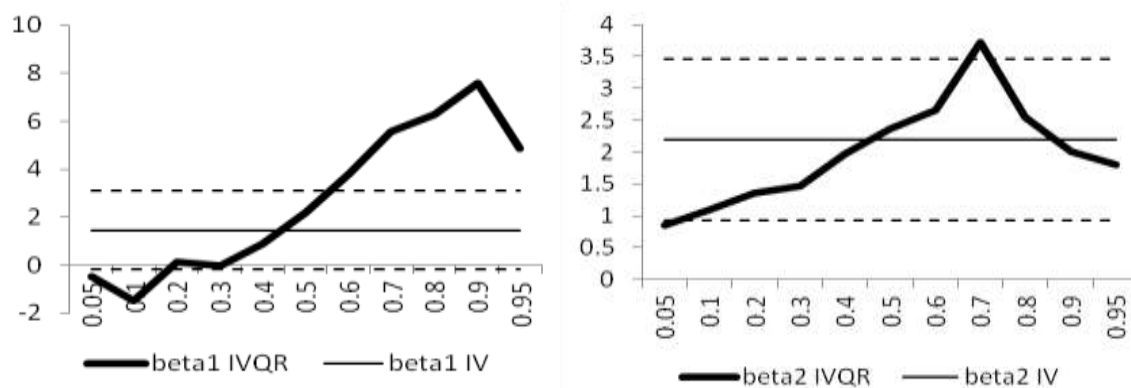


Figure 2 Long-term response of the Selic rate to inflation (β_1) and to output gap (β_2) for reaction function (8). Note: Dashed lines denote a 90% CI for the coefficients estimated by IV.

Table 4 shows the short-term coefficients of monetary rule (13) estimated for the quantiles and their respective standard errors (in brackets). The short-term response of the Selic rate to the expected inflation gap is statistically different from quantile 0.4 onwards. Results also demonstrate that this response has an uptrend as we move towards the right side of the conditional Selic rate distribution. Moreover, note that from quantile 0.6, the estimate of $\beta'_1(\tau)$ is higher than the estimate obtained by IV. Nonetheless, this difference is not significant, as the confidence intervals of the estimates at the quantiles include the point IV estimate. Finally, when we compare these results with those shown in Table 3, we verify that the short-term response of the Selic rate to the expected inflation gap is stronger than that to the current inflation gap between quantiles 0.4 and 0.95. This indicates that the CBB's forward-looking behavior

²³ The standard errors of the long-term responses of the Selic rate may be provided by the authors upon request.

²⁴ Chevapatrakul et al. (2009) solve this problem by estimating the original Taylor rule, i.e., without the smoothing parameter. However, as the short-term interest rate smoothing is observed in CBB's monetary policy, we opted not to follow Chevapatrakul et al. (2009), as we would have misspecification of the reaction function to be estimated.

is observed not only in the conditional mean, but also in most of the Selic rate distribution.

Table 4 – IVQR estimates for reaction function (13)

| Quantile | β_1 | β_2 | θ_1 | θ_2 |
|----------|---------------------------------|---------------------------------|---------------------------------|----------------------------------|
| 0.05 | 0.027 (0.099) | 0.065 ^{***} (0.024) | 1.787 ^{***} (0.135) | -0.829 ^{***} (0.138) |
| 0.1 | -0.042 (0.088) | 0.045 ^{**} (0.023) | 1.765 ^{***} (0.136) | -0.797 ^{***} (0.142) |
| 0.2 | 0.052 (0.064) | 0.040 ^{**} (0.016) | 1.640 ^{***} (0.138) | -0.660 ^{***} (0.141) |
| 0.3 | 0.060 (0.046) | 0.044 ^{***} (0.013) | 1.687 ^{***} (0.108) | -0.705 ^{***} (0.109) |
| 0.4 | 0.092 ^{**} (0.041) | 0.035 ^{***} (0.012) | 1.652 ^{***} (0.098) | -0.664 ^{***} (0.097) |
| 0.5 | 0.114 ^{***} (0.034) | 0.034 ^{***} (0.010) | 1.584 ^{***} (0.087) | -0.593 ^{***} (0.086) |
| 0.6 | 0.142 ^{***} (0.028) | 0.035 ^{***} (0.010) | 1.563 ^{***} (0.076) | -0.573 ^{***} (0.075) |
| 0.7 | 0.133 ^{***} (0.032) | 0.032 ^{***} (0.011) | 1.572 ^{***} (0.076) | -0.578 ^{***} (0.075) |
| 0.8 | 0.134 ^{***} (0.047) | 0.030 ^{**} (0.015) | 1.530 ^{***} (0.089) | -0.534 ^{***} (0.088) |
| 0.9 | 0.154 ^{**} (0.073) | 0.037 [*] (0.022) | 1.505 ^{***} (0.138) | -0.506 ^{***} (0.133) |
| 0.95 | 0.201 [*] (0.109) | 0.036 (0.026) | 1.663 ^{***} (0.167) | -0.668 ^{***} (0.159) |

Note: *** Significant at 1%. ** Significant at 5%. * Significant at 10%.

The response of output gap is significant between quantiles 0.05 and 0.9 and shows a downtrend along the conditional interest rate distribution. With respect to interest rate smoothing, it should be noted that the coefficient $\theta_1(\tau)$ has a downtrend whereas the coefficient $\theta_2(\tau)$ exhibits the opposite behavior. As with monetary rule (8), the sum $\theta_1(\tau) + \theta_2(\tau)$ indicates larger smoothing at the upper quantiles of the Selic rate distribution.

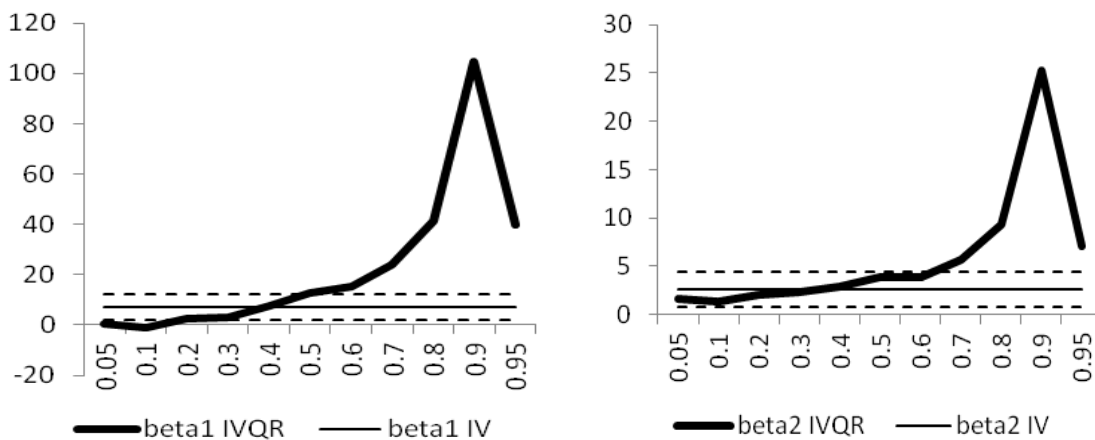


Figure 3 Long-term responses of the Selic rate to Dj_t (β_1) and output gap (β_2) for reaction function (13). Note: Dashed lines denote a 90% CI for the coefficients estimated by IV.

Figure 3 displays the long-term responses of the Selic rate to Dj_t and to the output gap for specification (13). Note that the response of the interest rate to these variables is increasing along the conditional distribution. However, the standard errors allow us to say that the estimate of $\beta_1(\tau)$ is significant only at quantile 0.6 (15.41 with a standard error of 8.87). On the other hand, the estimates of the coefficient of output gap (β_2) were significant at quantiles 0.05 (1.56 with a standard error of 0.80) and 0.3 (2.33 with a standard error of 1.10), but insignificant at the other quantiles of the conditional distribution.

4.4 Robustness of the results

In this section, we check the robustness of the results by performing two exercises: i) we use different output gap measures; ii) we include the exchange rate in the CBB's reaction function.

4.4.1 Different output gap measures

Table 5 shows the results estimated by IV, GMM, and IVQR for reaction function (8) for two different output gap measures. In the first half of the table, we consider the output gap (y_{TL}) obtained from a linear trend model, whereas in the second one, we use the output gap (y_{TQ}) calculated from a quadratic trend model for the natural log of output. For these specifications, we identify similarities to the results that consider the output gap obtained with the HP filter. For both specifications, the short-term response of the interest rate to inflation is increasing along the distribution. In addition, in the upper tail of the conditional distribution, this response has been stronger than the results estimated by IV and GMM and statistically different from zero between quantiles 0.5 and 0.9. Regarding the short-term response to the output gap, it is significant from quantile 0.05 to quantile 0.7 and shows an uptrend.

As far as long-term responses of the Selic rate are concerned, two results should be highlighted. First, the response of inflation gap in the conditional mean and between quantiles 0.5 and 0.7 is statistically different from zero and satisfies the Taylor (1993) principle. Second, the response of the Selic rate to output gap is statistically different from zero up to quantile 0.7. Nevertheless, all the significant part of the IVQR is within the confidence interval estimated by IV for the conditional mean. Thus, we may infer that the long-term response of the Selic rate to output gap is more stable than that of the inflation gap along the distribution of this policy instrument.

Table 6 shows the results obtained by IV, GMM, and IVQR for reaction function (13) taking into account different output gap measures (linear trend and quadratic trend). As demonstrated above, there are nonlinearities in the short-term response of the Selic rate to expected inflation. Particularly, we note that the CBB's short-term response to expected inflation is significant between quantiles 0, 3 and 0.95, but not in the extreme tail of the distribution. In turn, the response of the interest rate to output gap is not statistically different from zero at the quantiles below 0.2 and above 0.8.

Table 5 – IVQR estimates for reaction function (8)

| Quantile | β_1 | β_2 | θ_1 | θ_2 | β_1 | β_2 |
|------------|-----------------------------|-----------|------------|------------|-----------|-----------|
| | Specification with y_{TL} | | | | | |
| <i>IV</i> | 0.036* | 0.022*** | 1.768*** | -0.792*** | 1.560* | 0.945*** |
| | (0.019) | (0.007) | (0.063) | (0.064) | (0.797) | (0.237) |
| <i>GMM</i> | 0.038* | 0.022*** | 1.706*** | -0.728*** | 1.714* | 0.997*** |
| | (0.021) | (0.007) | (0.060) | (0.059) | (0.873) | (0.280) |
| 0.05 | -0.023 | 0.030** | 1.784*** | -0.819*** | -0.662 | 0.870* |
| | (0.045) | (0.012) | (0.107) | (0.105) | (1.595) | (0.492) |
| 0.1 | -0.029 | 0.030* | 1.791*** | -0.822*** | -0.905 | 0.955** |
| | (0.034) | (0.009) | (0.109) | (0.108) | (1.220) | (0.439) |
| 0.2 | 0.009 | 0.019** | 1.828*** | -0.853*** | 0.360 | 0.769* |
| | (0.032) | (0.009) | (0.137) | (0.134) | (1.226) | (0.400) |
| 0.3 | 0.026 | 0.016* | 1.755*** | -0.780*** | 1.060 | 0.652* |
| | (0.025) | (0.008) | (0.117) | (0.114) | (0.942) | (0.344) |
| 0.4 | 0.026 | 0.015* | 1.758*** | -0.777*** | 1.394 | 0.783* |
| | (0.023) | (0.009) | (0.091) | (0.089) | (1.087) | (0.402) |
| 0.5 | 0.048** | 0.016* | 1.694*** | -0.716*** | 2.160** | 0.722** |
| | (0.024) | (0.009) | (0.084) | (0.082) | (0.942) | (0.313) |
| 0.6 | 0.067* | 0.019** | 1.702*** | -0.720*** | 3.664*** | 1.042*** |
| | (0.026) | (0.009) | (0.081) | (0.079) | (1.351) | (0.364) |
| 0.7 | 0.083* | 0.028*** | 1.666*** | -0.683*** | 4.884** | 1.645*** |
| | (0.030) | (0.010) | (0.083) | (0.082) | (2.125) | (0.600) |
| 0.8 | 0.093* | 0.019 | 1.630*** | -0.645*** | 6.583 | 1.342 |
| | (0.033) | (0.014) | (0.090) | (0.089) | (4.206) | (0.887) |
| 0.9 | 0.081* | 0.008 | 1.631*** | -0.641*** | 8.315 | 0.864 |
| | (0.042) | (0.021) | (0.121) | (0.116) | (13.41) | (1.744) |
| 0.95 | 0.099 | 0.011 | 1.707*** | -0.716*** | 10.62 | 1.164 |
| | (0.067) | (0.026) | (0.186) | (0.178) | (26.38) | (2.884) |
| Quantile | Specification with y_{TO} | | | | | |
| <i>IV</i> | 0.032* | 0.023*** | 1.771*** | -0.791*** | 1.691* | 1.190*** |
| | (0.019) | (0.008) | (0.064) | (0.064) | (0.968) | (0.363) |
| <i>GMM</i> | 0.034* | 0.023*** | 1.705*** | -0.724*** | 1.887* | 1.288*** |
| | (0.020) | (0.007) | (0.060) | (0.059) | (1.075) | (0.427) |
| 0.05 | -0.034 | 0.028** | 1.785*** | -0.811*** | -1.263 | 1.057 |
| | (0.044) | (0.014) | (0.108) | (0.106) | (2.317) | (0.742) |
| 0.1 | -0.033 | 0.032*** | 1.793*** | -0.820*** | -1.218 | 1.202* |
| | (0.034) | (0.010) | (0.109) | (0.109) | (1.465) | (0.662) |
| 0.2 | 0.009 | 0.021** | 1.786*** | -0.807*** | 0.406 | 0.999* |
| | (0.030) | (0.009) | (0.133) | (0.131) | (1.378) | (0.578) |
| 0.3 | 0.018 | 0.023*** | 1.756*** | -0.777*** | 0.885 | 1.124** |
| | (0.024) | (0.008) | (0.115) | (0.113) | (1.084) | (0.520) |
| 0.4 | 0.027 | 0.018** | 1.745*** | -0.761*** | 1.618 | 1.078* |
| | (0.021) | (0.009) | (0.090) | (0.088) | (1.188) | (0.544) |
| 0.5 | 0.045* | 0.017* | 1.701*** | -0.720*** | 2.379** | 0.895** |
| | (0.023) | (0.009) | (0.084) | (0.082) | (1.124) | (0.437) |
| 0.6 | 0.065*** | 0.020** | 1.703*** | -0.718*** | 4.238** | 1.286** |
| | (0.025) | (0.010) | (0.080) | (0.079) | (1.794) | (0.584) |
| 0.7 | 0.075*** | 0.023** | 1.689*** | -0.701*** | 6.191* | 1.941* |
| | (0.028) | (0.010) | (0.082) | (0.082) | (3.386) | (1.037) |
| 0.8 | 0.084*** | 0.016 | 1.649*** | -0.656*** | 11.33 | 2.182 |
| | (0.032) | (0.013) | (0.091) | (0.091) | (13.13) | (2.727) |
| 0.9 | 0.080** | 0.009 | 1.631*** | -0.638*** | 10.34 | 1.186 |
| | (0.040) | (0.021) | (0.120) | (0.117) | (19.39) | (2.780) |
| 0.95 | 0.100 | 0.010 | 1.713*** | -0.714*** | 139.8 | 14.35 |
| | (0.066) | (0.027) | (0.181) | (0.174) | (4690.0) | (472.1) |

Note: *** Significant at 1%. ** Significant at 5%. * Significant at 10%.

Table 6 – IVQR estimates for reaction function (13)

| Quantile | β_1 | β_2 | θ_1 | θ_2 | β_1 | β_2 |
|------------|---------------------------------|---------------------------------|---------------------------------|----------------------------------|---------------------------------|---------------------------------|
| | Specification with y_{TL} | | | | | |
| <i>IV</i> | 0.149 ^{***} (0.029) | 0.031 ^{***} (0.007) | 1.613 ^{***} (0.070) | -0.634 ^{***} (0.071) | 7.165 ^{***} (2.268) | 1.465 ^{***} (0.407) |
| <i>GMM</i> | 0.165 ^{***} (0.026) | 0.033 ^{***} (0.006) | 1.582 ^{***} (0.061) | -0.605 ^{***} (0.063) | 7.380 ^{***} (2.051) | 1.472 ^{***} (0.392) |
| 0.05 | 0.052 (0.105) | 0.037 ^{**} (0.017) | 1.754 ^{***} (0.168) | -0.804 ^{***} (0.170) | 1.049 (2.089) | 0.752 [*] (0.391) |
| 0.1 | 0.021 (0.102) | 0.020 (0.014) | 1.750 ^{***} (0.158) | -0.800 ^{***} (0.163) | 0.425 (2.052) | 0.409 (0.311) |
| 0.2 | 0.135 (0.073) | 0.030 ^{***} (0.011) | 1.586 ^{***} (0.156) | -0.612 ^{***} (0.157) | 5.087 (3.371) | 1.104 [*] (0.591) |
| 0.3 | 0.119 ^{**} (0.046) | 0.028 ^{***} (0.008) | 1.650 ^{***} (0.117) | -0.671 ^{***} (0.116) | 5.858 ^{**} (2.767) | 1.359 ^{**} (0.606) |
| 0.4 | 0.127 ^{***} (0.039) | 0.021 ^{***} (0.008) | 1.655 ^{***} (0.101) | -0.671 ^{***} (0.099) | 7.878 ^{**} (3.383) | 1.316 ^{**} (0.597) |
| 0.5 | 0.151 ^{***} (0.033) | 0.023 ^{***} (0.006) | 1.608 ^{***} (0.092) | -0.624 ^{***} (0.090) | 9.168 ^{***} (2.707) | 1.386 ^{***} (0.503) |
| 0.6 | 0.155 ^{***} (0.029) | 0.022 ^{***} (0.006) | 1.556 ^{***} (0.078) | -0.569 ^{***} (0.077) | 11.71 ^{***} (4.095) | 1.654 ^{**} (0.686) |
| 0.7 | 0.160 ^{***} (0.031) | 0.023 ^{***} (0.007) | 1.574 ^{***} (0.070) | -0.584 ^{***} (0.069) | 17.24 [*] (9.359) | 2.488 [*] (1.424) |
| 0.8 | 0.172 ^{***} (0.046) | 0.023 ^{**} (0.010) | 1.511 ^{***} (0.092) | -0.513 ^{***} (0.090) | 102.2 (534.8) | 13.52 (70.15) |
| 0.9 | 0.150 [*] (0.090) | 0.014 (0.020) | 1.530 ^{***} (0.165) | -0.539 ^{***} (0.156) | 15.80 (28.12) | 1.440 (2.510) |
| 0.95 | 0.214 [*] (0.120) | 0.027 (0.025) | 1.665 ^{***} (0.193) | -0.658 ^{***} (0.182) | -30.37 (93.41) | -3.821 (12.94) |
| Quantile | Specification with y_{TO} | | | | | |
| <i>IV</i> | 0.141 ^{***} (0.029) | 0.032 ^{***} (0.007) | 1.622 ^{***} (0.071) | -0.638 ^{***} (0.072) | 8.714 ^{**} (3.398) | 1.966 ^{***} (0.693) |
| <i>GMM</i> | 0.156 ^{***} (0.026) | 0.034 ^{***} (0.007) | 1.585 ^{***} (0.063) | -0.602 ^{***} (0.064) | 9.127 ^{***} (3.239) | 1.994 ^{***} (0.690) |
| 0.05 | 0.025 (0.105) | 0.027 (0.018) | 1.770 ^{***} (0.161) | -0.815 ^{***} (0.163) | 0.537 (2.290) | 0.591 (0.448) |
| 0.1 | -0.016 (0.101) | 0.031 (0.016) | 1.782 ^{***} (0.153) | -0.826 ^{***} (0.159) | -0.363 (2.313) | 0.711 (0.454) |
| 0.2 | 0.095 (0.072) | 0.028 ^{**} (0.011) | 1.680 ^{***} (0.152) | -0.708 ^{***} (0.155) | 3.365 (3.022) | 0.997 [*] (0.594) |
| 0.3 | 0.106 ^{**} (0.046) | 0.028 ^{***} (0.008) | 1.660 ^{***} (0.116) | -0.675 ^{***} (0.116) | 6.908 (4.326) | 1.829 [*] (1.038) |
| 0.4 | 0.102 ^{***} (0.039) | 0.022 ^{***} (0.009) | 1.685 ^{***} (0.098) | -0.700 ^{***} (0.097) | 7.142 [*] (3.773) | 1.523 [*] (0.791) |
| 0.5 | 0.151 ^{***} (0.033) | 0.025 ^{***} (0.007) | 1.607 ^{***} (0.093) | -0.620 ^{***} (0.091) | 12.21 ^{**} (4.869) | 2.036 ^{**} (0.946) |
| 0.6 | 0.149 ^{***} (0.029) | 0.024 ^{***} (0.007) | 1.573 ^{***} (0.078) | -0.581 ^{***} (0.078) | 18.89 (11.55) | 3.068 (2.041) |
| 0.7 | 0.159 ^{***} (0.031) | 0.025 ^{***} (0.008) | 1.559 ^{***} (0.072) | -0.565 ^{***} (0.072) | 27.31 (23.72) | 4.303 (3.922) |
| 0.8 | 0.166 ^{***} (0.047) | 0.023 ^{**} (0.010) | 1.527 ^{***} (0.095) | -0.525 ^{***} (0.093) | -77.32 (325.8) | -10.789 (45.58) |
| 0.9 | 0.149 [*] (0.085) | 0.014 (0.021) | 1.531 ^{***} (0.163) | -0.537 ^{***} (0.155) | 23.66 (60.67) | 2.175 (5.598) |
| 0.95 | 0.197 [*] (0.118) | 0.026 (0.025) | 1.662 ^{***} (0.193) | -0.646 ^{***} (0.184) | -12.74 (19.02) | -1.651 (2.969) |

Note: *** Significant at 1%. ** Significant at 5%. * Significant at 10%.

4.4.2 Exchange rate effects

Several studies have investigated whether central banks react directly to exchange rate movements. Clarida et al. (1998) revealed that the central banks of Germany and of Japan include the real exchange rate in their reaction functions, even though the magnitude of the reactions is negligible. Mohanty & Klau (2004) estimated modified Taylor rules and found that several central banks in emerging countries (e.g., Brazil and Chile) react to exchange rate movements. Lubik & Schorfheide (2007) estimated a DSGE model for Australia, New Zealand, Canada, and the United Kingdom and verified that only the central banks of the first two countries react to exchange rate movements. In line with Lubik & Schorfheide (2007), Furlani et al. (2010) observed that the CBB does not change the Selic rate in response to exchange rate movements. Mello & Moccero (2009) revealed that the monetary policy instrument reacts to exchange rate in Mexico, but not in Brazil, Chile, and Colombia. Aizenman et al. (2011) and Ostry et al. (2012) demonstrated that the central banks of several emerging markets that adopted the inflation-targeting regime react to exchange rate movements.

Many are the reasons that may lead the monetary authority to show deep concern for the exchange rate. First, in an economy with part of the debt denominated in foreign currency, exchange rate devaluations may increase debt service, hinder the balances of firms and banks, limit credit, expand the number of bankruptcy filings, and reduce employment and aggregate output. Haussmann et al. (2001) and Calvo & Reinhart (2002) highlight that the effects on economic agents' balances has been the major reason why central banks seek to avoid currency devaluations in the presence of external shocks. On the other hand, Aghion et al. (2009) developed a theoretical model to show that exchange rate appreciations may reduce firms' gains and, consequently, their capacity to take loans and make innovations. This would negatively affect long-term output growth, with a larger impact on economies with a less developed financial system. Aizenman et al. (2011) proposed a simple macroeconomic model to assess monetary policy in a small open economy. They verified that a large weight on exchange rate volatility in the central bank's loss function strengthens the reaction of the policy instrument to the exchange rate and may bring welfare gains. These authors also argue that these gains may be larger in emerging economies or in those which export commodities, are more vulnerable to shocks on the terms of trade, and have a poorly developed financial system.

To check whether the CBB has reacted to exchange rate movements, we assumed that this policymaker's discretionary policy consists in choosing the Selic rate at t so as to minimize the loss function:

$$L_t = \frac{1}{2} \left[(\pi_t - \pi^*)^2 + \lambda_y y_t^2 + \lambda_e e_t^2 + \lambda_i (i_t - i^*)^2 + \lambda_{\Delta i} (i_t - i_{t-1})^2 \right] \quad (15)$$

subject to constraints

$$y_t = E_t y_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}) + \alpha e_t + u_t^d \quad (16)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \gamma e_t + u_t^s \quad (17)$$

$$e_t = E_t (e_{t+1}) - [i_t - E_t (\pi_{t+1})] + u_t^q \quad (18)$$

where e_t is the effective real exchange rate gap (i.e., the deviation of the natural log of the effective real exchange rate from its trend, estimated by the HP filter), λ_e is the relative weight of the real exchange rate gap in the CBB's loss function, $E_t(e_{t+1})$ is the

expected real exchange rate for $t+1$, u_t^q is a white noise error term that represents the impact of other exchange rate determinants (such as risk premium movements), while parameters α and γ are positive constants.^{25,26} In equations (16) and (17), note that an exchange rate devaluation has a positive effect on inflation output gap. Equation (18) shows the exchange rate is determined by the uncovered interest rate parity (UIP).²⁷

The first-order conditions arising from the minimization of loss function (15) subject to constraints (16)-(18) can be combined in order to obtain the following interest rate rule:

$$i_t = (1 - \theta_1) \left[\beta_0 + \beta_1 E_{t-1}(\pi_t - \pi_t^*) + \beta_2 E_{t-1}(y_t) + \beta_3 E_{t-1}(e_t) \right] + \theta_1 i_{t-1} \quad (19)$$

where

$$\beta_0 = i^*; \beta_1 = \left[\kappa(\sigma^{-1} + \alpha) + \gamma \right] / \lambda_i; \beta_2 = \lambda_y (\sigma^{-1} + \alpha) / \lambda_i; \beta_3 = \lambda_e / \lambda_i; \theta_1 = \lambda_{\Delta i} / (\lambda_i + \lambda_{\Delta i}).$$

To estimate interest rate rule (19), we made the changes described in Section 3.1 and obtained the following specification:

$$i_t = \beta'_0 + \beta'_1(\pi_t - \pi_t^*) + \beta'_2 y_t + \beta'_3 e_t + \theta_1 i_{t-1} + \theta_2 i_{t-2} + \varepsilon_t \quad (20)$$

In this case, the CBB's reaction function at quantile τ can be expressed as:

$$q_\tau(i_t | \pi_t - \pi_t^*, y_t, e_t, i_{t-1}, i_{t-2}) = \beta'_0(\tau) + \beta'_1(\tau)(\pi_t - \pi_t^*) + \beta'_2(\tau) y_t + \beta'_3(\tau) e_t + \theta_1(\tau) i_{t-1} + \theta_2(\tau) i_{t-2} \quad (21)$$

Besides interest rate rules (20) and (21), we also estimated two specifications with variable Dj_t by substituting the current inflation gap, namely:

$$i_t = \beta'_0 + \beta'_1 Dj_t + \beta'_2 y_t + \beta'_3 e_t + \theta_1 i_{t-1} + \theta_2 i_{t-2} + \varepsilon_t \quad (22)$$

$$q_\tau(i_t | Dj_t, y_t, e_t, i_{t-1}, i_{t-2}) = \beta'_0(\tau) + \beta'_1(\tau) Dj_t + \beta'_2(\tau) y_t + \beta'_3(\tau) e_t + \theta_1(\tau) i_{t-1} + \theta_2(\tau) i_{t-2} \quad (23)$$

The results of specifications (20)-(23) are shown in Table 7. For both specifications, the responses of interest rates were similar to those obtained previously without including the exchange rate. The short-term response of the Selic rate to the current inflation gap was not different from zero for the rule estimated in the conditional mean, but was increasing and, in general, significant at the upper quantiles of the conditional distribution. Conversely, the coefficient that measures the short-term response to output gap was significant in the conditional mean, as well as at almost all quantiles of the Selic rate distribution. Results also reveal that the CBB has a positive response to real exchange rate both in the conditional mean and along the interest rate distribution. This is consistent with the evidence provided by Soares & Barbosa (2006), who found a positive response of the Selic rate to real exchange rate, and by Palma & Portugal (2014), who show that the CBB has given a positive weight to real exchange rate in its loss function. Finally, results indicate that the response to real exchange rate is usually stronger in the upper tail of the conditional Selic rate distribution for both specifications.

²⁵ We used the series (no. 11752) of the effective real exchange rate - IPCA provided by the CBB.

²⁶ Note that, in new Keynesian models for a small open economy, output gap and consumer inflation depend on the current and/or expected movement of the real exchange rate (see, for instance, Areosa & Medeiros, 2007; Leitimo & Söderström, 2008; Divino, 2009). Here, we included only the current exchange rate in the Phillips and IS curves to make it easier to obtain the optimal monetary rule.

²⁷ We followed some studies and adjusted the external (exogenous) variables to zero (see, for instance, Bonomo & Brito, 2002; Leitimo & Söderström, 2008).

Table 7 – IVQR estimates for the reaction function with the exchange rate

| Quantile | β_1 | β_2 | β_3 | θ_1 | θ_2 | β_1 | β_2 |
|------------|------------------------------|---------------------|---------------------|---------------------|----------------------|-------------------|--------------------|
| | Specifications (20) and (21) | | | | | | |
| <i>VI</i> | 0.005 (0.021) | 0.070*** (0.015) | 0.021*** (0.006) | 1.645*** (0.080) | -0.656*** (0.080) | 0.421 (1.638) | 5.724 (3.461) |
| <i>GMM</i> | 0.014 (0.023) | 0.068*** (0.015) | 0.020*** (0.005) | 1.609*** (0.088) | -0.620*** (0.088) | 1.181 (1.790) | 5.931* (3.562) |
| 0.05 | -0.063 (0.033) | 0.045 (0.028) | 0.021** (0.034) | 1.653*** (0.116) | -0.684*** (0.144) | -2.042 (1.711) | 1.464 (1.011) |
| 0.1 | -0.069 (0.034) | 0.063*** (0.021) | 0.021** (0.028) | 1.678*** (0.128) | -0.701*** (0.128) | -2.977 (2.222) | 2.731* (1.624) |
| 0.2 | -0.020 (0.035) | 0.042** (0.021) | 0.011 (0.029) | 1.701*** (0.160) | -0.718*** (0.161) | -1.100 (1.977) | 2.234 (1.744) |
| 0.3 | -0.004 (0.024) | 0.039** (0.020) | 0.007 (0.026) | 1.706*** (0.138) | -0.722*** (0.138) | -0.227 (1.539) | 2.489 (1.967) |
| 0.4 | 0.029 (0.021) | 0.052*** (0.017) | 0.013** (0.023) | 1.619*** (0.111) | -0.631*** (0.111) | 2.492 (2.120) | 4.485 (3.322) |
| 0.5 | 0.034** (0.017) | 0.053*** (0.017) | 0.013*** (0.021) | 1.647*** (0.084) | -0.659*** (0.084) | 2.899 (1.767) | 4.564 (2.983) |
| 0.6 | 0.040** (0.017) | 0.054*** (0.019) | 0.012** (0.023) | 1.639*** (0.090) | -0.650*** (0.090) | 3.656* (2.072) | 4.918 (3.574) |
| 0.7 | 0.041* (0.023) | 0.074*** (0.022) | 0.020*** (0.027) | 1.527*** (0.110) | -0.539*** (0.110) | 3.315 (2.073) | 6.051 (4.741) |
| 0.8 | 0.038 (0.032) | 0.079*** (0.026) | 0.028*** (0.033) | 1.496*** (0.116) | -0.501*** (0.114) | 8.264 (15.48) | 17.41 (40.73) |
| 0.9 | 0.062* (0.036) | 0.087*** (0.032) | 0.035*** (0.043) | 1.392*** (0.153) | -0.405*** (0.150) | 4.753 (4.494) | 6.679 (7.757) |
| 0.95 | 0.103** (0.045) | 0.149 (0.100) | 0.053 (0.155) | 1.147*** (0.313) | -0.166 (0.326) | 5.482 (10.03) | 7.914 (19.55) |
| | Specifications (22) and (23) | | | | | | |
| <i>VI</i> | 0.052 (0.034) | 0.068*** (0.014) | 0.019*** (0.005) | 1.597*** (0.084) | -0.611*** (0.087) | 3.754 (2.992) | 4.928* (2.820) |
| <i>GMM</i> | 0.055* (0.033) | 0.070*** (0.014) | 0.020*** (0.005) | 1.578*** (0.083) | -0.590*** (0.085) | 4.539 (3.489) | 5.843* (3.463) |
| 0.05 | -0.003 (0.103) | 0.057* (0.034) | 0.011 (0.014) | 1.641*** (0.142) | -0.684*** (0.144) | -1.468 (3.476) | 1.538* (0.917) |
| 0.1 | -0.039 (0.095) | 0.072** (0.028) | 0.020 (0.012) | 1.628*** (0.148) | -0.677*** (0.152) | -0.640 (2.735) | 1.523** (0.763) |
| 0.2 | 0.011 (0.075) | 0.042* (0.023) | 0.009 (0.010) | 1.658*** (0.165) | -0.678*** (0.169) | 1.388 (5.138) | 1.963 (1.852) |
| 0.3 | -0.033 (0.050) | 0.058*** (0.019) | 0.012* (0.007) | 1.632*** (0.138) | -0.646*** (0.139) | 6.154 (6.182) | 4.426 (3.155) |
| 0.4 | 0.081** (0.041) | 0.056*** (0.018) | 0.010* (0.006) | 1.553*** (0.111) | -0.563*** (0.111) | 8.777 (8.818) | 5.708 (5.256) |
| 0.5 | 0.081** (0.032) | 0.055*** (0.016) | 0.011** (0.005) | 1.583*** (0.085) | -0.587*** (0.085) | 16.50 (16.83) | 7.755 (9.371) |
| 0.6 | 0.096*** (0.030) | 0.050*** (0.016) | 0.011** (0.005) | 1.595*** (0.080) | -0.600*** (0.080) | 16.01 (14.73) | 6.967 (8.323) |
| 0.7 | 0.082** (0.037) | 0.045** (0.019) | 0.017** (0.007) | 1.578*** (0.095) | -0.586*** (0.095) | 23.62 (39.45) | 14.44 (28.35) |
| 0.8 | 0.083 (0.054) | 0.048* (0.026) | 0.017 (0.010) | 1.440*** (0.119) | -0.447*** (0.118) | 31.94 (66.63) | 20.49 (49.76) |
| 0.9 | 0.083 (0.070) | 0.082** (0.032) | 0.034** (0.016) | 1.411*** (0.159) | -0.408*** (0.158) | -24.98 (113.5) | -25.56 (100.4) |
| 0.95 | 0.033 (0.090) | 0.110** (0.049) | 0.049* (0.027) | 1.368*** (0.248) | -0.358 (0.252) | -16.03 (91.82) | -24.69 (118.9) |

Note: *** Significant at 1%. ** Significant at 5%. * Significant at 10%.

5 Conclusions

In this paper, we sought to assess nonlinearities in the CBB's reaction function by using quantile regression. As the monetary policy rule has endogenous regressors, we followed the procedures suggested by Wolters (2012) and the inverse quantile regression (IVQR) method proposed by Chernozhukov & Hansen (2005, 2006) to estimate the CBB's quantile reaction function parameters for the inflation-targeting regime. This method allowed us to detect nonlinearities in the CBB's reaction function without having to make specific assumptions about the causal factors that underlie these nonlinearities.

The conditional mean results indicate an insignificant response of the Selic rate to the current inflation gap, but an otherwise positive one to the deviation of expected inflation from inflation targets. We also noted that the Selic rate reacted to output gap movements, and the smoothing of this policy instrument was around 0.98.

The quantile regression results show that the CBB's short-term response to current inflation was significant and increasing between quantiles 0.5 and 0.9. In turn, the short-term response of the Selic rate to output gap increased from quantile 0.2 to quantile 0.7 and was not statistically different from zero at the extreme quantiles of the conditional interest rate distribution. We also observed that the short-term response of the Selic rate to the expected inflation gap was significant from quantile 0.4 of the CBB's reaction function, exhibiting an uptrend. Concerning the long-term response, results suggest the reactions of the Selic rate to current and expected inflation were, in general, stronger when the interest rate was above its median. On the other hand, the long-term response to output gap was significant only at some quantiles on the interval [0.05, 0.7]. This suggests the CBB does not react to demand pressures when the interest rate is too high. When we included real exchange rate as a regressor for the interest rate rule, the CBB had a positive reaction to the real exchange rate both in the conditional mean and along the interest rate distribution. Moreover, results show the reaction to the real exchange rate was, in general, stronger in the upper tail of the conditional Selic rate distribution.

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