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Assessing Brazilian Macroeconomic Dynamics Using a Markov-Switching DSGE Model

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Resumo

O objetivo deste trabalho é avaliar o comportamento dos principais parâmetros da economia brasileira através da estimação de um modelo DSGE (*Dynamic Stochastic General Equilibrium*) de economia aberta usando métodos bayesianos e permitindo mudanças de regime markovianas de determinados parâmetros. Utilizando o modelo DSGE desenvolvido por Justiniano e Preston (2010) e o método de solução do modelo Markov *Switching* DSGE (MS-DSGE) proposto por Farmer *et al.* (2008), este trabalho encontrou superioridade nos ajustes dos dados dos modelos que incorporaram mudanças markovianas, rejeitando a hipótese de parâmetros constantes em modelos DSGE para a economia brasileira.

Palavras-chave: Modelo DSGE. Markov *Switching*. MS-DSGE.

Abstract

The goal of this paper is to evaluate the behavior of the main parameters of the Brazilian economy through the estimation of an open-economy dynamic stochastic general equilibrium (DSGE) model using Bayesian methods and allowing for Markov switching of certain parameters. Using the DSGE model developed by Justiniano & Preston (2010) and the solution method of the Markov switching DSGE (MS-DSGE) model proposed by Farmer *et al.* (2008), this paper found a superior fit in the data of Markov switching models, rejecting the hypothesis of constant parameters in DSGE models for the Brazilian economy.

Keywords: DSGE Model. Markov Switching. MS-DSGE.

JEL: C51, E32, C11
Area of Research: Applied Microeconomics

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1 Introduction

This paper assesses the behavior of the major Brazilian economic parameters after the Real Plan using the Markov switching-dynamic stochastic general equilibrium (MS-DSGE) model.

Dynamic stochastic general equilibrium (DSGE) models have become a standard tool for macroeconomic analysis. The advancements made since the first real business cycle approach have provided models with an increasingly higher capacity of capturing the characteristics of macroeconomic series. Theoretically, new features have been added to DSGE models, such as currency, international trade, real and nominal price rigidity, wages, and several shocks. This has enabled a more in-depth analysis of the relationships between aggregate variables and the effects of economic policies.

In practice, the theoretical and empirical progress of DSGE models has aroused the interest of central banks, as monetary authorities need a tool upon which policy decisions can be hinged. Fukac & Pagan (2006) described the historical path of DSGE models and highlighted their use mainly by central banks in developed countries.

Studies conducted for Brazil on DSGE model estimations suffer from a major drawback: the assumption that Brazilian economic parameters are constant.[1]

Nonetheless, it is known that parameters related to the central bank's reactions to key macroeconomic variables, such as inflation, output, or exchange rate, for example, may oscillate over time. To follow these movements closely, one can use Markov switching models to check for possible changes in the parameters of interest. However, the use of Markov models is restricted to reduced-form structural models.

Therefore, the combination of both approaches – DSGE and Markov switching models – best known as Markov switching DSGE (MS-DSGE) models, blazes a trail in the analysis of macroeconomic models, as it contemplates parameter changes over time in a more complex model.

In the international literature, several works deal with MS-DSGE models and with the solutions of Markov switching rational expectations (MSRE)[2] models. The debate was sparked off after uncertainties about the parameters of microfounded models came up and eventually evolved by the introduction of Markov switching into DSGE models.

The main differences between studies on MS-DSGE models lie in the parameters believed to vary according to the Markov process, in the basic theoretical model and in the solution method.

Initially, studies considered Markov regime shifts only in volatility shocks (JUSTINIANO; PRIMICERI, 2008). Later, there was avid interest in monetary policy parameters such as inflation target (Schorfheide, 2005; Ireland, 2007; Liu *et al*., 2010), or in Taylor rule parameters (Bianchi, 2013; Foerster, 2013). Besides the parameters mentioned earlier, other ones from DSGE models with variation in the Markov switching regime (MS-DSGE models) were analyzed, such as technological growth rate and nominal price rigidity, Phillips curve parameters such as indexation rate or exchange rate pass-through effect or only the price rigidity parameter.[3]

In regard to the theoretical models that allow certain parameters to vary according to the Markov regime, we highlight those DSGE models proposed by Lubik

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[1] See, for instance, Silveira (2008), Carvalho & Valli, (2010), Vereda & Cavalcanti (2010) and Castro *et al.* (2011).

[2] See, for example, David & Leeper (2007), Farmer *et al.* (2008), Liu *et al.* (2011), Eo (2009), Cho (2011), Liu & Mumtaz (2011) and Bianchi (2013).

[3] See, for instance, Eo (2009), Liu & Mumtaz (2011) and Chen & Macdonald (2012).

& Schorfheide (2004), Christiano, Eichenbaum & Evans (2005), Gali & Monacelli (2005), An & Schorfheide (2007), Smets & Wouters (2007) and Justiniano & Preston (2010), among others.

With respect to MS-DSGE solution models, studies have basically used the method put forward by Farmer *et al.* (2008) or that suggested by David & Leeper (2007), or a variant of these. In addition, most of these studies have employed Bayesian estimation methods. Recently, Foerster *et al.* (2013) have proposed a new method that utilizes perturbations to make approximations to MS-DSGE solutions.

In general, results have been positive. Liu *et al.* (2011) tested several models with different numbers of regimes for the U.S. economy and found that Markov regime switching models outperform those with constant parameters. Moreover, results were better in the presence of two regime shifts. Liu & Mumtaz (2011) estimated the first open-economy MS-DSGE model for the UK and their results showed the presence of large parameter changes.

This paper contributes towards the discussion about parameter changes in the Brazilian economy and is the first, to our knowledge, to use the DSGE model with parameter changes in Markov switching regimes. Additionally, the introduction of regime shifts is related to agents' behavior, as agents are aware of the possible regime shifts and as this information is taken into account in their expectations. Thus, the law of motion of the variables of interest depends not only on microfounded parameters, but also on the beliefs about alternative regimes (Bianchi, 2013).

Based on the theoretical model introduced by Liu & Mumtaz (2011) which utilizes the open-economy model proposed by Justiniano & Preston (2010) and by adopting the method formulated by Farmer *et al.* (2008), this paper employs regime shifts in certain parameters, such as those of the monetary policy rule, of inflation persistence, and of volatility shocks on the Brazilian economy after the Real Plan, between 1996 and 2012. Presumably, these parameters were not constant over the analyzed period, given the changes in the Brazilian economy, such as the adoption of the inflation targeting system, replacement of the Central Bank of Brazil's president, and the swearing-in of the new Brazilian president.

Through the estimation of four models: time-invariant; regime shifts in volatility only; regime shifts in the Phillips curve and in volatility parameters; and regime shifts in the Taylor rule and volatility parameters, this paper demonstrates that regime switching models were superior to the time-invariant model. In particular, the model that contemplates Markov regimes in the monetary policy rule and in exogenous shock volatilities showed the best fit. Hence, the hypothesis of constant parameters for the Brazilian economy in the analyzed period was rejected.

Aside from the introduction, this paper is organized as follows. Section 2 introduces the open-economy DSGE model with the inclusion of Markov regimes. Section 3 deals with the MS-DSGE model solution and estimation methods. Section 4 analyzes the results, and Section 5 then concludes.

2 The MS-DSGE model

Based on Gali & Monacelli (2005) and Monacelli (2005), Justiniano & Preston (2010) introduced important features into DSGE models for a small open economy, such as incomplete asset market, habit formation, and price indexation to past inflation.

The empirical literature uses log-linear approximation of the model's optimality conditions around a non-stochastic steady state. In what follows, we present the equations pertaining to this analysis. All variables are construed as the log of deviations from the respective steady state values. The model proposed by Justiniano & Preston (2010) is shown next, according to the work of Liu and Mumtaz (2011).

The log-linear Euler equation, obtained from the households' intertemporal maximization problem, is expressed by:

$$(1+h)c_t = hc_{t-1} + E_t c_{t+1} - \frac{1-h}{\sigma}(r_t - E_t\pi_{t+1}) + \frac{1-h}{\sigma}\left(\epsilon_{g,t} - \rho_g \epsilon_{g,t}\right) \tag{1}$$

where the log of current consumption (c_t) depends on consumption at $t-1$, on the expected future consumption ($E_t c_{t+1}$) and on the real interest rate ($r_t - E_t\pi_{t+1}$). Parameter h is the degree of habit persistence, σ is the inverse of the intertemporal elasticity of substitution and $\epsilon_{g,t}$ is the preference shock. The log-linear approximation of the commodity market equilibrium condition is given by:

$$y_t = (1-\alpha)c_t + \alpha[\eta(s_t + q_t) + y_t^*] \tag{2}$$

where y_t is the domestic output, s_t denotes the terms of trade, q_t is the real exchange rate and y_t^* is the foreing output. Equation 15 shows that the domestic output is the sum of domestic consumption and exports, while parameter α represents the level of economic openness and η is the elasticity of substitution between domestic and imported goods. The law of one price is given by:

$$\psi_{F,t} = q_t - (1-\alpha)s_t \tag{3}$$

where $\psi_{F,t}$ is the deviation from the law of one price. The terms of trade are given by $s_t = p_{F,t} - p_{H,t}$. This implies:

$$s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t} \tag{4}$$

where $\pi_{F,t}$ is the imported inflation and $\pi_{H,t}$ is the domestic inflation. Thus, steady-state domestic consumption depends on domestic output and on three sources of external disturbances: the terms of trade, deviation from the law of one price, and foreign output.

The relationship between the real exchange rate and the terms of trade can be expressed by the equation:

$$q_t = e_t + p_t^* - p_t = \psi_{F,t} + (1-\alpha)s_t \quad \text{or}$$
$$\Delta e_t = q_t - q_{t-1} + \pi_t - \pi_t^* \tag{5}$$

The Phillips curve for domestic inflation is given by the following equation:

$$(1+\beta\delta_H)\pi_{H,t} = \delta_H \pi_{H,t-1} + \beta E_t \pi_{H,t+1} + \frac{(1-\theta_H)(1-\theta_H\beta)}{\theta_H} mc_t \tag{6}$$

where $mc_t = \varphi y_t - (1+\varphi)\epsilon_{a,t} + \alpha s_t + \sigma(1-h)^{-1}(c_t - hc_{t-1})$ is the real marginal cost function of each firm, $\epsilon_{a,t}$ is the technology shock, β is the intertemporal discount rate, δ_H measures the indexation rate, θ_H is the fraction of firms that do not adjust their prices every period and φ is the inverse of the elasticity of labor supply. Therefore, domestic inflation depends on past inflation ($\pi_{H,t-1}$) and on its expectation ($E_t\pi_{H,t+1}$) for the subsequent period, and on the current marginal cost (mc_t).

The Phillips curve for imported inflation is given by:

$$(1+\beta\delta_F)\pi_{F,t} = \delta_F\pi_{F,t-1} + \beta E_t\pi_{F,t+1} + \frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F}\psi_{F,t} + \epsilon_{cp,t} \quad (7)$$

Similarly to Eq. 6, Eq. 7 incorporates the deviations of the law of one price (Eq. 3) for imported goods, given the hypothesis that import retailers engage in monopolistic competition. Furthermore, an exogenous cost-push shock ($\epsilon_{cp,t}$) that captures inefficient variations in mark-ups is taken into account.

Current inflation relates to domestic and imported inflation as follows:

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} \quad (8)$$

The uncovered interest rate parity condition is given by Eq. 9:

$$E_t(q_{t+1} - q_{t+1}) = (r_t - E_t\pi_t) - (r_t^* - E_t\pi_{t+1}^*) + \chi a_t - \epsilon_{\phi,t} \quad (9)$$

where a_t is the level of foreign assets, χ is the elasticity of debt relative to the interest rate premium and $\epsilon_{\phi,t}$ is the risk premium shock. The flow of budgetary constraint of assets can be represented by

$$c_t + a_t = \frac{1}{\beta}a_{t-1} - \alpha(q_t + \alpha s_t) + y_t \quad (10)$$

Finally, monetary policy is assumed to be expressed by a Taylor rule. Through the interest rate, the central bank reacts to movements in inflation rate and in output. Additionally, the central bank can react to nominal exchange rate depreciation. Thus, the interest rate rule is given by:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)[\lambda_1\pi_t + \lambda_2 y_t + \lambda_3\Delta e_t] + \sigma_m\epsilon_{m,t} \quad (11)$$

Equation 11 demonstrates that the nominal interest rate reacts to the past inflation rate, to current inflation, to output, to nominal exchange rate movement, and to an interest rate shock or, in general, to a monetary policy shock ($\epsilon_{m,t}$). Parameter $0 < \rho_r < 1$ represents the degree of interest rate smoothing, and λ_1, λ_2 and $\lambda_3 \geq 0$ are the inflation reaction, output, and exchange rate movement coefficients, respectively. Shock $\epsilon_{m,t}$ can be interpreted as a non-systematic component of the monetary policy.

Therefore, the model contains 22 variables (X_t) including four terms of expectation. The exogenous processes of model (Z_t) are: preference shock ($\epsilon_{g,t}$); technology shock ($\epsilon_{a,t}$); cost-push shock ($\epsilon_{cp,t}$); risk premium shock ($\epsilon_{\phi,t}$); monetary policy shock ($\epsilon_{m,t}$); foreign output shock ($\epsilon_{y^*,t}$); foreign inflation shock ($\epsilon_{\pi^*,t}$); and foreign interest rate shock ($\epsilon_{r^*,t}$). All disturbances are assumed to be independent AR (1) processes, except for ($\epsilon_{m,t}$) which follows an i.i.d. process. Hence, we have:

$$\begin{aligned} \epsilon_{j,t} &= \rho_j\epsilon_{j,t-1} + \sigma_j\eta_{j,t} \text{ for } j = g, a, cp\ e\ \phi \\ v_t &= \rho_v v_{j,t-1} + \sigma_v\eta_{v,t} \text{ for } v = y^*, \pi^*\ e\ r^* \end{aligned} \quad (12)$$

2.1 Matrix form

Rewriting the DSGE model in matrix form, we get:

$$\Gamma_0 X_{t+1} = \Gamma_1 X_t + \Psi Z_t + \Pi \eta_t \tag{13}$$

where X is the $n \times 1$ vector of the endogenous variables, Z is the vector of exogenous processes ($k \times 1$) and η_t are disturbances ($\ell \times 1$). Γ_0, Γ_1, Ψ and Π are matrices with the model's parameters.

The representation of the model in Eq. 13, with all parameters kept constant, allows solving it with rational expectations algorithms, such as the Gensys solution method proposed by Sims (2001). Note that the solution will be determinate (unique solution), indeterminate (multiple solutions) or explosive (no solution) given some conditions on matrices Γ_0, Γ_1, Ψ and Π. The existence of a single solution requires that endogenous shock η_t be adjusted every period in order to maintain the system in a linear subspace so that solutions remain bounded, and that depends upon the properties of matrices Ψ and Π, as well as upon the generalized eigenvalues of matrices Γ_0 and Γ_1. This method yields the following unique solution:

$$X_t = G(\Phi) X_{t-1} + A Z_t \tag{14}$$

where Φ represents the model's parameters. Combining Eq. 14 with an observation equation denoted as

$$Y_t = H X_t \tag{15}$$

where Y_t is a vector containing observed data and H is the loading matrix. In this case, the Kalman filter algorithm can be used to assess the likelihood function and to estimate the model's parameters. However, there is some interest in letting certain parameters vary over time under the Markov regime.

2.2 Introducing Markov regimes

As shown by Liu & Mumtaz (2011), for the specification of the MS-DSGE model, the vector of parameters Φ is split into three blocks:

$$\Phi = \{\Phi^S; \Sigma^s; \bar{\Phi}\},$$

where Φ^S is the block of parameters subject to regime shifts, Σ^s is the block of variances in regime-switching volatilities and $\bar{\Phi}$ contains the time-invariant parameters. Superscripts S and s represent two state variables. Superscript S denotes the unobserved regime associated with the parameters subject to regime shift and takes on discrete values $S = 1, 2$. Superscript s is associated with volatilities and also assumes discrete values $s = 1, 2$ and is independent from S.

Both S and s are assumed to follow a first-order Markov chain with the following transition matrix:

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \text{ and } Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$$

where $P_{ij} = p(S_t = j|S_{t-1} = i)$ and $Q_{ij} = p(s_t = j|s_{t-1} = i)$. P_{ij} stands for the probability of being in regime j at t given that one was in regime i in the previous period. The analysis of Q_{ij} is the same, but for volatilities instead.

The MS-DSGE model for regime S can be rewritten as follows:

$$\begin{pmatrix}\Gamma_{0,1}^S \\ \Gamma_{0,2}\end{pmatrix} X_{t+1} = \begin{pmatrix}\Gamma_{1,1}^S \\ \Gamma_{1,2}\end{pmatrix} X_t + \begin{pmatrix}\Psi_1^S \\ 0\end{pmatrix} Z_t + \begin{pmatrix}0 \\ \Pi\end{pmatrix} \eta_t \tag{16}$$

As an example, Eq. 11, which presents the Taylor rule under regimes S and s, is given by:

$$r_{St} = \rho_{rS} r_{t-1} + (1 - \rho_{rS})[\lambda_{S1}\pi_t + \lambda_{S2} y_t + \lambda_{S3}\Delta e_t] + \sigma_{ms}\epsilon_{ms,t} \tag{11'}$$

Another example is the Euler equation for consumption under regime s:

$$(1 + h)c_t = hc_{t-1} + E_t c_{t+1} - \frac{1-h}{\sigma}(i_t - E_t\pi_{t+1}) + \frac{1-h}{\sigma}\left(\epsilon_{gs,t} - \rho_{gs}\epsilon_{gs,t}\right) \tag{1'}$$

in which $\epsilon_{gs,t} = \rho_{gs}\epsilon_{gs,t-1} + \sigma_{gs}\eta_{gs,t}$ where $s = 1,2$, $\sigma_{g1} > \sigma_{g2}$.

3 MS-DSGE Model Solution and Estimation Procedures

Farmer *et al.* (2008) proposed a method to solve the model represented in Eq. 16. The option for this method served a dual purpose: computational efficiency, i.e., in general, the algorithm converges quickly; and presence of necessary conditions for the existence of a solution.[4] The solution procedure proposed allows rewriting the MS-DSGE model as a fixed-parameter model in an extended state vector:

$$\bar{\Gamma}_0 \bar{X}_{t+1} = \bar{\Gamma}_1 \bar{X}_t + \bar{\Upsilon} u_t + \bar{\Pi}\eta_t \tag{17}$$

where

$$\bar{\Gamma}_0 = \begin{pmatrix} diag(\Gamma_{0,1}^1, \Gamma_{0,1}^2) \\ \Gamma_{0,2} \\ \Phi \end{pmatrix}, \bar{\Gamma}_1 = \begin{pmatrix} diag(\Gamma_{1,1}^1, \Gamma_{1,1}^2) \\ \Gamma_{1,2} \\ 0 \end{pmatrix}, \bar{\Upsilon} = \begin{pmatrix} I & diag(\Psi_1^1, \Psi_1^2) \\ 0 & 0 \end{pmatrix},$$

$$\bar{\Pi} = \begin{pmatrix} 0 \\ \Pi \\ 0 \end{pmatrix}, \Phi = e_2 \otimes \Phi_{S=2}$$

The matrices of parameters $\bar{\Gamma}_0$, $\bar{\Gamma}_1$, $\bar{\Upsilon}$ and $\bar{\Pi}$ are functions of the parameters and of the transition probabilities. The extended state vector $\bar{X}_t$ is defined as:

$$\bar{X}_t = \begin{pmatrix} X_t^{S=1} \\ X_t^{S=2} \end{pmatrix} \tag{18}$$

Shocks u_t are:

[4] For alternative methods, see David & Leeper (2007), Svensson & Williams (2007) and Bikbov (2013).

$$u_t = \begin{pmatrix} \Xi^S(e_{S_{t-1}} \otimes (1 \otimes I))\bar{X}_{t-1} \\ e_{S_t} \otimes Z_t \end{pmatrix} \tag{19}$$

where $\Xi^S = (diag[\Gamma_{1,1}^1, \Gamma_{1,1}^2]) \times ((e_i 1' - P) \otimes I)$, $e_i, i = 1,2$ is the ith column of the identity matrix. According to Farmer *et al.* (2008), Eq. 19 contains two types of shocks. The first block, represented by the first element of u_t, includes the Markov-switching shocks. The second block, second element of u_t, contains the shocks to structural equations. Moreover, the authors mentioned above showed that both types of shocks have zero mean.

The solution to the Markov switching model, defined by Farmer *et al.* (2008), is represented as a stochastic process $\{x_t, \eta_t\}_{t=1}^{\infty}$such that:
1. η_t satisfies property $E_{t-1}(\eta_t) = 0$;
2. x_t is bounded in expectation $\|E_t(x_{t+s})\| < M_t$ for all $s > 0$;
3. $\{x_t, \eta_t\}$ satisfies Eq. 17.

These conditions can be satisfied by a broad set of fundamental and non-fundamental solutions (also known as *sunspot* or *bubbl*e). Nevertheless, Farmer *et al.* (2008) focus on fundamental solutions or, following the nomenclature used by the authors, minimal state variable (MSV) solutions which, unlike fundamental solutions, have a finite number and rely only on state variables (Cho & Moreno, 2010). In addition, the authors proved that the MSV solution to the system represented in Eq. 17 solves the model from Eq. 16, i.e., the MSV solution of the fixed-parameter model with an extended state vector is also the solution to the MS-DSGE model.

In this case, matrix Φ plays a crucial role in the determination of the MSV solution. To ensure the stochastic process $\{x_t, \eta_t\}_{t=1}^{\infty}$ is bounded, the solution should be in a linear subspace. For that to occur, Farmer *et al.* (2008) define a matrix z such that $z' X_t = 0$. This way, matrix z guarantees that constraints are imposed on $X_t^{S=1}$ in the case of regime 1, whereas the definition of Eq. 17 makes sure that $X_t^{S=2} = 0$ in the case of regime 1.

The definition of matrix Φ for imposition of the constraints occurs as follows: first, an initial value is set for $\Phi_{S=2}(\Phi_{S=2}^0)$ and matrices $\bar{\Gamma}_0^0$ and $\bar{\Gamma}_1$ are calculated, where the superscript refers to the iteration steps; second, the decomposition QZ of $\{\bar{\Gamma}_0^0, \bar{\Gamma}_1\}$: $Q^0 T^0 Z^0 = \bar{\Gamma}_1$ and $Q^0 S^0 Z^0 = \bar{\Gamma}_0^0$ is computed; third, matrices T^0 and S^0 are ordered such that the number of elements on the diagonal (T_{ii}^0 / S_{ii}^0) is rearranged in increasing order, in this case, $T_{ii}^0 / S_{ii}^0 < 1$ for $1 < q$ and $T_{ii}^0 / S_{ii}^0 > 1$ when $1 > q$, where q is a whole number; fourth, if z_u is formed by the last $np - p$ rows of $Z^0 = \{z_1, z_2\}$, then $\Phi_{S=2}^1 = z_2$ which is reiterated until it converges.

If the iteration process converges, it means that the solution to Eq. 17 is the solution to the original model – Eq. 16. *Gensys* is applied to verify the existence and unicity and also to compute the solution to Eq. 16. If a single solution exists, this solution can be rewritten in the form of a Markov switching VAR (MS-VAR):

$$X_t = G^S X_{t-1} + A^S Z_t \tag{20}$$

Combining Eq. 20 with an observation equation (Eq. 15), we have a state space model with Markov switching:

$$\begin{aligned} X_t &= G^S X_{t-1} + A^S Z_t \\ Y_t &= H X_t \end{aligned} \tag{21}$$

where $Z_t \sim N(0, Q^S)$ and Markov states S and s are independent with transition matrices P and Q, respectively.

Therefore, the solution method proposed by Farmer *et al.* (2008) allows rewriting the MS-DSGE model as a fixed-parameter model in an extended state vector, as shown in Eq. 17. The MSV solution not only satisfies the definition of the solution to the models, but also can be rewritten as an MS-VAR. Combining this solution to an observation equation, we have a state space model with Markov switching, expressed by Eq. 21. In what follows, the estimation of this model is discussed.

The state space model with Markov switching contains unobserved states X_t and also unobserved Markov states. The presence of these two sets of unobservable variables implies that the standard Kalman filter cannot be applied, as it will not be possible to make inference $X_t$and to calculate transition probabilities at the same time. However, with unobserved Markov states, the inference can be conditioned on the current and past values of S and s. As pointed out by Kim & Nelson (1999), each iteration of the filter implies that the number of cases increases in M, where M stands for the number of regimes. This makes the problem with finding the solution to the model computationally intensive.

That being said, Kim & Nelson (1999) proposed an approximation to make the filter more operational. This approximation causes a limited number of states to be taken along iterations in each period and to be "collapsed" at the end of each iteration. To apply the approximation, a new state variable is defined, S_t^*, which indexes both S_t and s_t and whose transition matrix is given by $P^* = P \otimes Q$, where $\otimes$ represents the Kronecker product. According to Kim and Nelson (1999), S_t^*, S_{t-1}^* and S_{t-2}^* can be traced out, which implies the existence of $4^3 = 64$ possible paths for the state variables in each time period. Intuitively, Kim & Nelson's (1999) algorithm runs the Kalman filter for each one of the paths and, thereafter, a weighted average is obtained using the weights given by the probabilities of each path.

The Bayesian approach is used to estimate the model following two steps. The first one combines the likelihood function, obtained from Kim & Nelson's (1999) algorithm, with prior distribution for the parameters. A combination of numerical maximizers is used to calculate the approximate posterior mode. In this case, the initial values are refined by the simplex algorithm and employed in the CSMINWEL optimization routine, proposed by Christopher Sims. The posterior mode is used as initial value for the Metropolis Hastings algorithm with 100,000 iterations. The second step consists in utilizing the mean and variance of the last 1,000 iterations from the first step to run the main Metropolis Hastings algorithm. This step consists of 200,000 iterations.[5]

The models to be estimated have the following specifications:

- Model 0: Rational expectations with no regime switching;
- Model 1: Rational expectations with two regime shifts in the volatility of the exogenous shocks;
- Model 2: Rational expectations with two regime shifts in the volatility of the exogenous shocks and in parameters δ_H (indexation rate) and θ_H (fraction of firms that do not alter their prices) of the Phillips curve (Eq. 6);

[5] The computer routine were kindly provided by Philip Liu. The codes can be obtained at caio.soares@hotmail.com.br.

- Model 3: Rational expectations with two regime shifts in the volatility of the exogenous shocks and in the parameters of the Taylor rule equation for open economies (Eq. 11).

The prior distribution of the model's parameters was determined based on studies available in the Brazilian literature, such as in Kanczuk (2002), Araújo *et al.* (2006), Silveira (2008), Furlani *et al.* (2010), Palma & Portugal (2011) and Castro *et al.* (2011). Table 1 shows the distribution for each parameter, in addition to the mean, standard deviation, and bounds . The economic openness parameter (α) was set at 0.23 according to the volume of exports/imports vis-à-vis the domestic output in the analyzed period. Given that the discount factor is calibrated in such a way that the value assumed by the average real long-term interest rate is equal to $\frac{1}{\beta} - 1$, with an average interest rate of 10.8% p.a., the intertemporal discount rate (β) was calibrated at 0.975.

Table 1: Prior distribution of parameters

	Parameters	Distribution	Mean	Standard Deviation	Lower bound	Upper bond
β	Intertemporal discount rate	-	0.975	-	-	-
α	Level of economic openness	-	0.23	-	-	-
σ	Inverse of the intertemporal elasticity of substitution	Gamma	1.2	0.2	0	10
φ	Inverse of the elasticity of labor supply	Gamma	2	0.35	0	10
θ^H	Fraction of non-optmizing producers	Beta	0.5	0.2	0	1
θ^F	Fraction of non-optmizing importers	Beta	0.5	0.2	0	1
η	Elasticity of substitution between domestic and imported goods	Gamma	0.6	0.25	0	10
h	Habit persistence parameter	Beta	0.7	0.1	0	1
δ^H	Backward-looking price-setting of domestic goods	Beta	0.7	0.2	0	1
δ^F	Backward-looking price-setting of imported goods	Beta	0.7	0.2	0	1
ρ_r	Degree of interest rate smoothing	Beta	0.6	0.15	0	1
λ_1	Inflation reaction coefficient	Gamma	0.5	0.09	0	10
λ_2	Output reaction coefficient	Gamma	0.5	0.09	0	10
λ_3	Exchange rate reaction coefficient	Gamma	0.5	0.09	0	10
χ	Elasticity of debt relative to the interest rate risk premium	Gamma	0.01	0.02	0	10
ρ_a	AR(1) parameter of technology shock	Beta	0.5	0.25	0	1
ρ_g	AR(1) parameter of preference shock	Beta	0.5	0.25	0	1
ρ_ϕ	AR(1) parameter of risk premium shock	Beta	0.5	0.25	0	1
ρ_{cp}	AR(1) parameter of cost-push shock	Beta	0.5	0.25	0	1
ρ_{π^*}	AR(1) parameter of foreign inflation shock	Beta	0.5	0.25	0	1
ρ_{y^*}	AR(1) parameter of foreign output shock	Beta	0.5	0.25	0	1
ρ_{i^*}	AR(1) parameter of foreign interest rate shock	Beta	0.5	0.25	0	1
σ_{π^*}	Standard deviation of foreign inflation shock	Inverse gamma	0.5	10	0	10
σ_{y^*}	Standard deviation of foreign output shock	Inverse gamma	0.5	10	0	10
σ_{i^*}	Standard deviation of foreign interest rate shock	Inverse gamma	0.5	10	0	10
σ_a	Standard deviation of technology shock	Inverse gamma	0.5	10	0	10
σ_{m_i}	Standard deviation of monetary policy shock	Inverse gamma	0.5	10	0	10
σ_g	Standard deviation of preference shock	Inverse gamma	0.5	10	0	10
σ_ϕ	Standard deviation of risk premium shock	Inverse gamma	0.5	10	0	10
σ_{cp}	Standard deviation of cost push shock	Inverse gamma	0.5	10	0	10
P_{11}	Probability of parameters $P_{11} = p(S_t = 1 \mid S_{t-1} = 1)$	Dirichelet*	18	1	-	-
P_{22}	Probability of parameters $P_{22} = p(S_t = 2 \mid S_{t-1} = 2)$	Dirichelet	18	1	-	-
Q_{11}	Probability of volatilities $Q_{11} = p(s_t = 1 \mid s_{t-1} = 1)$	Dirichelet	18	1	-	-
Q_{22}	Probability of volatilities $Q_{22} = p(s_t = 2 \mid s_{t-1} = 2)$	Dirichelet	18	1	-	-

Note: * The Dirichelet distribution $(\alpha_1 + \alpha_2)$ has $\alpha_1 = 18$ and $\alpha_2 = 1$, in which the mean is $E(x_i) = \alpha_i / \alpha_0$ where $\alpha_0 = \alpha_1 + \alpha_2$.

Consonant with Palma & Portugal (2011), the inverse of the elasticity of labor supply (φ) is assumed to follow a gamma distribution with a mean of 2 and a standard deviation of 0.35. Other parameters defined according to these authors were: the fractions of firms that do not adjust their prices ($\theta^H$and θ^F); the elasticity of substitution between domestic goods (η); the degree of habit persistence; and indexation rates (δ_H and δ_F). It was assumed that the inverse of the intertemporal elasticity (σ) follows a gamma distribution with a mean of 1.2 and a standard deviation of 0.2.

As to the degree of interest rate smoothing (ρ_r), it is believed to follow a beta distribution with a mean of 0.6 and a standard deviation of 0.15, as pointed out by Castro *et al*. (2011). The other Taylor rule parameters (λ_1, λ_2 and λ_3) are assumed to follow a gamma distribution with a mean of 0.5 and a standard deviation of 0.09 (Palma & Portugal, 2011). In line with Liu & Mumtaz (2011), it is assumed that the elasticity of debt relative to the interest rate premium follows a gamma distribution with a mean of 0.01 and a standard deviation of 0.02.

All of the autoregressive parameters of the exogenous disturbances were assumed to follow the beta distribution with a mean of 0.5 and a standard deviation of 0.25. This assumption was also employed by Castro *et al.* (2011) in the stochastic analytical model with a Bayesian approach (SAMBA). The parameters of the standard deviations of the shocks follow an inverse gamma distribution with a mean of 0.5 and a standard deviation of 10. The probabilities of the transition matrices were based on Liu & Mumtaz (2011) and these parameters were assumed to have a Dirichelet distribution with a mean of 18 and a unit standard deviation. According to these authors, this mean and this standard deviation indicate that the probability of remaining in the same regime is equal to 0.95.

4 Results

4.1 Data description

The following quarterly time series were used to estimate the MS-DSGE model for the Brazilian economy: gross domestic product (GDP) at market price (chain-linked and seasonally adjusted index); actual real exchange rate (end of period – index); exports (FOB US$ million); imports (FOB US$ million); broad consumer price index - IPCA (mean – index); import price index (mean – index); Selic interest rate (mean – % p.a.); U.S. real gross domestic product - US$ billion – seasonally adjusted); U.S. federal fund rate – % p.a.; and U.S. inflation (consumer price index – seasonally adjusted – end of period). The quarterly data span from the first quarter of 1996 to the fourth quarter of 2012. The data were obtained from the Brazilian Institute of Geography and Statistics (IBGE), Center for Foreign Trade Studies Foundation (Funcex), IMF International Financial Statistics (IFS) and the St. Louis Fed FRED database.

When treating the series, seasonality was removed by the X12-ARIMA technique and the HP filter was used in the domestic and foreign output series. Finally, all variables were log-transformed, except for the domestic and foreign interest rate series, and were redefined so that they had zero mean throughout the sample. Before proceeding, we verified whether the series were stationary on the ADF tests. Table 2 describes the results of these tests with and without intercept for the series in the level and in first differences. Given a 5% significance level, the results indicated that the domestic and foreign output series are stationary and that the other ones have a unit root in the level, being stationary only in first differences. Therefore, we used the first difference of these series.

Table 2: ADF unit root test (p value)

Variables	Series in the level		In first differences	
	With intercept	W/o intercept	With intercept	W/o intercept
y	0.0007***	0.0000***	-	-
π_H	0.8561	0.6483	0.0000***	
π_F	0.9341	0.5873	0.0001***	0.0000***
e	0.3773	0.0670	0.0000***	0.0000***
r	0.8637	0.4655	0.0017***	0.0001***
y^*	0.0172**	0.0011***	-	-
π^*	0.8903	0.5233	0.0000***	0.0000***
r^*	0.3284	0.0518	0.0095***	0.0005***

Note: *Significant at 10%. ** Significant at 5%. *** Significant at 1%.

4.2 Time-invariant rational expectations model

Table 3 shows the mean of the models' estimated parameters, the main diagonal of the transition matrices, and the 95% Bayesian credibility intervals below the respective parameter.

The first column of Table 3 exhibits the results for the time-invariant model (model 0). The intertemporal elasticity of substitution was estimated at 1.24, which is greater than the value observed by Castro *et al.* (2011) and Palma & Portugal (2011), but lower than those found by Silveira (2008). The mean for the inverse of the elasticity of labor supply was estimated at 2.46 (elasticity of 0.41). This value is higher than that obtained by Palma & Portugal (2011) – which was equal to 1.71 (elasticity of 0.58). The posterior for the elasticity of substitution between domestic and foreign goods was estimated at 0.33, greater than the value recorded by Palma & Portugal (2011) – 0.13, suggesting a small possibility of substitution between domestic and foreign goods.

The domestic (θ^H) and imported goods (θ^F) price rigidity parameters yielded 0.68 and 0.97, respectively, which suggests that domestic prices are adjusted approximately every other quarter and that imported goods prices are much higher and adjusted every eight years. Palma & Portugal (2011) found 0.66 and 0.87 for these parameters. With regard to the backward-looking parameters of the domestic Phillips curve (δ^H) and of imported goods (δ^F), the estimates were equal to 0.89 and 0.16, respectively, but Palma & Portugal (2011) found 0.31 and 0.07 for the same parameters. Thus, according to the estimations in this paper, following Eq. 6, the impact of past domestic price inflation on the current inflation was 0.47. Analogously, past imported goods prices inflation has a 0.14 impact on current price inflation. The average habit formation was estimated at 0.33, differently from what was observed by Silveira (2008), i.e., 0.55 to 0.81.

The interest rate smoothing coefficient was estimated at 0.60 and the monetary policy reaction parameters were estimated at 0.56, 0.48 and 0.17 relative to inflation, output and exchange rate deviations, respectively. While the Taylor (1993) principle is not satisfied, as the increase in the interest rate is less than proportional to inflation rate movements, this finding implies that the Central Bank of Brazil places a heavier weight on inflation control compared to output and exchange rate stabilization. Note that some exogenous shocks exhibit high persistence, especially process ρ_g, which was greater than 0.95.

Table 3: Posterior distribution of the estimated parameters

		Model 1		Model 2		Model 3	
Parameters	Model 0	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
$\boldsymbol{\sigma}$	1.24	0.40	-	0.41	-	0.41	-
	0.34 1.72	0.27 0.53		0.30 0.54		0.29 0.54	
$\boldsymbol{\varphi}$	2.46	2.43	-	2.14	-	2.38	-
	1.72 3.31	1.72 3.13		1.62 2.63		1.87 2.96	
$\boldsymbol{\theta^H}$	0.68	0.31	-	0.52	0.21	0.33	-
	0.11 0.78	0.16 0.48		0.33 0.74	0.05 0.37	0.17 0.49	
$\boldsymbol{\theta^F}$	0.97	0.85	-	0.85	-	0.85	-
	0.88 1.00	0.79 0.91		0.79 0.91		0.79 0.91	
$\boldsymbol{\eta}$	0.36	0.69	-	0.68	-	0.70	-
	0.22 0.51	0.54 0.96		0.55 0.84		0.56 0.88	
$\boldsymbol{h}$	0.33	0.31	-	0.36	-	0.32	-
	0.15 0.85	0.19 0.44		0.24 0.48		0.20 0.44	
$\boldsymbol{\delta^H}$	0.89	0.36	-	0.79	0.39	0.28	-
	0.56 1.00	0.08 0.85		0.51 1.00	0.08 0.80	0.04 0.78	
$\boldsymbol{\delta^F}$	0.16	0.10	-	0.09	-	0.09	-
	0.02 0.34	0.02 0.21		0.01 0.19		0.02 0.21	
$\boldsymbol{\rho_r}$	0.60	0.50	-	0.51	-	0.46	0.55
	0.34 0.76	0.38 0.61		0.41 0.61		0.34 0.57	0.26 0.84
$\boldsymbol{\lambda_1}$	0.56	1.36	-	1.37	-	1.43	0.53
	0.33 1.35	1.18 1.56		1.17 1.57		1.24 1.64	0.37 0.70
$\boldsymbol{\lambda_2}$	0.48	0.33	-	0.34	-	0.28	0.48
	0.33 0.66	0.21 0.47		0.23 0.47		0.19 0.39	0.34 0.66
$\boldsymbol{\lambda_3}$	0.17	0.14	-	0.17	-	0.15	0.53
	0.10 0.24	0.09 0.19		0.12 0.21		0.11 0.20	0.38 0.71
$\boldsymbol{\chi}$	0.0002	0.0003	-	0.0003	-	0.0016	-
	0.0001 0.0005	0.0001 0.0006		0.0001 0.0009		0.0001 0.0050	
$\boldsymbol{\rho_a}$	0.23	0.58	-	0.63	-	0.72	-
	0.04 0.61	0.44 0.72		0.53 0.72		0.63 0.81	
$\boldsymbol{\rho_g}$	0.98	0.38	-	0.36	-	0.31	-
	0.67 1.00	0.18 0.57		0.24 0.52		0.17 0.49	
$\boldsymbol{\rho_\phi}$	0.91	0.95	-	0.93	-	0.95	-
	0.71 1.00	0.92 0.97		0.90 0.96		0.92 0.98	

(Continued)

		Model 1		Model 2		Model 3	
Parameters	Model 0	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
$\boldsymbol{\rho_{cp}}$	0.41	0.81	-	0.80	-	0.79	-
	0.13 0.79	0.67 0.92		0.65 0.91		0.67 0.91	
$\boldsymbol{\rho_{\pi^*}}$	0.81	0.77	-	0.77	-	0.79	-
	0.73 0.88	0.68 0.86		0.71 0.83		0.71 0.85	
$\boldsymbol{\rho_{y^*}}$	0.06	0.23	-	0.27	-	0.15	-
	0.00 0.16	0.07 0.46		0.13 0.42		0.01 0.33	
$\boldsymbol{\rho_{i^*}}$	0.55	0.43	-	0.41	-	0.55	-
	0.21 0.90	0.05 0.91		0.05 0.84		0.16 0.97	
$\boldsymbol{\sigma_{mp}}$	1.18	1.72	0.44	2.47	0.44	4.26	0.47
	0.94 1.56	0.94 2.91	0.31 0.61	1.50 3.43	0.31 0.61	3.00 5.32	0.34 0.64
$\boldsymbol{\sigma_{g}}$	3.80	6.59	1.11	8.09	1.29	8.54	1.11
	1.61 9.45	3.84 9.71	0.62 1.88	7.23 8.86	0.69 2.18	7.33 9.43	0.67 1.65
$\boldsymbol{\sigma_{a}}$	5.67	2.26	0.50	1.79	0.49	2.12	0.49
	1.19 8.75	1.25 3.81	0.33 0.72	1.15 2.81	0.34 0.69	1.61 2.54	0.35 0.69
$\boldsymbol{\sigma_{cp}}$	1.74	1.40	0.50	1.58	0.51	1.95	0.52
	1.00 2.51	0.65 2.76	0.32 0.76	1.00 2.13	0.34 0.73	1.32 2.76	0.35 0.76
$\boldsymbol{\sigma_{\phi}}$	1.33	0.84	0.37	1.47	0.37	0.88	0.36
	0.33 3.00	0.42 1.68	0.26 0.50	0.57 2.53	0.27 0.50	0.50 1.51	0.27 0.48
$\boldsymbol{\sigma_{i^*}}$	0.20	0.35	0.29	0.36	0.28	0.37	0.27
	0.17 0.23	0.26 0.48	0.22 0.36	0.26 0.50	0.22 0.35	0.27 0.52	0.21 0.35
$\boldsymbol{\sigma_{y^*}}$	0.68	0.61	0.40	0.61	0.38	0.66	0.38
	0.59 0.81	0.39 0.91	0.28 0.52	0.39 0.89	0.27 0.49	0.45 0.94	0.31 0.48
$\boldsymbol{\sigma_{\pi^*}}$	0.58	0.49	0.38	0.47	0.40	0.48	0.39
	0.49 0.68	0.33 0.71	0.28 0.51	0.32 0.65	0.31 0.52	0.31 0.68	0.29 0.52
$\boldsymbol{P_{11}}$	-	-	-	0.9197	-	0.9971	-
				0.8270 0.9920		0.9878 1.0000	
$\boldsymbol{P_{22}}$	-	-	-	0.9838	-	0.9408	-
				0.9726 0.9964		0.8273 0.9994	
$\boldsymbol{Q_{11}}$	-	0.8848	-	0.8683	-	0.8462	-
		0.7724 0.9745		0.8165 0.9142		0.7059 0.9557	
$\boldsymbol{Q_{22}}$	-	0.9224	-	0.8605	-	0.9231	-
		0.8413 0.9832		0.8248 0.8965		0.8551 0.9778	

4.3 Rational expectations model with regime switching in the variance

The results of this model were somewhat different from those of the time-invariant model. As shown in Table 3, the intertemporal elasticity of substitution was estimated at 0.40 – lower than that described in the Brazilian literature and also lower than in the model without regime switching. However, according to Silveira (2008), there is no final conclusion about this parameter given the variability of results demonstrated in the Brazilian literature.

The values of parameters θ^{H} and θ^{F} were 0.31 and 0.85, respectively, suggesting adjustment in domestic prices slightly over a quarter for domestic prices and six quarters for imported goods prices. Regarding the other parameters of the inflation equation, the means of δ^H and δ^F were 0.36 and 0.10, respectively. This result implies that a one percentage-point increase in past domestic price inflation pushes the current inflation up by 0.27 percentage points. This impact, in relation to imported goods price inflation, corresponds to 0.33 percentage points.

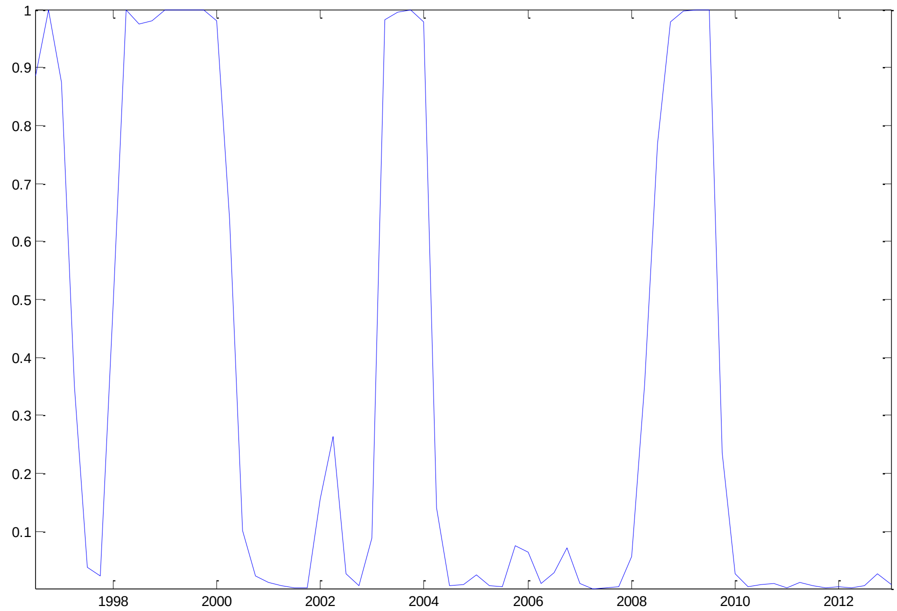
The average interest rate smoothing was equal to 0.50, i.e., lower than that observed in the Brazilian empirical literature, such as in Castro *et al.* (2011), who obtained 0.79. The elasticity of substitution between domestic and foreign goods yielded 0.69. This raises the possibility of substitution between the two types of goods compared to the time-invariant model. Moreover, the risk premium shock (ρ_ϕ) exhibited a persistence greater than 0.95 in this model.

However, the results obtained for the time-invariant model and this one that allows for Markov shifts in volatilities were similar. This was the case of the parameter that represents habit formation in the Brazilian economy, estimated at 0.31 and the inverse of the elasticity of labor supply equal to 2.43 (elasticity of 0.41), also suggesting high rigidity in the labor market. In addition, the Taylor rule parameters have the same pattern observed for the DSGE without regime switching. However, with different magnitudes, the coefficients of monetary policy reaction to inflation, to output, and to the exchange rate yielded 1.36, 0.33 and 0.14, respectively. In this case, the Taylor (1993) principle is satisfied, with smaller reactions than those found in Taylor, 1.36 against 1.5 for the reaction of interest rates to inflation and 0.33 against 0.5 for the output gap.

The lower end of Table 3 shows the probabilities of the main diagonal of the transition matrix. The probabilities estimated for this model suggest that regime 2 is more persistent than regime 1, given that, on average, persistence in regime 1 corresponds to approximately nine quarters against 13 quarters in regime 2. In addition, regime 1 is associated with larger volatilities for all exogenous shocks. Thus, it may be said that regime 1 represents high volatility while regime 2 indicates the opposite.

Fig. 1 depicts the filtered probabilities for the high-volatility state (regime 1), which is confirmed at the beginning of the analyzed period, followed by a shift in 1997, and then predominating again from 1998 to late 2000. High volatility was detected again in the Brazilian economy in another two periods – throughout 2003 and in 2008 to 2010 as a result of the U.S. crisis.

Figure 1: Filtered Probabilities for Model 1 (regime 1 - $p(s_t = 1)$)



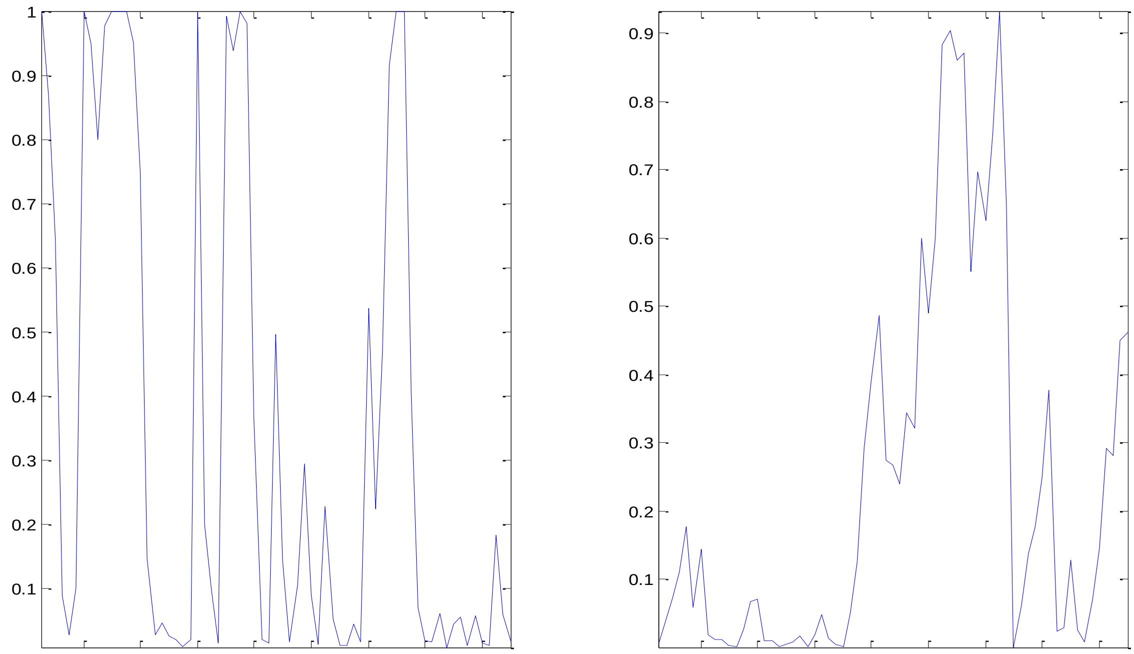
4.4 Rational expectations model with regime switching in the Phillips curve

Table 3 also displays the results for the Markov switching volatility model and for the Phillips curve parameters. The estimated parameters were mostly similar to the ones obtained by the previous model (M_1), except for the cases of the inverse of the elasticity of labor supply, whose mean was equal to 2.14 (elasticity of 0.47) and, therefore, lower than that described in the previous models, the backward-looking parameter price-setting of domestic goods (0.79) and the fraction of the non-optmizing producers (0.52).

With respect to parameters that vary across Markov regimes, parameter δ^H corresponded to 0.79 in regime 1 and to 0.39 in regime 2. Hence, regime 1 can be considered to have high indexation whereas regime 2 exhibits the opposite behavior. Nonetheless, even though the point statistic is relatively dissimilar between the two regimes, the confidence intervals suggest that the change in parameter δ^H is weak for the analyzed period. As for parameter θ^H, it was equal to 0.52 in regime 1 against 0.21 in regime 2. This implies that domestic prices are reoptimized approximately every 2.1 quarters in regime 1 and every 1.3 quarters in regime 2. In other words, regime 1 can be interpreted as having the largest domestic price rigidity while regime 2 has the smallest rigidity. Note also that both regimes are persistent, with probabilities P_{11} and P_{22} estimated at 0.992 and 0.984, respectively.

Model 2 showed the same volatility pattern as model 1. Thus, regime 1 is clearly identified as having high volatility while regime 2 has low volatility. High volatility prevailed in the following periods: (i) from 1998 to 2000; (ii) throughout 2003; and (iii) from 2008 to 2010. Nevertheless, the difference between this model and the previous one is that the latter indicated a quick shift in the high-volatility regime in the last quarter of 2001.

Figure 2: Filtered Probabilities for Model 2 (regime 1)



(a) regime 1 - $p(s_t = 1)$

(b) regime 1 - $p(S_t = 1)$

The period in which regime 1 predominated coincides with the periods of turmoil in the Brazilian economy. For example, between 1998 and 2001, Brazil was affected by foreign crises, such as the Mexican crisis in 1995, the Asian crisis in 1997, and the Russian crisis in 1998. The sizeable loss of reserves due to external vulnerability eventually led to the redefinition of the exchange rate regime, with the adoption of the floating exchange regime in January 1999 as the substitute for exchange rate bands. Moreover, in June 1999, the inflation-targeting regime was formally introduced in Brazil.

In early 2003, when president Lula took office, there was uncertainty in the Brazilian economic scenario about the new policies the new government would adopt. Another period of high volatility in the Brazilian economy occurred in late 2007 when the first signs of the U.S. crisis appeared, producing effects on the Brazilian economy until late 2009.

Fig. 2(b) displays the filtered probabilities concerning regime switching in the parameters. Note that the Brazilian economy faced the highest domestic price rigidity within a few quarters between 2006 and 2008. Therefore, in the analyzed period, the Brazilian economy revealed lower domestic price rigidity most of the time.

4.5 Rational expectations model with regime switching in the Taylor rule

The last estimated model allowed for Markov regime shifts in the monetary policy rule parameters and also in the volatilities of exogenous shocks. Table 3 shows the results obtained. The posterior mean of the estimated parameters was similar to that of the previous models.

The value of p_{11} suggests that regime 1 is highly persistent. This regime is characterized by strong reaction of the Selic rate to inflation, with a parameter estimate of 1.43. Conversely, the estimate of this parameter for regime 2 was 0.53, indicating a weak reaction of monetary policy to inflation. By contrast, output and exchange rate parameters were equal to 0.28 and 0.15 in regime 1 and to 0.48 and 0.53 in regime 2. Thus, considering Eq. 11 and the respective estimated parameters, the following monetary policy rules are obtained for regimes 1 and 2, respectively:

$$\begin{aligned} r_t &= 0{,}46r_{t-1} + 0{,}77\pi_t + 0{,}15y_t + 0{,}08\Delta e_t \qquad (regime\ 1) \\ r_t &= 0{,}55r_{t-1} + 0{,}24\pi_t + 0{,}22y_t + 0{,}24\Delta e_t \qquad (regime\ 2) \end{aligned} \tag{22}$$

These results demonstrate that, under regime 1, the increase of one percentage point in inflation rate raises the interest rate by 0.77 percentage points; the increase of one percentage point in output gap augments the interest rate by 0.15 percentage points; and an increase of one percentage point in exchange rate depreciation raises the interest rate by 0.08 percentage points. Under regime 2, the impacts on interest rate are similar for the variables the interest rate accounts for, yielding 0.24 for one-percentage-point increases in inflation and in exchange rate depreciation and 0.22 for output.

Therefore, regime 1 shows strong reaction to inflation and weak reaction to output and to the exchange rate. Conversely, regime 2 has a weaker reaction to inflation and stronger reactions to output and to the exchange rate than regime 1. Note also that regime 2 is characterized by a larger smoothing coefficient and that the 95% confidence intervals overlap.

Fig. 3 illustrates the posterior distribution of the monetary policy equation parameters in both regimes. Observe the distinction between the regimes as well as their characteristics, as pointed out earlier. The graphs on the right show the impacts on the monetary policy rule.

Figure 3: Posterior distribution for the Taylor rule coefficients

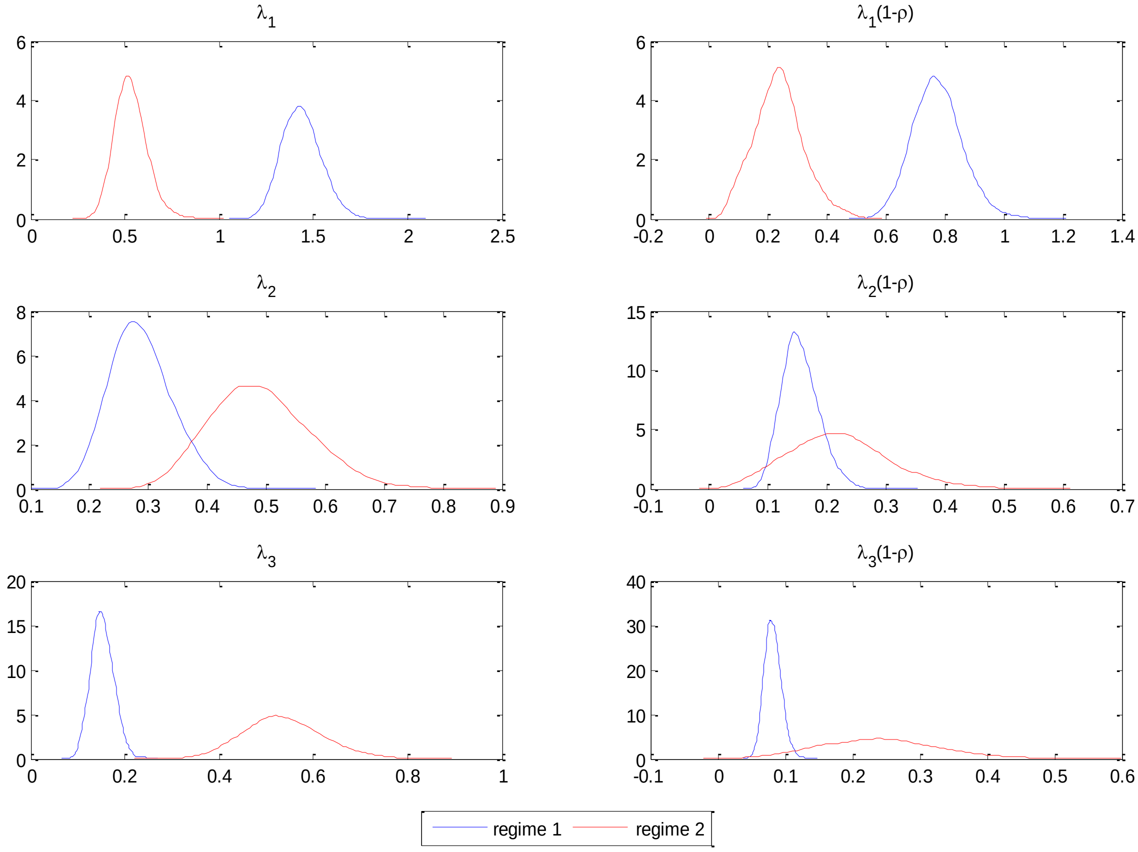
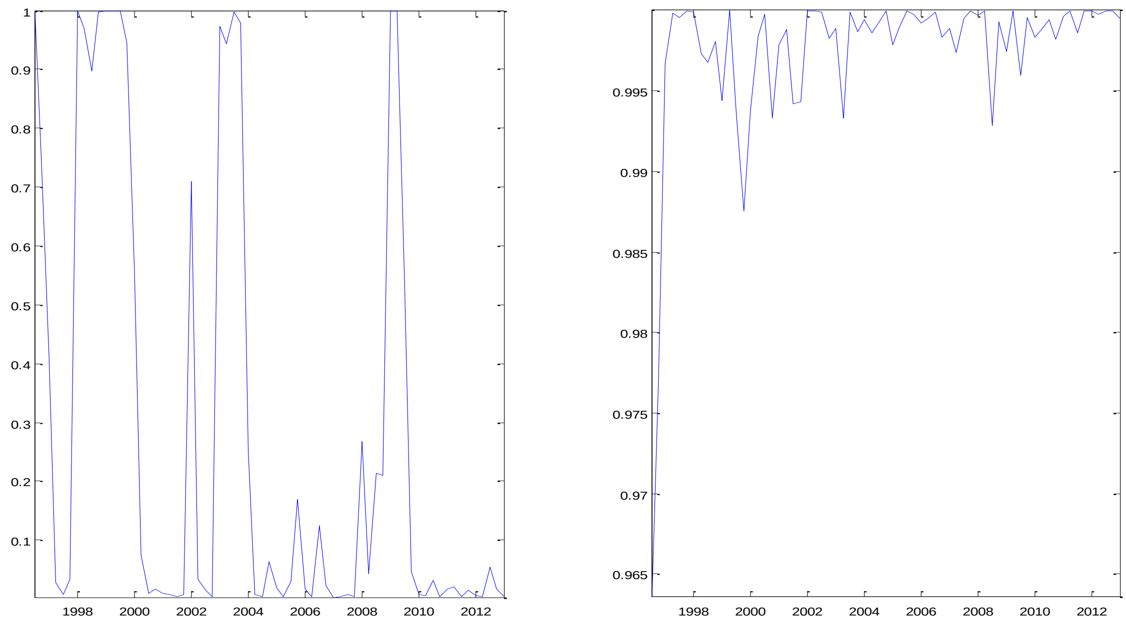


Fig. 4 displays the filtered probabilities for the volatility regime and for the parameters. Similarly to model 2, model 3 indicates high volatility for regime 1 and low volatility for regime 2. As with models 1 and 2, model 3 also identified three periods in which high volatility was predominant. The difference in this model is that the last period showed high volatility only in late 2008 and in early 2009 (see Fig. 4(a)).

Figure 4: Filtered probabilities for Model 3 (regime 1)



(a) regime 1 - $p(s_t = 1)$ (b) regime 1 - $p(S_t = 1)$

Regarding the regime with strong reaction to inflation (state 1), and weak reaction to inflation (state 2), there was predominance of regime 1 throughout the analyzed period and filtered probabilities were greater than 0.95 in all quarters (see Fig. 4(b)).

4.6 Comparison of models

The empirical relevance of Markov switching models was determined by comparing the models using marginal likelihood and Akaike (AIC) and Bayesian (BIC) information criteria for each model. Table 4 displays these results.

Table 4: Comparison of models

Model	Regime switching	Marginal Log Likelihood	AIC	BIC
Model 0	No shifts	-919.73	1897.50	1961.40
Model 1	Only in volatilities	-875.60	1833.20	1923.60
Model 2	Volatilities and Phillips curve	-873.40	1832.80	1927.60
Model 3	Volatilities and Taylor rule	-865.33	1820.70	1919.90

As shown in Table 4, the model with regime switching in volatilities and in the monetary policy rule (model 3) was able to fit the data more properly – higher log likelihood (-865.33). Model 2 and model 3 had the best fits. Thus, Markov switching models, i.e., MS-DSGE models, demonstrated fitted the data more precisely than did the DSGE model. Even when AIC and BIC criteria were used, penalizing a larger number of parameters, despite the inversion of model 1 and 2 following the order of the best results, Markov switching models outperformed the time-invariant model.

Therefore, in the analyzed period, MS-DSGE models were superior to the DSGE model. This finding is consistent with the studies conducted for other economies as pointed out by Liu *et al.* (2011) for the U.S. economy and by Liu & Mumtaz (2011) for the UK economy.

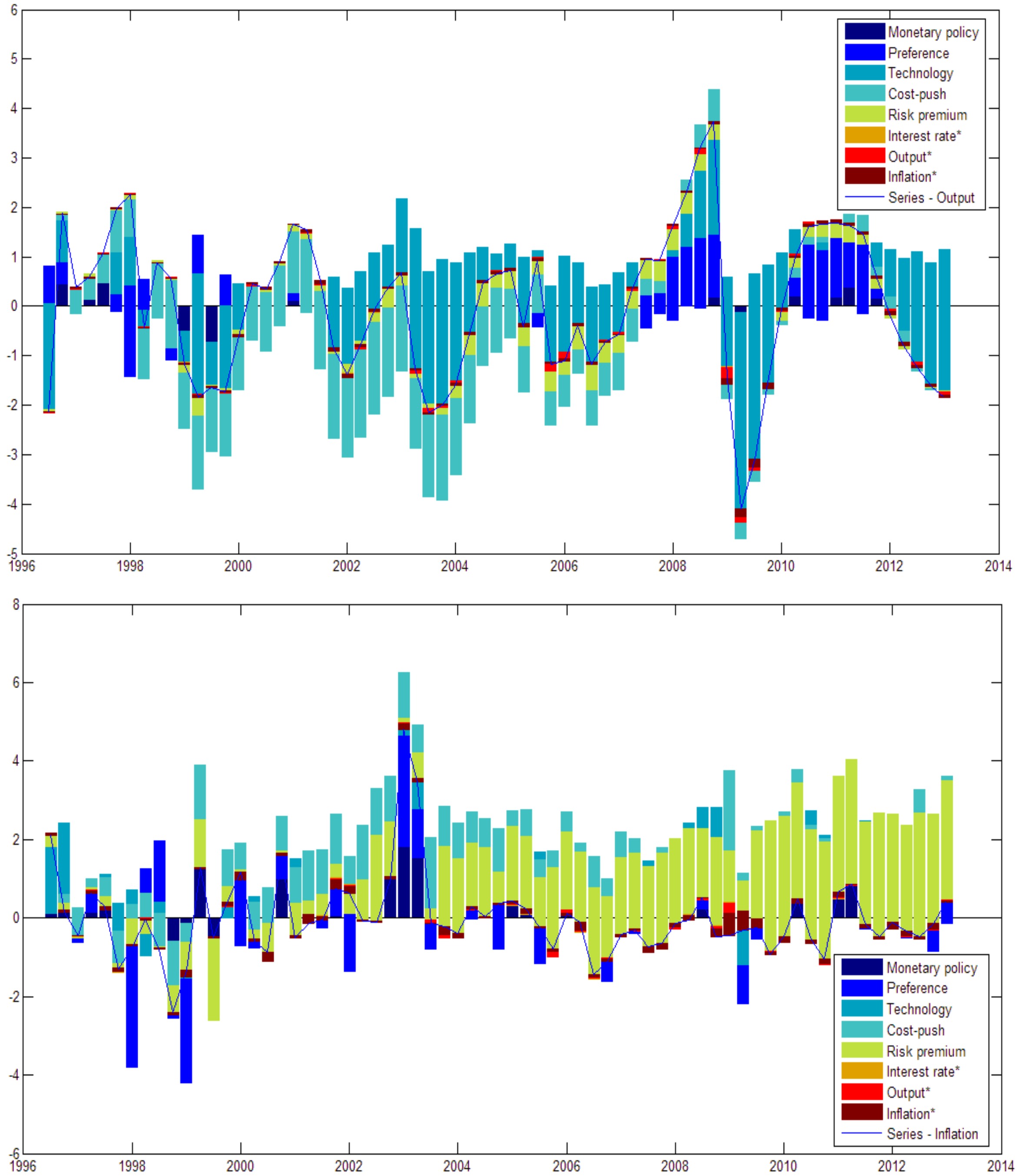
4.7 Dynamics of the Brazilian economy

To assess the role of shocks on Brazil's economic performance, the historical decomposition of shocks and impulse response functions were used, based on the model with the best fit, i.e., the MS-DSGE model with changes in volatilities and in the monetary policy rule (model 3).

The historical decomposition of shocks was computed by the smoothing algorithm proposed by Kim and Nelson (1999) applied to the model in the form of Eq. 20 to estimate exogenous shocks. From these results, it was possible to calculate the contribution of shocks to the observed variables. Fig. 5 shows the results for the output and inflation rate deviation series. According to the model's structure, eight shocks were taken into account: preference shock ($\epsilon_{g,t}$); technology shock or productivity shock ($\epsilon_{a,t}$); cost-push shock ($\epsilon_{cp,t}$); risk premium shock ($\epsilon_{\phi,t}$); monetary policy shock ($\epsilon_{m,t}$); foreign output shock ($\epsilon_{y^{*},t}$); foreign inflation shock ($\epsilon_{\pi^{*},t}$); and foreign interest rate shock ($\epsilon_{r^{*},t}$).

As depicted on the first graph of Fig. 5, technology shocks are associated with output movements, especially concerning the downward movements of the mean for the period. However, other shocks play a role in output dynamics, such as the cost-push shock, mainly between 2001 and 2007, and the preference shock, especially before and after the U.S crisis. In addition, risk premium shocks and monetary policy shocks are more relevant in particular quarters of the series.

Figure 5: Historical decomposition of the output and inflation rate series



Regarding domestic price indices, risk premium shocks, cost-push shocks, preference shocks, monetary policy shocks and, taken in isolation, technology shock were the shocks that mostly had an impact on the dynamics of the price series. Note also in Fig. 5 that the monetary policy shock is always related to movements above the average inflation rate. Additionally, preference shocks contributed negatively to the inflation series most of the time. Nonetheless, in specific cases as in early 2003, this demand shock increased inflation, as well as did the monetary policy shock and supply shocks (technology and cost-push shocks).

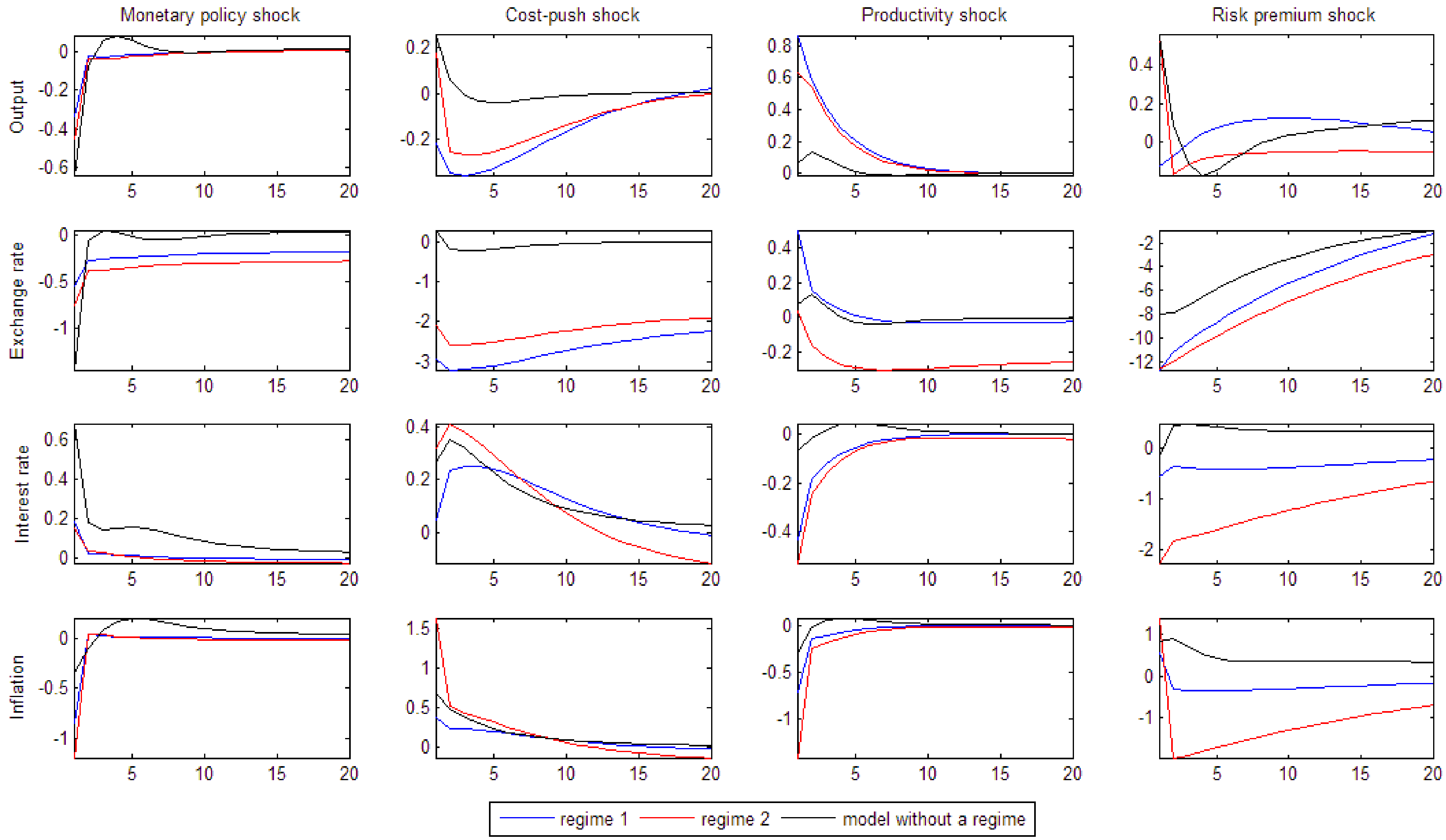
Figure 6 shows the effects of impulse response functions for the monetary policy, cost-push, productivity or technology and risk premium shocks on output, exchange rate, interest rate and inflation variables in both regimes in model 3 (regime 1 – strong reaction to inflation and regime 2 – weak reaction to inflation). Although regime 1 predominates, regime 2 results allow observing what the effects would be if the Central Bank of Brazil opted for a weaker reaction to inflation. Moreover, for the sake of comparison, the impulse response functions of the time-invariant DSGE model were included (model 0).

Results indicate that, between the two regimes in the MS-DSGE model, the dynamics of the effect of the monetary policy shock on the selected variables are similar. However, the magnitude is different, as shown in Fig. 7.

The movement of the analyzed variables is higher in regime 2 than in regime 1, except for a few cases. In relation to the monetary policy shock, the responses of the regime 1 are very close to the regime 2. Furthermore, regardless of the shock, in regime 1, inflation is minimally influenced and reverts to the mean in a few quarters. This differs from regime 2, in which the difference in magnitude is remarkable, as the reaction of inflation submitted to a cost-push shock is about three times greater in regime 2 than in regime 1.

As for the reactions obtained by the DSGE model and taking into consideration that the MS-DSGE model with regime switching in volatilities of the exogenous processes and in the monetary policy rule had a better fit, the time-invariant model overestimated or underestimated the reactions of the selected macroeconomic variables. However, in some cases, they were similar to those of regime 1, which predominated in the analyzed period.

Figure 6: Impulse responses for monetary policy, cost-push, productivity and risk premium shocks in selected variables



5 Conclusion

Unlike other studies that addressed time-invariant parameters, the present paper analyzed the behavior of the major Brazilian economic variables using open-economy DSGE models and allowing for Markov switching in certain parameters estimated by Bayesian methods. The method proposed by Farmer *et al.* (2008) was used to solve the MS-DSGE model. The method consists in rewriting the model so that it includes fixed parameters with extended states, whose MSV solution written as an MS-VAR solves the original model.

The open-economy DSGE model used, developed by Justiniano & Preston (2010), contemplates the interaction of households, domestic and import firms, and the central bank. The model also incorporates characteristics such as habit formation, indexation to past inflation (e.g., sources of rigidity), in addition to a monopolistic competition with sticky prices for both types of firms.

Using a two-step estimation process with Metropolis Hastings algorithm, four models were estimated for the Brazilian economy with quarterly data for 1996 to 2012,: Model 0 - no regime shifts; Model 1 – two regime shifts in the volatility of exogenous shocks; Model 2 – two regime shifts in the volatility of exogenous shocks and in parameters δ_H (indexation rate) and θ_H (fraction of firms that do not adjust their prices) of the Phillips curve; and Model 3 – two regime shifts in the volatility of exogenous shocks and in the Taylor rule equation parameters.

By comparing the estimated models, those with Markov switching outperformed the time-invariant model, rejecting the hypothesis of constant parameters in DSGE model in the Brazilian economy in the analyzed period.

Among Markov switching models, model 3 with regime shifts in the volatilities and in the Taylor rule had the best fit. Hence, this model was used to analyze the macroeconomic dynamics in Brazil.

Results reveal that the Central Bank of Brazil placed a heavier weight on inflation stabilization to the detriment of output and exchange rate throughout the analyzed period. As no regime shifts were found in Taylor rule parameters, this paper confirms that studies on the Taylor rule for the Brazilian economy that do not contemplate regime switching are on the right path. Nevertheless, the models identified three periods of high volatility in Brazil: (i) from 1998 to 2000; (ii) throughout 2003; and (iii) from 2008 to 2010, period of the U.S. crisis, with some differences as to its beginning and end. Thus, the change in exogenous shock parameters should be taken into account in models for the Brazilian economy. In addition, the Phillips curve parameters were not constant over time (model 2).

The historical decomposition revealed that technology shocks are correlated with output movements below the mean during the period. Additionally, cost-push and preference shocks play an important role in the Brazilian output dynamics. Risk premium shocks have a major role in the behavior of inflation rate, and monetary policy shock has always been related to inflation rate movements above the mean.

With respect to impulse response functions, the main result is that, even though the dynamics is similar, there are differences in the magnitude of reactions when both regimes (strong and weak reaction to inflation) are compared. In general, the magnitude of the effects of shocks in the regime with weak reaction to inflation is greater than in the regime with strong reaction to inflation.

The analysis made in this paper focused on the model with shifts in the monetary policy rule parameters, in the Phillips curve, including independent regime shifts in the volatilities of exogenous shocks, and on a model with Markov switching only in volatilities. However, other regime shifts in other Brazilian economic parameters could be investigated. The extension of the analysis to other parameters of the model used herein or even to other DSGE models is a suggestion for future studies. The same applies to solution methods; despite the fact that the MSV solution used by Farmer *et al.* (2008) is large, it is not exhaustive.

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