Exchange Rate Pass-Through in Brazil: a Markov switching DSGE estimation for the inflation targeting period

Fabrizio Almeida Marodin*

Marcelo Savino Portugal**

Abstract

This paper investigates the nonlinearity of exchange rate pass-through in the Brazilian economy during the floating exchange rate period (2000-2018) using a Markov-switching new Keynesian DSGE model. We find evidence of two distinct regimes for the exchange rate pass-through and for the volatility of shocks to inflation. Under the so-called "normal" regime, the long-run pass-through to consumer prices inflation is estimated at near zero value, only 0.00057 percentage point given a 1% exchange rate shock. Comparatively, the expected pass-through under a "crisis" regime is of 0.1035 percentage point to inflation, for the same exchange rate shock. The Markov-switching (MS) model outperforms the fixed parameters model according to several comparison criteria. The results allowed us to identify the occurrence of four distinct cycles for the exchange rate pass-through during the inflation targeting period in Brazil.

Keywords: Exchange-rate pass-through, New Keynesian model, DSGE, Regime Switching, Markov Chain.
JEL codes: E31, F31, C3.

* Department of Economics. University of California Irvine. 3151 Social Sciences Plaza, Irvine, CA 92697-5100, USA. E-mail: fmarodin@uci.edu
** Graduate Program in Economics, Universidade Federal do Rio Grande do Sul, Porto Alegre/RS, Brazil. E-Mail: msp@ufrgs.br

1. Introduction

This paper assesses the exchange rate pass-through to inflation in the Brazilian economy during the floating exchange rate period using a Markov-switching (MS) new Keynesian Dynamic Stochastic General Equilibrium (DSGE) model. Our aim is to check for nonlinear behavior of the exchange rate pass-through, given the possibility it could further amplify inflation during periods of external sector distress, using an innovative methodological framework.

The research is aligned with the recent literature in structural parameter drifting, or nonlinear behavior of structural parameters. According to Hamilton (2014), nonlinear mechanisms that trigger macroeconomic regime shifts are some of the most noteworthy contemporary issues in macroeconomics. Current economies are subject to remarkable changes, recurrent crises, recessions, and financial stress. These events produce "dramatic breaks" in macroeconomic time series and, consequently, lead agents to create expectations under different regimes.

The traditional approach to analyze changes in structural parameters is based on Hamilton's (1989) business cycle model. In this method, some parameters in the reduced form linear model may vary freely according to a Markov processes. Sims & Zha (2006) is a pioneer work on modeling regime shifts in the U.S. monetary policy by the use a structural MS-VAR model. However, less work has been done with structural new Keynesian, or Dynamic Stochastic General Equilibrium (DSGE), models with regime-switching mechanisms as the topic is relatively new. Justiniano & Primicieri (2008), for example, assess regime switching only in the volatility of shocks whereas Fernández-Villaverde, Guerrón-Quintana & Rubio-Ramírez (2010), Bianchi (2013), Baele et al. (2015), and Iboshi (2016) focus on changes in the Taylor rule parameters and their consequences for macroeconomic equilibrium. These studies are usually interested in identifying periods during which the U.S. monetary policy has an "active" vs. a "passive" behavior towards inflation.

Our motivation derives from the risk of underestimating the effect of an exchange rate shock to inflation when DSGE models are being used. The international literature has found meaningful evidence of nonlinear exchange rate pass-through in several developing and developed economies (Albuquerque & Portugal 2005, Correa & Minella 2010, Holmes 2009, Khemiri & Ali 2012, Donayre & Panovska 2016, Baharumshah et al. 2017). However, to the best of our knowledge, there is no published work on the estimation of DSGE models allowing for non-linear exchange rate pass-through. When policymakers are using linearized DSGE models to forecast inflation they may incur in greater than expected forecast error, specially during external sector crisis events, where policy decisions are of the highest importance.

Nearly all the estimated DSGE models for the Brazilian economy have assumed constant parameters, as in Silveira (2008), Furlani, Portugal & Laurini (2010), De Castro et al. (2015), and Palma & Portugal (2014). An exception is Gonçalves, Portugal & Arágon (2016), who investigated regime switching in the Taylor rule. On the other hand, a rich strand of research evaluates changes in structural parameters of the Brazilian economy by applying conventional regime switching models such as Hamilton (1989). Fasolo & Portugal (2004) find changes in the Phillips curve parameters whereas Vieira & Pereira (2013) describe differences in the business cycle dynamics. More recently, Rodrigues & Mori (2015) have identified different monetary policy regimes using a model with changes in the Taylor rule parameters, and Oliveira & Feijó (2015) have investigated the nonlinearity between unemployment and inflation using a Phillips curve with Markov switching. We therefore assume that the paucity of empirical studies on regime switching new Keynesian DSGE models in Brazil is due mainly to their recent development rather than to the belief that this economy is subject to fixed structural parameters. Hence, the contribution of this paper is to fill this gap and propose a novel empirical strategy to quantify the effect of non-linear exchange rate pass-through in a DSGE estimation.

The nonlinear behavior of the exchange rate pass-through is theoretically endorsed by Dixit (1989) and Taylor (2000). Dixit (1989) attributes the differences in pass-through to firms decision-making uncertainty. The more uncertain are firms about the steady state of the exchange rate level, the greater their incentive to adopt a wait-and-see strategy before adjusting prices, given adjustment (menu) and reputation costs. Thus, if the exchange rate shock is seen as permanent, agents would respond with a higher pass-through to prices, compared to cases of temporary shocks. Taylor (2000) proposes a different argument, in which differences in the level of pass-through are related to price rigidity. In periods of higher inflation, firms transfer their costs more frequently, including costs associated with imported inputs, as overall price rigidity is smaller. Figueiredo & Gouvea (2011) support Taylor's viewpoint and provide empirical evidence of heterogeneity in the pass-through in Brazil related to the level of price rigidity.

As mentioned before, there is a well stablished empirical literature on variable, or nonlinear, exchange rate pass-through which does not relate yet to DSGE estimation. Goldfajn & Werlang (2000) confirm that the intensity of pass-through in the cases of exchange rate depreciation is not fixed, but depends upon a series of economic state variables. The key factors would be the cyclical component of output, the initial overvaluation of the real exchange rate, the initial inflation rate, and the level of economic openness. In the Brazilian economy, Albuquerque & Portugal (2005) asseverate that the intensity of exchange rate pass-through varies over time and depends on macroeconomic factors, which is also supported by Tombini & Alves (2006). Minella et al. (2003) and Kohlscheen (2010) affirm that exchange rate volatility is associated with variance of inflation and with higher pass-through. Moreover, the nonlinear or asymmetric behavior of exchange rate pass-through is verified in the empirical studies undertaken by Correa & Minella (2010), Nogueira Jr (2010), and Pimentel, Modenesi & Luporini (2015). In other countries, Holmes (2009) and Khemiri & Ali (2012) assess regime switching in exchange rate pass-through by means of regressions based on the Phillips curve for Tunisia and New Zealand, respectively. Donayre & Panovska (2016) gather strong evidence of nonlinear behavior between the pass-through and economic activity for Canada and Mexico. In particular, the authors find a higher pass-through in expansionary periods, corroborating Goldfajn & Werlang (2000). The influence of the macroeconomic environment and of inflation stability on the observation of smaller pass-through is also advocated by Winkelried (2014) in an empirical study for Peru.

In this paper, our goal is to estimate a basic new Keynesian DSGE model subject to regime switching in the exchange rate pass-through parameter and in the volatility of shocks to inflation by applying the methods developed by Baele et al. (2015) with further developments. One of the peculiarities of this method is the use of survey data on market expectations, which relaxes the assumption of rational expectations and, at the same time, simplifies model solution and estimation. Baele et al. (2015) suggest Cho's (2014) recursive solution method for regime switching rational expectations models, which circumvents some problems of convergence observed in Farmer, Waggoner & Zha (2011). We differ from Baele et al. (2015), who investigate regime switching in the monetary policy rule, as our focus lies in the exchange rate pass-through. In order to achieve that we expand the original model by adding new elements to the AS curve and a new equation for exchange rate dynamics.

The MS model estimation using the sample from years 2000 until 2015 allows us to identify two possible regimes for the exchange rate pass-through and the volatility of shocks to inflation. During a "normal" cycle, the expected long-run pass-through to consumer prices inflation is very close to zero. We estimate only 0.00057 percentage point change given a 1% shock to the level of the exchange rate. Comparatively, the expected effect during a "crisis" cycle is much higher, of 0.1035 percentage point to inflation, given the same exchange rate shock. In addition, we verified that the volatility of shocks to inflation is larger during the "crisis" period. The MS model outperforms the linear model according to several comparison criteria. In the last part of the paper, we present a prediction exercise for the years 2016 until 2018/q2 that offers further insight about the relationship of central bank credibility and the pass-through regime. We understand that the results are relevant to enrich models of inflation forecasting and to address economic policy analysis more precisely during periods of distress.

The paper is organized into five sections, apart from this introduction. Section 2 describes the basic new Keynesian model and its extensions, introduce regime switching, assess the equilibrium conditions and presents our identification strategy. Section 3 presents the data and the estimation method. Section 4 describes the results and their implications. Finally, Section 5 makes the concluding remarks.

2. The Model

This section describes the basic macroeconomic model, with the introduction of exogenous exchange rate shocks. We expand the model by adding a regime-switching mechanism. Then, we assess the rational expectations equilibrium and finally describe the strategy for including survey expectations.

2.1 The new Keynesian model

Consider the benchmark new Keynesian model with three variables as in Baele et al. (2015):

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda y_t + \epsilon_{\pi,t} \qquad \epsilon_{\pi,t} \sim N(0, \sigma^2_{AS}) \qquad (1a)$$

$$y_t = \mu E_t y_{t+1} + (1-\mu) y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \epsilon_{y,t} \qquad \epsilon_{y,t} \sim N(0, \sigma^2_{IS}) \qquad (2)$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{i,t} \qquad \epsilon_{i,t} \sim N(0, \sigma^2_{MP}) \qquad (3)$$

where π_t is the inflation rate, y_t is the output gap and i_t is the nominal interest rate. The operator E_t refers to conditional expectations. Each equation is amenable to unexpected shocks, respectively: $\epsilon_{\pi,t}$ is the aggregate supply shock (AS shock); $\epsilon_{y,t}$ is the aggregate demand shock (IS shock); $\epsilon_{i,t}$ is the monetary policy shock (MP shock).

As to the structural parameters of the model, δ and μ stand for the forward-looking behavior of firms (AS curve) and consumers (IS curve), respectively. The model allows for endogenous persistence if these parameters are different from 1, with weight attached to the past values of each variable. Parameter λ is the response of inflation to the output gap whereas ϕ is the response of output to the real interest rate. The monetary authority's reaction function is a Taylor rule with smoothing parameter ρ_i, which reacts to inflation expectation with response β and to deviations in output gap with parameter γ. It is assumed that the monetary policy should not react to temporary shocks, which affect only the current inflation rate without affecting its future path.

The equations presented in this simple rational expectations (RE) model are derived from the first-order log-linearized conditions of the optimization problems of each representative agent: consumers, firms, and monetary authority. The model describes the dynamics of endogenous macroeconomic variables, in which current decisions are a function of future expectations for these variables and their past values. As a closed economy model, it does not deal with exchange rate pass-through. We found two alternatives to circumvent this problem.

The first option would be to adopt the full dynamics of a small open economy model, such as Adolfson et al. (2007). Further adding the regime-switching and transition probabilities parameters would result in an even more complex specification, with dozens of parameters to be estimated or calibrated. Since our focus is on a relatively short time period we choose a less complex strategy. We model the exchange rate shock as an unexpected change in the price of imported inputs, which affects, on impact, only the supply curve. In this way, we prioritize the direct effects on inflation by the adjustment of input prices as a consequence of exchange rate fluctuations. We disregard the indirect effects on aggregate demand, such as the change in relative prices of domestic versus imported goods, or the effect on the domestic interest rate. We justify our decision by two reasons. First, exchange rate depreciations in relatively closed economies, such as Brazil, tend to cause relatively smaller changes in the spending shares of domestic versus imported goods. This argument is sustained by Albuquerque & Portugal (2005) and discussed in Goldfajn & Werlang (2000). Second, the estimation of a model using short observed time series and multiple regimes would be hindered as the number of parameters increases, making all the endeavor impractical.

We define Δe as the first difference of the nominal price of foreign currency e, and κ as the effect on inflation, namely the exchange rate pass-through. The Phillips curve in equation (1a) becomes, thus:

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda y_t + \kappa \Delta e_t + \epsilon_{\pi,t} \qquad \epsilon_{\pi,t} \sim N(0, \sigma_{AS}^2)$$

We assume that exchange rate fluctuations at time *t* will affect inflation with a lag, following empirical arguments (Goldfajn & Werlang, 2000). The appropriate number of lags will be checked upon estimation. In addition, we assume that the change in the level of the exchange rate follows a first-order autoregressive process. We add a forth equation to the model to describe the path of Δe_t, where $\epsilon_{e,t}$ is an identically distributed exogenous exchange rate shock. The model is then represented, with one possible lag in the pass-through, as:

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda y_t + \kappa_1 \Delta e_{t-1} + \epsilon_{\pi,t} \qquad \epsilon_{\pi,t} \sim N(0, \sigma_{AS}^2) \quad (1)$$

$$y_t = \mu E_t y_{t+1} + (1-\mu) y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \epsilon_{y,t} \qquad \epsilon_{y,t} \sim N(0, \sigma_{IS}^2) \quad (2)$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{i,t} \qquad \epsilon_{i,t} \sim N(0, \sigma_{MP}^2) \quad (3)$$

$$\Delta e_t = \rho_e \Delta e_t + \epsilon_{e,t} \qquad \epsilon_{e,t} \sim N(0, \sigma_e^2) \qquad (4)$$

Our model differs from that of Baele et al. (2015) in equation (1) as we included the supply shock, and in equation (4) for the inclusion of the exchange rate path. Note that the Phillips curve represented by equation (1) is similar to the specifications used in previous studies on exchange rate pass-through in the Brazilian economy, such as Carneiro, Monteiro & Wu (2004), Correa & Minella (2010), Tombini & Alves (2006) and Nogueira Jr (2010). Our approach, however, differs as it considers cross-equation structural and equilibrium constraints derived from model-consistent expectations[1].

In matrix notation, the model can be written as:

$$AX_t = BE_t X_{t+1} + DX_{t-1} + \epsilon_t \qquad \epsilon_t \sim N(0, \Sigma) \qquad (6)$$

Here, X_t is the vector of macroeconomic variables and ϵ_t is the vector of structural shocks. Matrices A, B, D contain the values of the structural parameters and Σ represents the diagonal matrix with the variances of ϵ_t. In our case, we have $X_t = [\pi_t \; y_t \; i_t \; \Delta e_t]'$.

We follow Baele et al. (2015) considering that the rational expectations equilibrium (REE) of the model is the one that depends solely on minimal state variables, also known as fundamental solution. The solution to model (6) follows the VAR(1) law of motion, where matrices Ω and Γ are highly nonlinear functions of the structural parameters:

$$X_t = \Omega X_{t-1} + \Gamma \epsilon_t \qquad \epsilon_t \sim N(0, \Sigma) \qquad (7)$$

A model in this format can be solved by several methods, such as Sims (2002) or Cho & Moreno (2011). The inclusion of regime shifts in the model, however, requires a new characterization of the rational expectations equilibrium.

[1] Some small open economy models include the exchange rate variation in the Taylor rule, as for example Furlani, Portugal & Laurini (2010). Other single equation estimations of the Central Bank of Brazil reaction function, such as Rodrigues & Mori (2015), find statistical significance for the reaction to the exchange rate, during some periods of time. We have tried this type of specification for equation (3). However, we do not find stable solutions to the rational expectations model in this case.

We use the identification and estimation strategy proposed by Baele et al. (2015), which employs observed survey-based expectations, instead of assuming the rational expectations are not observed but part of the states to be estimated. The use of survey-based expectations for the estimation of new Keynesian models is rather uncommon, even though it is relatively simple. Admittedly, market surveys may contain missing information bias or reflect the opportunistic behavior from agents. Notwithstanding, for Baele et al. (2015), survey-based expectations represent different perceptions of economic agents based on a potentially richer set of information, and hence they could be useful to improve estimation. The authors mention the high predictive power of surveys conducted with professional forecasters. In Brazil, the inflation forecast exercise of Altug & Çakmakli (2016) do confirm the high predictive power of survey-based expectations. On the practical side, as will be seen further ahead, the calculation of the likelihood function and the identification of regime shifts become a lot easier with this method, since a smaller number of states variables are considered unobserved.

2.2 Introducing regime switching

We allow two possible regimes for both the exchange rate effect on inflation and the volatility of structural shocks on the aggregate supply curve. Thus, we define the discrete unobserved variable S_t, which takes on two possible values $S_t^{\pi} = [0, 1]$ and serves as an indicator of the state of the economy in period t. The variable S_t evolves according to a first-order Markov process, where $P[S_t = 0|S_{t-1} = 0] = p_{00}$; $P[S_t = 1|S_{t-1} = 0] = p_{10} = (1 - p_{00})$; $P[S_t = 1|S_{t-1} = 1] = p_{11}$; $P[S_t = 0|S_{t-1} = 1] = p_{01} = (1 - p_{11})$. The model is called fixed transition probabilities (Hamilton ,1989; Kim & Nelson, 1999).

The Markov-Switching (MS) model is written as:

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda y_t + \kappa_{1S_t} \Delta e_{t-1} + \epsilon_{\pi,t} \qquad \epsilon_{\pi,t} \sim N(0, \sigma_{AS}^2(S_t^{\pi})) \quad (8)$$

$$y_t = \mu E_t y_{t+1} + (1-\mu) y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \epsilon_{y,t} \qquad \epsilon_{y,t} \sim N(0, \sigma_{IS}^2) \quad (9)$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{i,t} \qquad \epsilon_{i,t} \sim N(0, \sigma_{MP}^2) \quad (10)$$

$$\Delta e_t = \rho_e \Delta e_{t-1} + \epsilon_{e,t} \qquad \epsilon_{e,t} \sim N(0, \sigma_e^2) \quad (11)$$

Note that the regime shift is considered only in the first equation (aggregate supply) in parameters κ_{1S_t}, representing the exchange rate pass-through, and $Var(\epsilon_{\pi,t}|X_{t-1}, S_t^\pi) = \sigma_{AS}^2(S_t^\pi)$. These two parameters jointly depend on the state of the economy S_t^π. We are already assuming that the exchange rate shock impacts inflation with one lag, but this was checked during estimation.

For notation reasons, we assume that regime $S_t^\pi = 0$ will have the smallest volatility in aggregate supply curve shocks: $\sigma_{AS}^2(S_t^\pi = 0) < \sigma_{AS}^2(S_t^\pi = 1)$. The model includes the possibility that regimes may occur recurrently through transition probabilities. There is no ex-ante restriction to a higher pass-through period occurring on states $S_t^\pi = 0$ or $S_t^\pi = 1$.

The representation of the model in matrix notation, with the introduction of dependent variables, is as follows:

$$AX_t = BE_t X_{t+1} + D(S_t) X_{t-1} + \epsilon_t \qquad \epsilon_t \sim N\big(0, \Sigma(S_t)\big) \qquad (12)$$

Where matrix $D(S_t)$ takes on a different value in each regime, and so do the variance-covariance matrices between structural shocks $\Sigma(S_t)$. Note that the regime shift could also occur in matrices A and B, however, this is not necessary in our case.

One of the advantages of the representation method adopted by Baele et al. (2015) lies in its simplicity. Liu & Mumtaz (2010) and Gonçalves, Portugal & Arágon (2016), for example, follow the more complex state-space representation and solution method of Farmer, Waggoner & Zha (2011). We follow the method proposed by Baele et al. (2015), which is based on Farmer, Waggoner & Zha (2011) and Cho (2014), to characterize the stability and determinacy of the rational expectations equilibrium of the model. During the estimation process, described below, each maximum likelihood solution will be tested for determinacy conditions. If the model in indeterminate when evaluated at the parameters $\hat{\theta}$, then the likelihood function is heavily penalized in order to discard this point in estimation. For further details, we refer the reader to Cho (2014) or to Appendix A in Baele et al. (2015).

2.4 Identifying the model by using survey expectations

As previously described, our identification and estimation strategy for the MS model follows Baele et al. (2015) and make use of observed survey-based market expectations. Survey expectations for inflation and output gap follow the law of motion below:

$$\pi_t^f = \alpha E_t \pi_{t+1} + (1-\alpha)\pi_{t-1}^f + w_t^\pi \qquad w_t^\pi \sim N\left(0, \sigma_f^\pi\right) \qquad (15)$$

$$y_t^f = \alpha E_t y_{t+1} + (1-\alpha) y_{t-1}^f + w_t^y \qquad w_t^y \sim N\left(0, \sigma_f^y\right) \qquad (16)$$

The equations allow for a slow adjustment mechanism in expectations formation, in which survey expectations potentially react to rational expectations one to one only when parameter α is equal to 1. Otherwise, the adjustment of expectations is slower and depends on past values. The process is inspired in Mankiw & Reis's (2002) model of the Phillips curve in which the information disseminates slowly.

Baele et al. (2015) simplify the estimation mechanism by assuming that the volatility of shocks σ_f^π and σ_f^y in the equations for expectations movement is equal to zero. In this case, the survey-based expectations are the exact function of current expectations and of the past values from the survey. Substituting both equations above into our main model, we have:

$$\pi_t = \frac{\delta}{\alpha}\left(\pi_t^f - (1-\alpha)\pi_{t-1}^f\right) + (1-\delta)\pi_{t-1} + \lambda y_t + \kappa_{1S_t}\Delta e_{t-1} + \epsilon_{\pi,t} \qquad (17)$$

$$y_t = \frac{\mu}{\alpha}\left(y_t^f - (1-\alpha) y_{t-1}^f\right) + (1-\mu) y_{t-1} - \phi i_t + \frac{\phi}{\alpha}\left(\pi_t^f - (1-\alpha)\pi_{t-1}^f\right) + \epsilon_{y,t} \quad (18)$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\frac{\beta}{\alpha}\left(\pi_t^f - (1-\alpha)\pi_{t-1}^f\right) + \gamma y_t\right] + \epsilon_{i,t} \qquad (19)$$

$$\Delta e_t = \rho_e \Delta e_t + \epsilon_{e,t} \tag{20}$$

Where $\epsilon_{\pi,t} \sim N(0, \sigma_{AS}^2(S_t))$, $\epsilon_{y,t} \sim N(0, \sigma_{IS}^2)$, $\epsilon_{i,t} \sim N(0, \sigma_{MP}^2)$, $\epsilon_{e,t} \sim N(0, \sigma_e^2)$. Note that when $\alpha = 1$, it is assumed that the rational expectations are equivalent to the survey expectations. Defining $X_t^f = [\pi_t^f \; y_t^f]'$, we can write the model in matrix form:

$$AX_t = BX_t^f + DX_{t-1}^f + G_{S_t}X_{t-1} + \epsilon_t \qquad \epsilon_t \sim N\big(0, \Sigma(S_t)\big) \tag{21}$$

Where the matrices are specified as follows:

$$A = \begin{bmatrix} 1 & -\lambda & 0 & 0 \\ 0 & 1 & \phi & 0 \\ 0 & -(1-\rho_i)\gamma & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad G_{S_t} = \begin{bmatrix} (1-\delta) & 0 & 0 & \kappa_{1S_t} \\ 0 & (1-\mu) & 0 & 0 \\ 0 & 0 & \rho_i & 0 \\ 0 & 0 & 0 & \rho_e \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\delta}{\alpha} & 0 \\ \frac{\phi}{\alpha} & \frac{\mu}{\alpha} \\ \frac{(1-\rho_i)\beta}{\alpha} & 0 \\ 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} -\frac{\delta(1-\alpha)}{\alpha} & 0 \\ -\frac{\phi(1-\alpha)}{\alpha} & \frac{-\mu(1-\alpha)}{\alpha} \\ \frac{-(1-\rho_i)(1-\alpha)\beta}{\alpha} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Sigma(S_t) = \begin{bmatrix} \sigma_{AS}^2(S_t) & 0 & 0 & 0 \\ 0 & \sigma_{IS}^2 & 0 & 0 \\ 0 & 0 & \sigma_{MP}^2 & 0 \\ 0 & 0 & 0 & \sigma_e^2 \end{bmatrix}$$

If we establish the condition that $\alpha \neq 0$ and assure the invertibility of matrix A, we can multiply each side of the equation by $A^{-1}$and write the following reduced form, which will be used for the estimation:

$$X_t = \Omega_1 X_t^f + \Omega_2 X_{t-1}^f + \Omega_3(S_t)X_{t-1} + \Gamma\epsilon_t \qquad \epsilon_t \sim N\big(0, \Sigma(S_t)\big) \tag{22}$$

In this equation, we have $\Omega_1 = A^{-1}B$, $\Omega_2 = A^{-1}D$, $\Omega_3(S_t) = A^{-1}G_{S_t}$ and $\Gamma = A^{-1}$. Baele et al. (2015) highlight that the advantage of this approach is that the matrices that determine the law of motion of vector X_t are simple analytical functions of the structural parameters, which makes the calculation of the likelihood function relatively easy. Market expectations add new information, which is absent from the other variables and from the structure of the original model and which will contribute to estimation. It will not be necessary to compute the rational expectations equilibrium with multiple regimes, solving the model in each step of the likelihood optimization, as in Farmer, Waggoner & Zha (2011) and Liu & Mumtaz (2010). Otherwise, unobserved regimes will be inferred by the conventional multivariate methods proposed by Hamilton (1989) and Kim & Nelson (1999). More specifically, we will maximize the log-likelihood function of a structural VAR (SVAR) model with regime switching, in which structural restrictions stem from the new Keynesian model and are given by matrices $A, B, D, G_{S_t}, \Sigma(S_t)$. The estimation of the likelihood function of the regime-switching VAR model follows the description of Hamilton (1994), Bellone (2005), and Krolzig (1997), and the algorithm of inference about regimes is the conventional Hamilton filter, which was implemented according to Kim & Nelson (1999).

3. Data and Estimation

This section describes the observed data series used for estimation and the maximum likelihood method adopted.

3.1 Descriptive statistics and stationarity tests

Model estimation requires six observed variables: inflation, output gap, interest rate, exchange rate movement, and the survey expectations for inflation and output gap. Sixty-four (64) quarterly observations – from 2000/1st quarter to 2015/4th quarter – are in the sample used for estimation. We opted to leave the year 1999 out of the sample due to the large fluctuations observed shortly after the transition to the floating exchange rate regime, and also because data on survey expectations are not readily available.

The data sources are briefly described here and more details are given in the Appendix. We use the consumer price index (IPCA) to measure inflation. We extract the output gap from GDP using an Hodrick-Prescott Filter. We measure the quarterly interest rate i_t as the nominal interest rate discounted for the long-run real interest rate. In this sense, we try to account for the fact that the Brazilian economy experienced a sharp reduction in its long-run real interest rate between 2005 and 2012. The data we are taking to the model is the nominal rate in excess of the long run interest rate, and we should obviously consider this characteristic when interpreting our estimation results. We measure exchange rate fluctuations from the first difference of the nominal exchange rate, BRL to USD at the end of the period. Finally, we use Central Bank of Brazil's survey of market expectations to extract next period inflation and output gap expectations[2]. Table 1 presents the descriptive statistics for the time series.

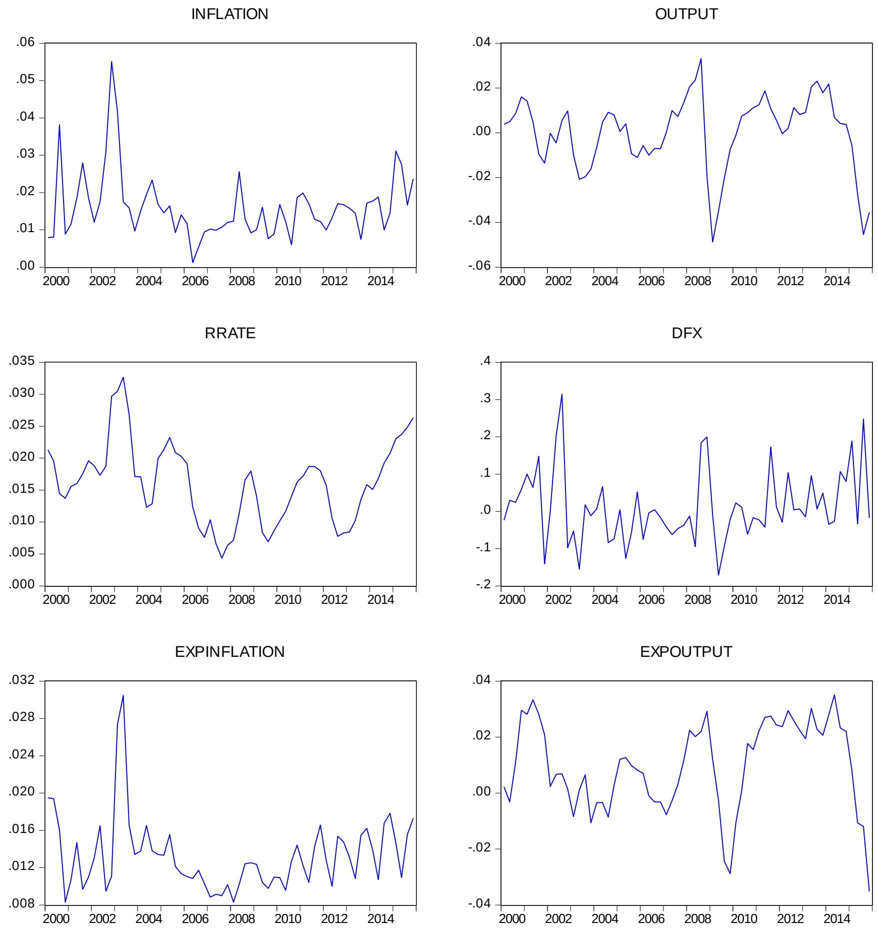
Table 1: Time series descriptive statistics. Units are percentage points / 100. Source: Authors' calculations.

	π_t	y_t	i_t	Δe_t	$E_t\pi_{t+1}$	$E_t y_{t+1}$
Mean	0.0161	0.0002	0.0161	0.0122	0.0133	0.0096
Median	0.0145	0.0041	0.0164	-0.0066	0.0126	0.0103
Maximum	0.0551	0.0331	0.0327	0.3143	0.0305	0.0351
Minimum	0.0012	-0.0488	0.0044	-0.1708	0.0083	-0.0352
Std. dev.	0.0089	0.0164	0.0063	0.0961	0.0040	0.0159
Skewness	1.9608	-0.9021	0.3704	0.8913	1.9747	-0.5447
Kurtosis	8.2738	3.9325	2.8547	3.9075	8.7659	2.8346
Jarque-Bera	115.1797	10.9999	1.5198	10.6694	130.2482	3.2383
Probability	0.0000	0.0041	0.4677	0.0048	0.0000	0.1981

We tested the series for stationarity using the conventional Augmented Dickey-Fuller (ADF) test with intercept and the Phillips-Perron (PP) test. The tests reject the presence of unit root at reasonable levels for all series. The exception is found for the output gap expectations series. We can only be reject non-stationarity when taking a slightly reduced sample, ignoring the last three quarters. The reader is referred to the Appendix for a detailed description and a possible explanation for this finding, but we conclude that the econometric estimation can proceed without restrictions. For the sake of illustration, Figure 1 displays the observed data series.

[2] Data on expectations observed on the first business day of the quarter is used to circumvent the endogeneity problem between expectations and the endogenous variable.

Figure 1: Observed series (from left to right, up to bottom): inflation, output gap, interest rate, exchange rate movement, inflation expectation, and output gap expectation. Units are percentage points / 100. Time interval is from 2000/q1 to 2015/q4. Source: Authors' calculations.



3.2 Estimation method

The reduced-form MS model in equation (22) will be estimated by maximum likelihood using the Hamilton filter and the likelihood function of a structural VAR model with Markov switching, following the methods described in Hamilton (1994), Kim & Nelson (1999), Bellone (2005), and Krolzig (1997). We follow the notation of Kim & Nelson (1999), where ψ_{t-1} is the set of information available at $t-1$, and the observed data yield $y_t = [X_t \; X_t^f]'$. The full set of parameters to be estimated is called θ, a line vector that contains the structural parameters of the MS model, including the volatilities of shocks and the transition probabilities between p, q. A total of 18 parameters need to be estimated:

$$\theta = \{\delta, \lambda, \kappa_{1S_t}(S_t = 0), \kappa_{1S_t}(S_t = 1), \mu, \phi, \rho_i, \beta, \gamma, \rho_e, \alpha, \sigma_{AS}(S_t = 0), \sigma_{AS}(S_t = 1), \sigma_{IS}, \sigma_{MP}, \sigma_e, p, q\}$$

The log-likelihood function is given by $\ln L = \sum_{t=1}^{T} \ln\big(f(y_t)\big)$, where $f(y_t)$ is expressed in terms of its parameters θ. The aim is to maximize the density function $\ln L(y_t;\ \theta)$. In the case of regime-switching models, we do not observe regimes S_t, but we can infer about them in every time period. The reader is referred to Kim & Nelson (1999) to a detailed description of the estimation algorithm. In what follows, we describe some peculiarities about our implementation.

We begin by assigning an initial value to each parameter of vector θ_0, and proceed by maximizing the log-likelihood function with a numerical constraint optimization algorithm. The initial values were chosen based on estimation results obtained by Baele et al. (2015). The constraints are applied on the bounds of each parameter, and stem from theoretical assumptions of the structural model. Our constraints are similar, but lighter than those of Baele et al. (2015). Initial values and constraints are detailed in the Appendix. At each optimization step, the parameters of the candidate vector θ_i are used for the construction of matrices $A, B, D, G_{S_t}, \Sigma(S_t)$, which represent the model in its structural form. We compute the model in reduced form, matrices $\Omega_1, \Omega_{2,}\Omega_3(S_t), \Gamma$. With this in hand and the candidate transition probabilities, the log-likelihood calculation is done by the Hamilton filter, using the sample likelihood function of an MS-VAR model[3] (Hamilton, 1994; Bellone, 2005; Krolzig, 1997). At last, we check whether the determinacy conditions for the rational expectations equilibrium are met with each candidate solution vector θ_i, and we penalize the objective function if that is not the case. This procedure will guarantee that the search will be made along a stable solution path.

4. Empirical results

In this section, we discuss the estimation results for the MS model and compare them with the conventional fixed coefficients model. Then, we analyze the impulse response functions. In the last part, we interpret the regimes identified by the MS model

[3] Details about the likelihood function are also available in the Appendix.

according to economic events which occurred in the Brazilian economy, and put the model to work in an out-of-sample prediction exercise.

4.1 Parameter estimation in the Markov Switching model

Recall that the estimation was made using the sample period from 2000/q1 to 2015/q4. Table 2 shows the estimates for each parameter in the MS model, as well as their standard deviation and corresponding p-value obtained in the conventional t test. The variance-covariance matrix of the maximum likelihood estimates was calculated using the information matrix outer product method (Hamilton ,1994). The solution offered by the model characterizes a fundamental stable rational expectations equilibrium, as discussed before.

Most parameters are statistically significant. Recall that the signs of the parameters are guaranteed by the constraints imposed on the likelihood function optimization and that no parameter was calibrated. The parameters for which joint regime switching was allowed were the pass-through coefficient κ_{1S_t} and the volatility of shocks $\sigma_{AS}(S_t^{\pi})$ to inflation.

In the aggregate supply (AS) equation of the MS model we estimated $\delta = 0.6971$, demonstrating a relatively heavier weight to inflation expectations in comparison to the endogenous persistence (backward-looking) term. This value is quite close to the estimates made by Silveira (2008), who found $\delta = 0.61$ in a price indexation model. In the demand curve (IS), however, a smaller weight was estimated to the expected output gap component, $\mu = 0.1234$. This fact suggests either higher output persistence or worst predictive power of market expectations about future output. Silveira (2008) found parameters that would correspond to $\mu = 0.26$, and a confidence interval that would include our value of $\mu = 0.12$. By and large, we consider the model provides evidence in favor of endogenous persistence of both output and inflation.

The response of inflation to the output gap is estimated at the value of $\lambda = 0.0722$, which is in line with Bayesian estimations of more complex new Keynesian models such as Gonçalves, Portugal & Arágon (2016) who obtain $\lambda = 0.0654$. Our result, however, is not statistically significant. Actually, several studies on the Phillips curve for the Brazilian economy do not demonstrate a statistically significant impact of the output gap, or of marginal cost, on inflation (Alves & Areosa, 2005; Areosa &

Medeiros, 2007; Arruda, Ferreira & Castelar, 2008), prompting Sachsida (2013) to put the validity of this assumption into question. An exception is Mazali & Divino (2010) who use GMM estimation. We tested for alternative measures of the output gap, such as the Beveridge-Nelson decomposition suggested by Tristão & Torrent (2015), but none of them resulted in improvements. Anyway, further investigation into this topic is not within the scope of this paper.

The MS model identifies two distinct regimes regarding the behavior of the exchange rate pass-through, confirming the major assumption of our paper. We refer to the regimes as $S_t = 0$ and $S_t = 1$, which correspond to low and high exchange rate pass-through periods, respectively. The value estimated in the AS curve for the pass-through in regime $S_t = 0$ is statistically zero, with $\kappa_{1S_t}(S_t = 0) = 0.0004$. The point estimate would correspond to a long-run effect of only 0.00057 percentage points on inflation, which is practically zero, considering a 1% exchange rate shock (depreciation of the domestic currency). On the other hand, the estimate for regime $S_t = 1$ is $\kappa_{1S_t}(S_t = 1) = 0.0722$, with strong statistical significance. The long-run effect, considering a 1% exchange rate shock during the high pass-through regime, is 0.1035 percentage points on inflation. Note that the point estimate, under regime $S_t = 1$, for the exchange rate pass-through is 10.35%, an order of magnitude higher, in comparison to 0.057% under the alternative regime.

Table 2: Parameters estimated for the Markov Switching model. Source: Authors' calculations.
Note: The first row contains the parameter estimation; the second row contains the standard deviation and p-values in brackets. Sample period for estimation: 2000/q1 to 2015/q4.

1. Parameters for the inflation curve			
δ	λ	$\kappa_{1S_t}(S_t^{\pi} = 0)$	$\kappa_{1S_t}(S_t^{\pi} = 1)$
0.6971	0.0722	0.0004	0.0722
0.1189 (0.000)	0.0508 (0.145)	0.0153 (0.397)	0.0316 (0.026)
2. Parameters for the output gap curve			
μ	ϕ		
0.1234	0.6740		
0.1056 (0.199)	0.3043 (0.037)		
3. Monetary policy parameters			
ρ_i	β	γ	
0.2515	0.7852	0.0117	
0.0711 (0.001)	0.1692 (0.000)	0.0680 (0.390)	
4. Parameters for exchange rate dynamics			5. Expectations formation
ρ_e			α
0.1488			0.9999

0.1685 (0.267)			0.2583 (0.000)	
6. Volatilities				
$\sigma_{AS}(S_t^{\pi}=0)$	$\sigma_{AS}(S_t^{\pi}=1)$	σ_{IS}	σ_{MP}	σ_e
0.0043	0.0096	0.0108	0.0043	0.0950
0.0024 (0.002)	0.0063 (0.028)	0.0040 (0.000)	0.0023 (0.001)	0.0425 (0.000)
7. Transition probabilities				
q	p			
0.9583	0.9559			
0.0608 (0.000)	0.0412 (0.000)			
8. Statistics				
R^2_{AS}	R^2_{IS}	R^2_{MP}	R^2_{e}	*Log-likelihood*
0.4728	0.5112	0.5384	0.0063	752.0262

In addition to a smaller pass-through regime $S_t = 0$ demonstrated smaller volatility in shocks to inflation, with a standard deviation estimated at $\sigma_{AS}(S_t^{\pi}=0) = 0.0043$, compared to $\sigma_{AS}(S_t^{\pi}=1) = 0.0096$. The transition probabilities reveal relatively high and very similar persistence for both regimes. Consequently, the economy is expected to remain for several quarters in one specific regime, once the transition occurs. Parameter $q = 0.9583$ corresponds to the probability of the economy remaining in regime $S_t = 0$ when it is already in it, i.e., $Pr[S_t = 0|S_{t-1} = 0]$. Additionally, parameter $p = 0.9559$ is equivalent to the probability of remaining in regime $S_t = 1$, that is $Pr[S_t = 1|S_{t-1} = 1]$. In brief, the model estimates that periods of high pass-through and high volatility in the shocks will be slightly shorter than periods of low pass-through and low volatility. For the sake of simplicity, we will, henceforth, refer to regime $S_t = 1$ as "crisis" and to regime $S_t = 0$ as "normal."

The MS model is superior to the fixed parameters model in terms of better fit (parameter R^2_{AS}), larger log-likelihood value, and higher value for the Schwartz criterion. Table 3 shows the comparison between the models, as suggested by Hamilton (2005). Moreover, by assuming regime switching in the volatility of shocks, we ran the Wald test on constraint $\kappa_{1S_{t=0}}(S_t = 0) = \kappa_{1S_t}(S_t = 1)$ and the result is the rejection at 5% significance. In other words, the test rejects the hypothesis of equal pass-through coefficients in both regimes. This results strengthens our argument for the superiority of a Markov switching representation.

Table 3: Comparison between selected models. Source: Authors' calculations. Note: Schwartz criterion calculated as $\mathcal{L} - (k/2)\log T$, where $\mathcal{L}$ is the log-likelihood, k is the number of parameters and T is the sample size (Hamilton, 2005).

Model	Number of parameters	Log-likelihood	Schwartz criterion
Markov switching (MS)	18	752.03	714.59
Linear (fixed parameters)	14	741.42	712.31

The values estimated for the exchange rate pass-through are consistent with earlier findings for the Brazilian economy, although the difference across sample periods does not allow strict comparisons. In particular, numerous studies include the first stage of the Real Plan (1994-1999), prior to the implementation of the inflation-targeting and floating exchange rate regime. In that initial phase, the Brazilian foreign exchange rate was highly controlled by the Central Bank, working as an anchor to prices while most of the macroeconomic shocks were absorbed by sharp moves on the interest rate.

Pimentel, Modenesi & Luporini (2015) estimate the exchange rate pass-through between 1999 and 2013 assuming an asymmetric effect during appreciations versus depreciations. Our estimate for the pass-through during the “crisis” regime, of 10.35%, is close to their estimate of pass-through during depreciation events, which is 11.38%. Correa & Minella (2010) investigated the pass-through between 1995 and 2005, having estimated an effect of 20% in inflation for large depreciation events. Conversely, the pass-through is statistically zero for periods with small exchange rate movements. Our findings are comparable to those of Correa & Minella (2010), but one must consider the large difference between sample periods. Carneiro, Monteiro & Wu (2004) analyzed the period from 1994 to 2001 and found a nonlinear effect of short-run pass-through ranging from 5.6% to 11%, comparable to our high pass-through period estimates. Tombini & Alves (2006) presented a variable estimate for exchange rate pass-through between 2002 and 2006, which varied from zero to approximately 8%, which is again consistent with our findings. Finally, our results are in line with the exchange rate pass-through estimate published by the Central Bank of Brazil (2015) in its several small scale linear projection models.

Note that our long-run pass-through estimate (10.35%), even in a “crisis” period, is considered relatively low by the criteria established by Goldfajn & Werlang (2000) and Belaisch (2003), which implies that the economy has exhibited reasonable capacity to absorb exchange rate shocks without direct pass-through to consumer inflation.

In order to analyze the results of the aggregate demand (IS) and the monetary policy (MP) curves we should recall that our measure of the interest rate i_t is calculated as the nominal interest rate discounted for the long term real interest rate. In other words i_t is the interest rate "in excess" of the long term real rate. With that in mind, we find that the IS curve demonstrates a strong response of output to the interest rate "in excess", with parameter $\phi = 0.64740$. Our estimated parameter is much higher than the calibration of Baele et al. (2015), of $\phi = 0.1$, or the estimation of Gonçalves, Portugal & Arágon (2016) who obtain $\phi = 0.4063$, as both of these works use purely the nominal interest rate as input to their models. Our findings indicate a strong reaction of aggregate demand to the interest rate "in excess" of the long term real rate, which, in turn, shows an efficient channel for monetary policy transmission in Brazil.

Regarding the monetary policy rule, we obtain an interest rate smoothing value of $\rho_i = 0.2515$, which is relatively small in comparison to Bayesian estimations of new Keynesian models such as Furlani, Portugal & Laurini (2010). Again, the difference in our interest rate input series could explain a much slower smoothing coefficient. The Central Bank would be aiming to smooth the nominal interest rate, which leads to a smaller smoothing of the interest rate that is "in excess" of the long term real rate. Parameter β, which stands for the response of the interest rate to inflation expectations, was estimated at 0.7852 and indicates an activist response to inflation as it is significantly different from zero.[4] The interpretation that arises, in our case, is that the Central Bank would be willing to raise the interest rate above the long term real rate for every positive shock in inflation expectations.

The response of monetary policy to output is estimated at $\gamma = 0.0117$, and it is not statistically different from zero. A positive value would indicate that the Central Bank responds to output gap deviation. Our estimate is smaller than those of Palma & Portugal (2014) and Gonçalves, Portugal & Arágon (2016) possibly due to difference on the time series used to measure interest rates.

The exchange rate dynamics shows some positive autocorrelation in exchange rate movements, with $\rho_e = 0.1488$, but it is not significant. The volatility of shocks to the exchange rate equation is by far the largest and the fit of the curve is almost irrelevant.

[4] Note that, if we used the nominal interest rate as i_t instead of the rate "in excess" of the long term real rate, the activist regime would be characterized by $\beta > 1$.

Parameter α describes the law of motion of market expectations, and is very close to one, implying that the model disregards market expectations assessed in the previous period. According to Baele et al. (2015), this finding indicates that market expectations fully adjust to rational expectations, and the slow dissemination of information does not appear to be important in this process.

Table 4 shows the estimation results for the new Keynesian model without regime switching, for the sake of comparison. The exchange rate pass-through coefficient, estimated in the AS curve, is $\kappa_1 = 0.0419$, which corresponds to a long-run effect of 7.62% to inflation. Naturally, this value is within the interval between the smallest and largest pass-through values estimated in the two-regime model. The other AS curve parameters have similar values to those obtained for the MS model.

Table 4: Parameters estimated for the model without regime switching. Source: Authors' calculations. Note: the first row contains the parameter estimation and the second row contains the standard deviation and *p*-values in brackets. Sample period for estimation: 2000/q1 to 2015/q4.

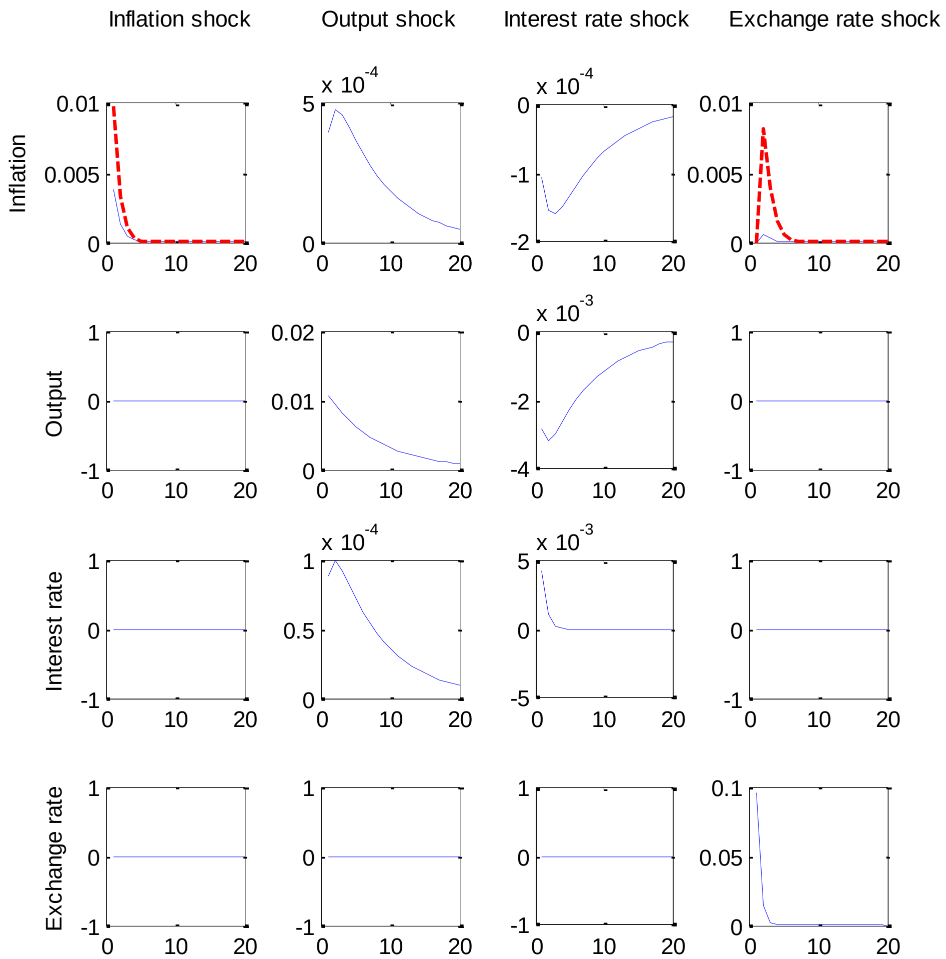
1. Parameters for the inflation curve				
δ	λ	κ_1		
0.5497	0.0887	0.0419		
0.1029 (0.000)	0.0776 (0.206)	0.0158 (0.014)		
2. Parameters for the output gap curve				
μ	ϕ			
0.1231	0.6718			
0.1015 (0.189)	0.3167 (0.044)			
3. Monetary policy parameters				
ρ_i	β	γ		
0.2513	0.7857	0.0110		
0.0711 (0.001)	0.1614 (0.000)	0.0541 (0.389)		
4. Exchange rate dynamics parameters			5. Expectations formation	
ρ_e			α	
0.1488			1	
0.1556 (0.250)			0.2819 (0.001)	
6. Volatilities				
σ_{AS}	σ_{IS}	σ_{MP}	σ_e	
0.0073	0.0108	0.0043	0.0950	
0.0000 (0.000)	0.0000 (0.000)	0.0000 (0.003)	0.0017 (0.000)	
7. Statistics				
R^2_{AS}	R^2_{IS}	R^2_{MP}	R^2_e	Log-likelihood
0.2688	0.5111	0.5383	0.0063	741.4227

We ran several specification tests on the residuals of each equation – serial autocorrelation, normality, and conditional variance. Serial autocorrelation was rejected in general, but not in the interest rate equation. We also submitted the Markov Switching model to the linearity test of Di Sanzo (2009), which is based on a bootstrap distribution of the likelihood ratio, but could not reject the null hypothesis of linearity. Full details of the specification and linearity tests are given in the Appendix.

4.2 Impulse response functions

The effect of independent structural shocks on endogenous variables are represented by impulse response functions in Figure 2.

Figure 2: Impulse response function for inflation, output, interest rate, and exchange rate shocks. Units are percentage points / 100. Source: Authors' calculations. Note: Shocks represent one standard deviation. Legend: A continuous blue line indicates the response in regime $S_t = 0$ ("Normal"); the dotted red line indicates the response in regime $S_t = 1$ ("Crisis").



The responses are in general consistent with standard new Keynesian models. An unexpected shock to inflation does not cause reaction in other variables, as it is assumed that both output and interest rate should react only to inflation expectations. Shock to output gap causes a rise in inflation and a response of monetary policy. There is high persistence, with an effect on the steady state even after 20 quarters. In turn, the unexpected shock to interest rate reduces output and inflation.

Regime switching affects the response of inflation given supply shocks and shocks to the exchange rate. A typical supply shock has higher relative effects during the “crisis” period simply because of higher estimated standard deviation. Regarding the exchange rate shock, the magnitude of pass-through is much larger in “crisis” regime, so the effect on inflation is stronger and lasts for approximately twice as long. The behavior of each variable was calculated by assuming no regime switching after the shock.

4.3 Identification of high exchange rate pass-through regimes

One of the major results of the MS model is the identification of macroeconomic regimes, which, in our case, correspond either to the “normal” regime (low exchange rate pass-through and lower volatility of shocks to inflation) or the “crisis” regime (high exchange rate pass-through and larger volatility of shocks to inflation).

As previously commented, both regimes demonstrate strong persistence, with a slightly higher value attributed to the “normal” regime. In effect, the expected duration for the “normal” cycle is $E(D|S_t = 0) = 24.0$ quarters against $E(D|S_t = 1) = 22.7$ quarters for the “crisis” cycle.

The graphs in Figure 3 show filtered and smoothed probabilities for each regime throughout the period 2000 to 2015. Note that the probabilities tend to concentrate around 1 or zero most of the time, allowing for a plausible identification of regimes and confirming the usefulness of the model. We are able to clearly identify two periods of high exchange rate pass-through and high volatility of shocks to inflation, with duration between 6 and 14 quarters. Table 5 summarizes the information on the beginning and end of each period, as well as on exchange rate movements and accumulated inflation. The “crisis” regime totals 20 quarters, whereas the “normal” one extends for 44 consecutive quarters.

Table 5: Periods of high exchange rate pass-through and high volatility of shocks to inflation identified by the MS model. Source: Authors' calculations.

Beginning	End	Duration (quarters)	Identification	Larger exchange rate depreciation (in one quarter)	Inflation accumulated in the period (IPCA)
2000/q1	2003/q2	14	Internet bubble (USA) and domestic electoral crisis	31.4%	36.3%
2014/q3	2015/q4	6	Domestic political crisis	24.7%	12.9%

During each cycle of the "crisis" regime, we find events of large exchange rate depreciations, between 24.7% and 31.4%, in at least one of the quarters. Exchange rate shocks above certain limits is one of the arguments presented by Correa & Minella (2010) for the nonlinear behavior of the exchange rate pass-through, and this feature appears to be relevant here. Note that mean exchange rate depreciation per quarter in "crisis" regimes reaches 5.22% against a mean appreciation of 0.6% in "normal" regimes. Taking the whole sample, we observe a mean depreciation of 1.2% per quarter. Notwithstanding, we identified some quarters in which large exchange rate depreciations (above 10%) were not enough to characterize a "crisis" regime in the pass-through. In other words, in a few cases, the economy appears to be able to absorb the exchange rate shock without significant pass-through to inflation. For example, we mention quarters 2011/q3 and 2012/q2, which had exchange rate depreciations of 17.2% and 10.3%, respectively. Mean inflation per quarter during the "crisis" regime was 2.19%, a much higher value than the overall sample mean of 1.60%, or the "normal" regime mean of 1.34%. Note that quarterly inflation exceeded 2% in only one occasion (2004/q3) out of 44 in which the "normal" regime was active.

The first "crisis" cycle begins in 2000/q1 and lasts until mid-2003. The period is characterized by several events that affected confidence in the Brazilian economy. First, the transition to the floating exchange rate regime and the resulting sharp exchange rate depreciation of 1999 resulted in a sudden increase in inflation for the following quarters. Besides, the burst of the US stock market bubble for high-tech companies in 2000 triggered considerable uncertainty in international financial markets. Emerging market economies such as Brazil suffered capital outflows, exchange rate depreciations and increased country risk premium. Added to this, the political crisis in the neighbor country Argentina, a major trade partner, was particularly severe in the years 2000-

2001. To make things worse, the Brazilian economy was hit by a systemic shortage of energy between 2001-2002 which forced the government to cut up to 20% of consumption in some regions. All this facts affected the perception of country risk in Brazil. For instance, in the third quarter of 2000, the Brazilian exchange rate depreciation amounted to 8.42%, with a peak in inflation of 3.82%. In 2001/q3, we observe another strong exchange rate shock with further increase in country risk premium[5].

Over the year of 2002, the Brazilian presidential elections marked another round of harsh confidence crisis. Markets were skeptical about the economic policy intentions of the labour's party presidential candidate, which was winning by far at the opinion polls. The country risk premium increased considerably, reaching its peak in October 2002.[6] The domestic currency experienced more than 50% depreciation in 2002 whereas yearly inflation exceeded 12%. Indeed, inflation volatility only decreased at the end of 2003, as the new government's monetary and economic policies became consolidated as orthodox and adherent to the principles of inflation-targeting.

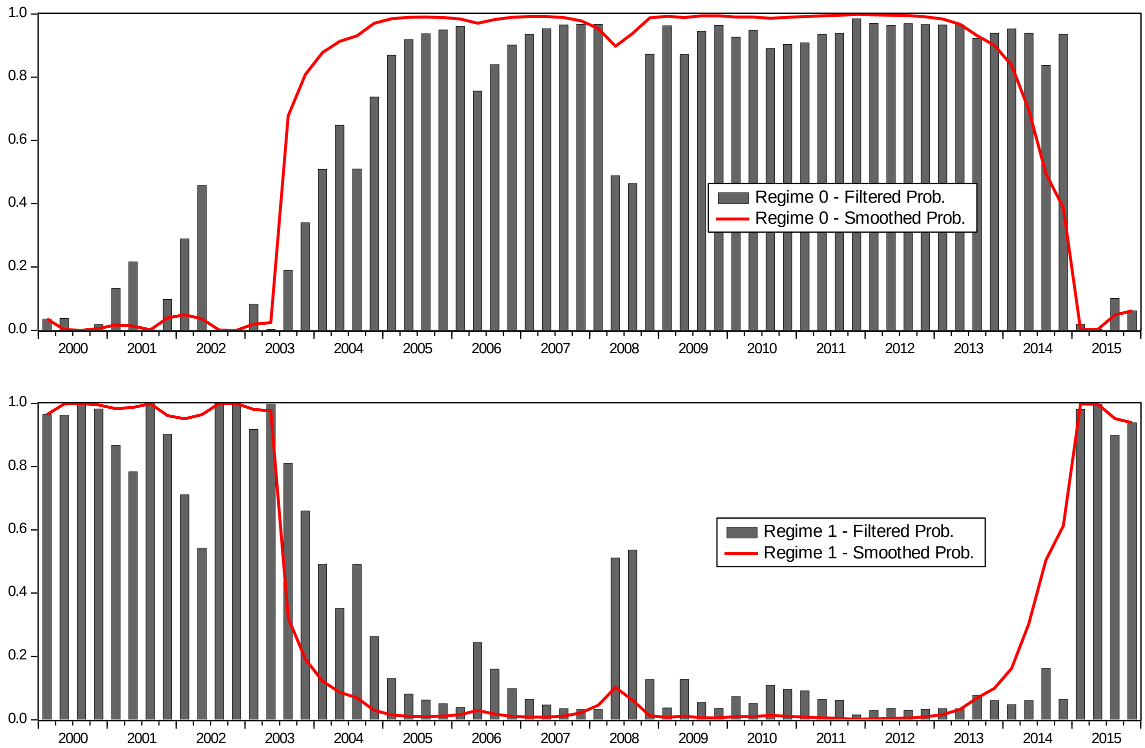
The following period, a long "normal" cycle, starts from mid-2003 and lasts until the end of 2014, totaling 44 quarters in a row. Note that the exchange rate had quite a significant appreciation from the beginning to the end of the period, 32% from 2003/q2 to 2014/q2, although we observe a few quarters with depreciations higher than 10%.

The fact that the exchange rate pass-through was low in that period is consistent with findings of Pimentel, Modenesi & Luporini (2015) about the asymmetric effect of exchange rate movements. According to their study, appreciations tend to exhibit a much lower pass-through level than depreciations of the same size. The total appreciation of 32% contributed to keep inflation in low levels during this period. What is a striking fact is how come depreciations greater than 10% in some quarters did not triggered a higher level of pass-through? For example, in the second and third quarters of 2008, during the onset of the international financial crisis, the MS model indicates an increase in the probability of a "crisis" cycle. The exchange rate depreciation reaches 42% between 2008/q3 and q4, even though inflation is kept at relatively low levels for both 2008 and the following year. As a result, the smoothed probabilities signal that the

[5] The Embi+BR country risk premium index, measured by JP Morgan, had an average monthly value of 1,165 basis points in October 2001.
[6] The Embi+BR index had an average monthly value of 2,039 basis points in October 2002.

economy appears to continue under a “normal” cycle. Thus, according to the model, the sharp depreciation was not enough in order to substantially raise the level of pass-through. This finding is in contrast with Correa & Minella’s (2010) threshold explanation. In our opinion, there are two possible reasons for this phenomena. First, the sudden negative shock in economic activity during 2008-2009 could be helping to alleviate inflation pressures, in line with the Phillips curve assumptions. Second, there appears to be a role for confidence. The “crisis” cycle is being signaled by the model during phases when the Brazilian economy appears to be in distress, as it was the case from 2000-2003 and during the final cycle (2015) described below. The 2008-2009 crisis events were originated in foreign economies and did not affected, specifically, the confidence in the Brazilian economic fundamentals or confidence in the inflation targeting regime.

Figure 3: Probabilities estimated for the MS model for Regime 0 (at the top) and Regime 1 (at the bottom). Source: Authors’ calculations. Note: Filtered probabilities in bars and smoothed probabilities in lines. Sample period: 2000/q1 to 2015/q4.

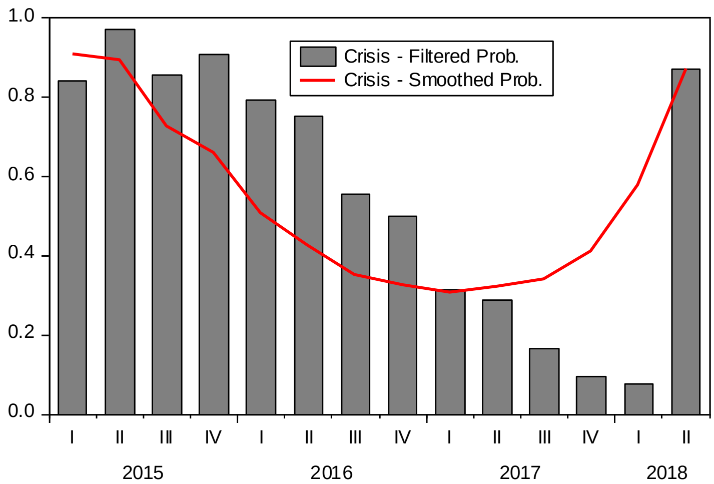


From the beginning of 2015, the MS model began to clearly indicate a new “crisis” cycle. We see a strong exchange rate depreciation (18.9%) in 2015/q1 and a constant rise in inflation levels. The period is characterized by a deep downturn on economic activity in Brazil, political crisis at the federal government level, and fiscal hardships. As a matter of fact, the high persistence of inflation combined with an expansionary government budget, though unsustainable, have hindered the actions of the monetary authority. Based on model findings, one can expect that the 24.7% exchange rate shock of 2015/q3 would drag substantial effect on inflation during the following quarters.

4.4 Putting the model to work: analyzing pass-through from 2016/q1 until 2018/q2

In order to test the MS model, we have predicted the probabilities of each regime for the out-of-sample period, from 2016/q1 until 2018/q2, using the model parameters previously estimated. In this exercise, at each point *t* in time, we feed the data series observed until *t* and calculate the implied probabilities for the "crisis" and "normal" regime of exchange rate pass-through.[7] The results are shown in Figure 4.

Figure 4: Probabilities of "Crisis" regime predicted by the MS model for out-of-sample period (2016/q1-2018/q2). Source: Authors' calculations. Note: Filtered probabilities in bars and smoothed probabilities in lines.



As mentioned before, the year 2015 is characterized by a deep downturn on economic activity in Brazil, political crisis at the federal government level, and fiscal hardships. The high persistence of inflation is fueled by expansionary fiscal policy and a

[7] The exercise is considered out-of-sample in the sense that we do not re-estimate the model at each step, but we use the point estimates obtained before.

delayed response of the monetary authority to inflation. The model is under a “crisis” cycle which began in the first quarter of 2015.

The out-of-sample prediction offers us new and insightful results. The crisis cycle lasts for seven quarters, until the 3rd quarter of 2016, according to the filtered probabilities. This date coincides with the change in the board of directors of the Central Bank of Brazil, as the new president took office in August 2016. Starting from the beginning of 2017 the model indicates a higher probability for a new normal cycle, which lasts until second quarter of 2018, almost the end of our sample. This evidence shows a possible role for the credibility of the monetary authority in moderating the effect of pass-through. A more credible central bank, in terms of its willingness to react to inflation, will be able to coordinate agents expectations to a lower level of inflation, for the same given exchange rate shock. Or in other words, when inflation expectations are well-anchored, firms will tend to moderate the degree of repricing, and exchange rate pass-through will be lower. Indeed, both inflation expectations and observed consumer price inflation drops sharply from the end of 2016 until basically the beginning of 2018. We see evidence that the new board was able to better anchor inflation expectations and thus achieve a lower pass-through.

Our finding is in line with recent research relating central bank credibility with a variable degree of exchange rate pass-through. de Mendonça & Tiberto (2017) measure central bank credibility in a panel data including 114 developing countries to find that credibility is able to counteract the effects of exchange rate depreciations on inflation. They argue that monetary authorities committed to anchor inflation expectations can obtain a gain in credibility that helps to reduce exchange rate pass-through. In a study focusing in the Brazilian economy, Montes & Ferreira (2018) find that central bank credibility is capable of mitigating the transmission of uncertainties about the exchange rate to uncertainties about the inflation and interest rates. This channel, of uncertainty reduction, could also be working for the benefit of reducing the degree of repricing by firms whenever the economy is subject to an exchange rate shock.

A note must be made regarding the last period of the out-of-sampler prediction. The model point to a high probability of the “crisis” cycle coming back during 2018/q2. In fact, the exchange rate has suffered a sharp depreciation, over 20% from 2017/q4 to 2018/q2 and observed consumer price inflation turned out to be unexpectedly high in 2018/q2. We consider two alternative hypothesis to explain this spike. The first one is related to a domestic supply shock. There was a huge disruption of the domestic supply chain in Brazil due to a national strike of truck drivers, and prices for food and other basic items went suddenly high. The second, alternative, explanation is that the Brazilian economy may have experienced a return to the "crisis" cycle of high pass-through. In any case, the sustained credibility of the central bank may allow the economy either to remain or come back to a low pass-through "normal" cycle, specially if it is combined with a positive a shock to fundamentals in the near term.

5. Conclusion

The present paper investigates the exchange rate pass-through in the Brazilian economy during the floating exchange rate period using a structural new Keynesian Markov-switching model. Our basic hypothesis is of nonlinear behavior in the pass-through coefficient, combined with regime switching in the volatility of shocks to the aggregate supply curve.

Our empirical results show that the exchange rate pass-through assumed two possible states, or regimes, during the years 2000 until 2015. In the first regime, conveniently referred to as “normal”, the pass-through is very low and statistically nonsignificant while, at the same time, the volatility of shocks to inflation is also relatively low. In the second regime, the pass-through is relevant and significant, of about 10.3% in the long run, while the volatility of shocks to inflation is also relatively higher. The high pass-through regime was named “crisis” as it appears to occur in periods when the Brazilian economy was facing different types of distress, or confidence crisis. The MS model outperformed the linear specification by some usual econometric criteria, such as the Schwartz criterion.

Our estimates for the nonlinear exchange rate pass-through in the Brazilian economy are consistent with those obtained by Carneiro, Monteiro & Wu (2004), Correa & Minella (2010), and Pimentel, Modenesi & Luporini (2015), taking into account the differences between the periods of interest and the estimation method. It should be highlighted that the long-run pass-through, of 10.3% even in the “crisis” regime, is relatively low according to the criteria set by Goldfajn & Werlang (2000) and Belaisch (2003), which implies some reasonable capacity of the economy to absorb exchange rate shocks without greater effect to consumer inflation.

The presence of regime switching in the volatility of shocks to inflation could be related to the heteroskedasticity of inflation itself (Engle, 1982; Brunner & Hess, 1993) or to theoretical arguments, such as Ball & Cecchetti's (1990) and Owyang's (2001). These authors point out that higher inflation levels lead to higher volatility and greater uncertainty over future inflation expectations. That is, unexpected inflation shocks increase uncertainty over future inflation and causes larger volatility in inflation in the subsequent periods. The system would tend to remain in a high volatility regime for some periods, which is well described in our findings.

The present study innovates in terms of methodology by using a Markov Switching DSGE model to identify nonlinear exchange rate pass-through in an emerging economy. For the case of Brazil, previous studies usually sought to measure exchange rate pass-through directly in the Phillips curve (Carneiro, Monteiro & Wu, 2004; Correa & Minella, 2010; Nogueira Jr, 2010; Pimentel, Modenesi & Luporini, 2015) or in regressions specifically derived from microfoundations, such as in Albuquerque & Portugal (2005). As econometric method, the literature uses nonlinear least squares (Carneiro, Monteiro & Wu, 2004), threshold models (Correa & Minella, 2010), smooth transition regression (Nogueira Jr, 2010), asymmetric SVAR models (Pimentel, Modenesi & Luporini, 2015), linear state-space models with variable parameters (Albuquerque & Portugal, 2005), or univariate Markov switching regressions on the Phillips curve (Baharumshah et al. 2017). Our review of the extant literature did not find previous publications using Markov switching DSGE models to assess exchange rate pass-through in emerging or advanced economies. At the same time, we differ from that of Baele et al. (2015) as they are focused on regime changes of monetary policy.

We make a novel contribution by identifying four phases, or cycles, for the pass-through behavior in the Brazilian economy during the inflation targeting period. Under our interpretation, two "crisis" cycles, one at the beginning of the sample period (2000-2003) and the other at the end (2015), are separated by a long "normal" cycle (2003-2014). In the last part of the sample (2016-2018/q2), analyzed in a prediction exercise, the economy appears to have gradually returned to a "normal" cycle. The "crisis" cycles occur during periods where the Brazilian economy is under different types of stress. In some cases, the economic strain was due to external factors, such as the emerging markets confidence runs of 2000-2001. In other cases, political facts such as the crisis of

2002, triggered by uncertainty during the presidential elections, or the political crisis of 2015, appear to be related to exchange rate shocks complemented with a higher level of pass-through.

We conclude that the overall results of the MS model are useful to the economic analysis and interpretation of the exchange rate pass-through dynamics and its nonlinear effects to a great extent. The model provides relevant information for inflation forecast, especially during large exchange rate shocks, when there is more uncertainty about the effect of the pass-through. For example, the econometrician forecasting inflation with a linear model would assume a 7.6% level of pass-through in the long run. On the other hand, if the econometrician is using a nonlinear model of our kind, there is room for a more subtle interpretation. If the economy is under a "crisis" regime, the MS model indicates that the expected pass-through is substantially higher, of 10.35%. However, under a "normal" regime, one could expect a long run pass-through roughly null, of 0.57%. The difference between the two model types carry obvious consequences for policy analysis and design. Finally, our findings underscore the importance of assessing regime switching in certain structural parameters of the Brazilian economy in new Keynesian models, corroborating, to some extent, Gonçalves, Portugal & Arágon (2016).

References

Adolfson, M.; Laséen, S.; Lindé, J.; Villani, M. (2007) Bayesian estimation of an open economy DSGE model with incomplete pass-through. *Journal of International Economics*, v.72, p. 481-455, 2007.

Albuquerque, C. R.; Portugal, M. S. (2005) Pass-through from exchange rate to prices in Brazil: an analysis using time-varying parameters for the 1980-2002 period. *Revista de Economia*, vol 12, n. 1, Banco Central del Uruguay, Mai. 2005.

Altug, S.; Çakmakli, C. (2016) Forecasting inflation using survey expectations and target inflation: evidence for Brazil and Turkey. *International Journal of Forecasting*, v. 32, issue 1, pp. 138-153, Jan.-Mar. 2016.

Alves, S. A. L.; Areosa, W. D. (2005) Targets and inflation dynamics. *Banco Central do Brasil Working Papers Series,* n. 100, Oct. 2005.

Areosa, W. D.; Medeiros, M. (2007). Inflation dynamics in Brazil: the case of a small open economy. *Brazilian Review of Econometrics*, Vol. 27, n. 1, 2007, pp.131-166.

Arruda, E.; Ferreira, R.; Castelar, I. (2008) Modelos lineares e não lineares da curva de Phillips para previsão da taxa de inflação no Brasil. *Annals of the XXXVI National Economic Meeting ANPEC*, Salvador, BA, Brazil. Dez. 2008.

Baele, L.; Bekaert, G.; Cho, S.; Inghelbrecht, K.; Moreno, A. (2015) Macroeconomic regimes. *Journal of Monetary Economics*, v. 70, Mar. 2015, pp. 51-71.

Baharumshah, A. Z., Soon, S. V., & Wohar, M. E. (2017). Markov-switching analysis of exchange rate pass-through: Perspective from Asian countries. *International Review of Economics & Finance*, 51, 245-257.

Ball, L.; Cecchetti, S. (1990) Inflation and uncertainty at short and long horizons. *Brookings Papers on Economic Activity*, v. 1, pp. 215-54, 1990.

Belaisch, A. (2003) Exchange rate pass-through in Brazil. *IMF Working Papers*, WP/03/141, Jul. 2003.

Bellone, B. (2005) Classical estimation of multivariate Markov-switching models using MSVARlib. *EconWPA*, Econometrics 0508017, 2005.

Bianchi, F. (2013). Regime switches, agents' beliefs, and post-World War II U.S. macroeconomic dynamics. *Review of Economic Studies*, 80 (2), pp. 463-490.

Brunner, A. D.; Hess, G. D. (1993) Are Higher Levels of Inflation Less Predictable? A State-Dependent Conditional Heteroscedasticity Approach. *Journal of Business & Economic Statistics*, vol. 11, Issue. 2, 1993.

Carneiro, D. D.; Monteiro, A. M. D.; WU, T. Y. H. (2004) Mecanismos não-lineares de repasse cambial para o IPCA. *Revista de Economia e Administração*, v.3, n.1, pp.01-14, Jan-Mar 2004.

de Castro, M. R.; Gouvea, S. N.; Minella, A.; Santos, R.; Souza-Sobrinho, N. F. SAMBA: Stochastic Analytical Model with a Bayesian Approach. *Brazilian Review of Econometrics*, v. 35, n. 2, 2015.

Central Bank of Brazil (2015). *Relatório de Inflação (Inflation Report)*. Brasilia: Banco Central do Brasil, v. 17, n. 2, Jun. 2015.

CHO, S. (2014) Characterizing Markov-switching rational expectation models. *University of Indiana Economics Department Spring Working Papers*. Available at: http: //www.indiana.edu/~econdept/workshops/Spring_2013_Papers/SEONGHOON%20CHO.pdf.

Cho, S.; Moreno, A. (2011) The forward method as a solution refinement in rational expectations models. *Journal of Economic Dynamics and Control*, v. 25 (3), pp. 257-327.

Correa, A. D. S.; Minella, A. (2010). Nonlinear mechanisms of the exchange rate pass-through: A Phillips curve model with threshold for Brazil. *Revista Brasileira de Economia*, *64*(3), 231-243.

Di Sanzo, S. (2009) Testing for linearity in Markov switching models: a bootstrap approach. *Statistical Methods and Applications*, v. 18, issue 2, Jul. 2009, pp. 153-168.

Dixit, A. (1989) Hysteresis, import penetration, and exchange rate pass-through. *The Quarterly Journal of Economics*, v. 104, n. 2, pp. 205-228, May 1989.

Donayre, L.; Panovska, I. (2016) State-dependent exchange rate pass-through behavior. *Journal of International Money and Finance*, v. 64, Jun. 2016, pp. 170-195.

Engle, R. F. (1982) Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, vol. 50, no. 4, July, 1982, pp.987-1007.

Farmer, R. E. A.; Waggoner, D. F.; Zha, T. (2011) Minimal state variable solutions to Markov-switching rational expectations models. *Journal of Economic Dynamics & Control*, v. 35, pp. 2150-2166, 2011.

Fasolo, A. M.; Portugal, M. S. (2004) Imperfect rationality and inflationary inertia: a new estimation of the Phillips curve for Brazsil. *Estudos Econômicos*, v. 34, n. 4, pp. 725-779, Out-Dez, 2004.

Fernández-Villaverde, J.; Guerrón-Quintana, P.; Rubio-Ramírez, J. F. (2010) Fortune or virtue: time-variant volatilities versus parameter drifting in U.S. data. *NBER Working Paper*, No. 15928, Apr. 2010.

Figueiredo, F. M. R.; Gouvea, S. (2011) Repasse Cambial para a Inflação: o papel da rigidez de preços. In: Banco Central do Brasil (2011). *Dez Anos de Metas para Inflação 1999-2009*. Brasília: Banco Central do Brasil, 2011.

Furlani, L. G. C.; Portugal, M. S.; Laurini, M. P. (2010) Exchange rate movements and monetary policy in Brazil: econometric and simulation evidence. *Economic Modelling*, v. 7, issue 1, PP. 284-295, Jan. 2010.

Goldfajn, I.; Werlang, S. R. C. (2000) The pass-through from depreciation to inflation: a panel study. *Banco Central do Brasil Working Papers Series,* n. 5, Set. 2000.

Gonçalves, C. C. S.; Portugal, M. S.; Arágon, E. K. S. B. (2016) Assessing Brazilian macroeconomic dynamics using a Markov-switching DSGE model. *EconomiA*, Mar. 2016.

Hamilton, J. D. (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, v. 57, n. 2, pp 357-384, Mar. 1989.

Hamilton, J. D. (1994) *Time Series Analysis*. Princeton University Press, 1994.

Hamilton, J. D. (2005) What's real about the Business Cycle? *Federal Reserve Bank of St. Louis Review*, 87 (4), pp. 435-452, Jul-Aug 2005.

Hamilton, J. D. (2014) Macroeconomic regimes and regime shifts, in Taylor, J. B. & Uhlig, H. (Eds.) *Handbook of Macroeconomics*, Vol. 2.

Holmes, M. J. (2009) The response of exchange rate pass-through to the macroeconomic environment. *The Open Business Journal*, v. 2, pp. 1-6, 2009.

Iboshi, H. (2016) Monetary policy regime shifts under the zero lower bound: an application of a stochastic rational expectations equilibrium to a Markov switching DSGE model. *Economic Modelling*, v. 52, part A, Jan. 2016, pp. 186-205.

Khemiri, R.; Ali, M. S. B. (2012) Exchange rate pass-through and inflation dynamics in Tunisia: a Markov-switching approach. *Economics E-journal Discussion Paper*, n. 2012-39, Aug., 2012.

Kohlscheen, E. (2010) Emerging Floaters: pass-throughs and (some) new commodity currencies. *Banco Central do Brasil Working Papers Series*, n. 224, Nov. 2010.

Krolzig, H. M. (1997) *Markov-Switching Vector Autoregressions: modelling, statistical inference and application to business cycle analysis*. Springer-Verlag: Berlin, 1997.

Liu, P.; Mumtaz, H. (2010) Evolving macroeconomic dynamics in a small open economy: an estimated Markov-switching DSGE model for the United Kingdom. *Bank of England Working Paper* No. 397.

Mankiw, N. G.; Reis, R. (2002) Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve. *Quarterly Journal of Economics*, 117 (4), pp. 251-301.

Mazali, A. A.; Divino, J. A. (2010) Real wage rigidity and the new Phillips curve: the Brazilian case. *Revista Brasileira de Economia*, v.64, n.3, pp. 291-306, Jul-Sep 2010.

de Mendonça, H. F., ; Tiberto, B. P. (2017). Effect of credibility and exchange rate pass-through on inflation: An assessment for developing countries. *International Review of Economics & Finance*, 50, 196-244.

Minella, A.; De Freitas, P. S.; Goldfajn, I.; Muinhos, M. K. (2003) Inflation Targeting in Brazil: Constructing Credibility under Exchange Rate Volatility. *Banco Central do Brasil Working Papers Series*, n. 77, Jul. 2003.

Montes, G. C., ; Ferreira, C. F. (2018). Does monetary policy credibility mitigate the effects of uncertainty about exchange rate on uncertainties about both inflation and interest rate?. *International Economics and Economic Policy*, 1-30.

Nogueira JR, R. P. (2010) Inflation environment and lower exchange rate pass-through in Brazil: is there a relationship? *Revista Brasileira de Economia*, v. 64, n. 1, pp. 49-56, Jan.-Mar. 2010.

Oliveira, L.; Feijó, F. T. (2015) Curva de Phillips com mudança de regime markoviano: uma análise da economia brasileira para o período 1995-2014. *Annals of the XLIII National Economic Meeting ANPEC*. Florianópolis, SC, Brazil. Dez., 2015.

Owyang, M. T. (2001) Persistence, excess volatility, and volatility clusters in inflation. *Federal Reserve Bank of St. Louis Review*, v. 83, n. 6, pp. 41-52, Nov-Dec 2001.

Palma, A. A.; Portugal, M. S. (2014) Preferences of the Central Bank of Brazil under the inflation targeting regime: estimation using a DSGE model for a small open economy. *Journal of Policy Modeling*, v. 36, issue 5, pp. 824-839, Sep-Oct 2014.

Pimentel, D. M.; Modenesi, A. M.; Luporini, V. (2015) Assimetrias no repasse cambial para a inflação: uma análise empírica para o Brasil (1999 a 2013). *Annals of the XVIII Economic Meeting of Southern Region ANPEC Sul*. Porto Alegre, RS, Brazil. Jun., 2015.

Rodrigues, W. G.; Mori, R. (2015) Mudanças de regimes na função de reação do Banco Central do Brasil: uma abordagem utilizando Markov regime switching. *Annals of the XLIII National Economic Meeting ANPEC*. Florianópolis, SC, Brazil. Dez., 2015.

Sachsida, A. (2013) Inflação, desemprego e choques cambiais: uma revisão da literatura sobre a Curva de Phillips no Brasil. *Revista Brasileira de Economia*, v. 67, n.4, pp. 521-531, Out.-Dez. 2013.

Silveira, M. A. C. (2008) Using a Bayesian approach to estimate and compare New Keynesian DSGE models for the Brazilian economy: the role for endogenous persistence. *Revista Brasileira de Economia*, v. 62, n. 3, pp. 333-357, Jul-Sep 2008.

Sims, C. (2002) Solving linear rational expectations models. *Computational Economics*, v. 20 (1-2), pp. 1-20.

Sims, C.; Zha, T. (2006) Were there regime switches in U.S. monetary policy? *American Economic Review*, 91(1), pp.54-81.

Taylor, J. (2000) Low inflation, pass-through and the pricing power of firms. *European Economic Review*, 44, pp. 1389-1408, 2000.

Tombini, A. A.; Alves, S. A. L. (2006) The recent Brazilian disinflation process and costs. *Banco Central do Brasil Working Papers Series*, n. 109, Jun. 2006.

Tristão, T. S.; Torrent, H. S. (2015) Relações não lineares na curva de Phillips: uma abordagem semi-paramétrica. *Economia Aplicada*, v. 19, n. 4, 2015, pp. 679-703.

Vieira, H. P.; Pereira, P. V. (2013) A study of the Brazilian business cycles (1900-2012). *Brazilian Review of Econometrics*, v. 33, no 2, pp. 123-143, Nov. 2013.

Winkelried, D. (2014) Exchange rate pass-through and inflation targeting in Peru. *Empirical Economics*, v. 46, issue 4, Jun. 2014, pp. 1181-1196.

Exchange Rate Pass-Through in Brazil: a Markov switching DSGE estimation for the inflation targeting period

Appendix

1. Data Sources and Stationarity Tests

The seasonally adjusted quarterly consumer price index IPCA (%) was used as a measure of inflation. First, the monthly series[8] was accumulated quarterly and then we applied a multiplicative moving average seasonal adjustment. The output gap was obtained from the quarterly GDP logarithm at seasonally adjusted market values[9] and the trend was estimated by the Hodrick-Prescott filter. The remaining component (business cycle) was considered to be the output gap. A broader window, beginning in 1996, was used for extracting the gap so as to avoid the tail effect at the beginning of the period. The extraction of output gap by Beveridge-Nelson in the AR(1) model was also tested, but we did not obtained the desired statistical properties. In turn, the quarterly exchange rate movement is calculated as the first difference of the nominal exchange rate value, R$ vis-à-vis US$, at the end of the period.[10]

We measured the quarterly interest rate i_t as the nominal interest rate discounted for the long-run real interest rate. In this sense, we try to account for the fact that the Brazilian economy experienced a sharp reduction in its long-run real interest rate between 2005 and 2012. Thus, the data we are taking to the model is the nominal rate in excess of the long run interest rate, and we should obviously consider this characteristic when interpreting our estimation results. In order to calculate our series i_t, we first took the quarterly equivalent of the monthly Selic Over rate (% p.a.)[11] at end of the period. The long-run real interest rate was estimated as the trend of an HP filter obtained with the difference of the nominal rate minus observed inflation. We then discount the estimated long-run real interest rate from our quarterly nominal interest rate.

---

[8] Source: Series 433 (monthly IPCA). Central Bank of Brazil Time Series.
[9] Source: Series 22109 (seasonally adjusted GDP). Central Bank of Brazil Time Series.
[10] Source: Series 3696 (free exchange rate). Central Bank of Brazil Time Series.
[11] Source: Monthly Over/Selic interest rate (% p.a.) series. IPEA Data System.

Finally, the Central Bank of Brazil's[12] survey-based market expectations were used to calculate inflation and output gap expectations for the subsequent quarter. Inflation expectation was measured as the median value of the survey for the consumer prices inflation (IPCA) for the three months of the subsequent quarter, observed on the first business day of the current quarter. The monthly values were accumulated to obtain the quarterly inflation expectation. Data observed on the first business day is used to circumvent the endogeneity problem between inflation in the current quarter and inflation expectations for the subsequent period, without having to rely on instrumental variables. In fact, one avoids the correlation between exogenous shock to inflation in the current period $\epsilon_{\pi,t}$ and future expectations $E_t\pi_{t+1}$ as basically information from the current period is not included in the measure. On the other hand, it was necessary to use a calculation procedure to obtain the output gap expectations for the subsequent quarter. The variable observed by the market survey is the real growth of GDP (% p.a.) for the subsequent quarter. The first step consisted in extracting the equivalent quarterly growth rate, and then estimating the real domestic product expected for *t+1*. The seasonally adjusted series observed in the past was included up to period *t*, under the value expected for *t+1*, forming a new series. The log of the complete new series was extracted and its trend was estimated by the HP filter. The value of the cycle in period *t+1* thus corresponds to an estimate of the output gap expectation.

Table A1 presents the descriptive statistics for each of the series.

Table A1: Time series descriptive statistics. Units are percentage points / 100. Source: Authors' calculations.

	π_t	y_t	i_t	Δe_t	$E_t\pi_{t+1}$	$E_t y_{t+1}$
Mean	0.0161	0.0002	0.0161	0.0122	0.0133	0.0096
Median	0.0145	0.0041	0.0164	-0.0066	0.0126	0.0103
Maximum	0.0551	0.0331	0.0327	0.3143	0.0305	0.0351
Minimum	0.0012	-0.0488	0.0044	-0.1708	0.0083	-0.0352
Std. dev.	0.0089	0.0164	0.0063	0.0961	0.0040	0.0159
Skewness	1.9608	-0.9021	0.3704	0.8913	1.9747	-0.5447
Kurtosis	8.2738	3.9325	2.8547	3.9075	8.7659	2.8346
Jarque-Bera	115.1797	10.9999	1.5198	10.6694	130.2482	3.2383
Probability	0.0000	0.0041	0.4677	0.0048	0.0000	0.1981

The six series are tested for stationarity. We provide the results for the conventional Augmented Dickey-Fuller (ADF) test, with intercept, and Phillips-Perron (PP) test, in Table A2. Of these six series, only one is in first difference (Δe), and the

[12] Source: Central Bank of Brazil Market Expectations System (Focus Report).

other ones are used in the level. As demonstrated, both the ADF and PP tests reject the presence of unit root at the 5% significance level for inflation, output gap, exchange rate movement, and inflation expectation. For the interest rate series, the ADF test rejects the presence of unit root at 5%, whereas the PP test cannot reject it, not even at 10%. Nevertheless, we do not consider this evidence strong enough to invalidate the use of this series.

Table A2: Unit root tests (full sample). Source: Authors' calculations.

	Augmented Dickey-Fuller			Phillips-Perron		
	t-Statistic	Prob.	Lag length	Adj. t-Statistic	Prob.	Bandwidth
π_t	-4.8340	0.0002	0	-4.7683	0.0002	4
y_t	-4.2781	0.0011	1	-2.9916	0.0411	2
i_t	-3.1076	0.0311	1	-2.4491	0.1328	2
Δe_t	-6.8397	0.0000	0	-6.7941	0.0000	6
$E_t\pi_{t+1}$	-2.9610	0.0044	2	-4.6421	0.0003	9
$E_t y_{t+1}$	-2.5837	0.1017	1	-2.4118	0.1426	3

It is not possible to reject the null hypothesis for the output gap expectation. However, when the unit root tests are taken with a slightly reduced sample, without the last three quarters (2015q2 to 2015q4), we get far better results. The ADF test rejects the unit root hypothesis at 5% and the PP rejects at 10%. We argue that the output expectations suffered a severe shock from the third quarter of 2015 onwards, which has not been totally reversed to its mean yet. Given the theoretical hypotheses of output gap stationarity and rational expectations, the reversion should take place in the long run. We conclude that the econometric estimation of the model can proceed without any restrictions.

2. Initial parameter values and parameter space

The initial values for maximum likelihood estimation were chosen based on estimation results obtained by Baele et al. (2015) and are shown in Table A3. From the value of θ_0, we maximize the log-likelihood function with a numerical constraint optimization algorithm. Parameters are restricted to a domain of possible values, also shown in Table A3, and which stem from the theoretical constraints of the original RE model. Our constraints are similar, but lighter than those of Baele et al. (2015).

Table A3: Initial parameters and restrictions of the MS model. Source: Authors' calculations.

Parameter	Initial value	Minimum	Maximum
δ	0.425	0.00001	1
λ	0.102	0.00001	$+\infty$
$\kappa_{1S_t}(S_t = 0)$	0.005	$-\infty$	$+\infty$
$\kappa_{1S_t}(S_t = 1)$	0.09	$-\infty$	$+\infty$
μ	0.675	0.00001	1
ϕ	0.10	0.00001	$+\infty$
ρ_i	0.834	0.00001	0.99999
β	1.10	0.00001	$+\infty$
γ	0.80	0.00001	$+\infty$
ρ_e	0.16	$-\infty$	0.99999
α	0.90	0.00001	1
$\sigma_{AS}(S_t = 0)$	0.0038	0.00001	$+\infty$
$\sigma_{AS}(S_t = 1)$	0.0098	0.00001	$+\infty$
σ_{IS}	0.0108	0.00001	$+\infty$
σ_{MP}	0.0043	0.00001	$+\infty$
σ_e	0.0950	0.00001	$+\infty$
p	0.90	0	1
q	0.76	0	1

3. Likelihood Function

Let $n = 4$ be the number of endogenous variables, $m = 8$ the number of regressors of the reduced model, and $T = 64$ the number of observations. Following Hamilton's (1994) notation, consider:

- $y_t = [X_t]$ the vector of endogenous variables, $nx1$;
- $x_t = [X_t^f \; X_{t-1}^f \; X_{t-1}]$ the vector containing the grouped regressors of the reduced model, $mx1$;

- $\Omega_{Var}(S_t) = \Gamma\, \Sigma(S_t)\Gamma'$ the variance-covariance matrix of the reduced model for each state obtained from $\Sigma(S_t)$ and from Γ, nxn;
- $\Pi(S_t)' = [\Omega_1\, \Omega_2\, \Omega_3(S_t)]$ the state-dependent coefficient matrix of the reduced model, nxm.

The same reduced model in equation (22) can be written as a regime-switching VAR model:

$$y_t = \Pi(S_t)' x_t + u_t \qquad\qquad u_t \sim N\big(0, \Omega_{Var}(S_t)\big) \tag{25}$$

After defining this notation for each filtering step, the marginal density of the VAR model, given θ, S_t, ψ_{t-1}, is as follows:

$$f(y_t|\theta, S_t, \psi_{t-1}) = (2\pi)^{-n/2}\sqrt{\left|\big(\Omega_{Var}(S_t)\big)^{-1}\right|}\exp\left\{-\frac{1}{2}[y_t - (\Pi(S_t)'x_t)]'\big(\Omega_{Var}(S_t)\big)^{-1}[y_t - (\Pi(S_t)'x_t)]\right\}$$

The log-likelihood maximization yields a vector of optimal estimated parameters $\hat{\theta}$.

4. Specification and linearity tests

We ran the basic univariate specification tests on the standardized residuals of each equation – serial autocorrelation, normality, and conditional variance – and linearity tests on the MS model. The results of the specification tests on standardized residuals are shown in Table A4. Serial autocorrelation was assessed by the Ljung-Box Q test for 20 lags, using $\min(20, T-1)$ as standard, as suggested by Box, Jenkins & Reinsel (1994).

Table A4: Specification tests on standardized residuals of the MS model (*p* values). Source: Authors' calculations. Note: *p* values in brackets.

Univariate statistical tests	**MS model**

	Inflation	Output gap	Monetary policy	Exchange rate
Serial autocorrelation (p values) standard 20 lags	(0.196)	(0.696)	(0.000)	(0.999)
Skewness	0.304	-1.717	0.392	0.745
Kurtosis	3.178	8.283	3.209	3.629
Jarque-Bera test (p values)	(0.500)	(0.001)	(0.296)	(0.030)
Serial autocorrelation of squared residuals (p values) standard 20 lags	(0.904)	(0.754)	(0.923)	(0.431)

The weakness of the MS model seems to be its inability to eliminate serial autocorrelation in residuals, especially in the equation for monetary policy response. For the other equations, the lack of autocorrelation is not rejected, at least for 20 lags. Baele et al. (2015) admit that these statistics may be biased in small samples, especially when the data-generating process is nonlinear as in our model. In their empirical study, the authors cannot prevent the rejection of the hypothesis of no serial autocorrelation in the residuals of the output gap equation in the MS rational expectations and unrestricted MS-VAR models, even when using critical test values obtained from a Monte Carlo simulation with a small sample. Our analysis included some attempts to change the specification of the model, inserting a larger number of lagged endogenous variables as regressors in all equations ($X_{t-2}, X_{t-3}, X_{t-4}$). Yet, it was not possible to eliminate the signs of serial autocorrelation, so we opted to keep the model simpler. A possible way to circumvent this problem would be to model the shocks in each curve as autoregressive processes. However, that would require a more complex estimation method.

As to the other tests, normal distribution is rejected for the residuals of the output gap and exchange rate equations due to high kurtosis. We checked for the existence of conditional variance through a Ljung-Box Q test in the squared residuals. There was no evidence of conditional variance in any of the error terms of the equations.

Table A5: Specification test on the residuals of the semi-structural linear model (*p* values). Source: Authors' calculations. Note: *p* values in brackets.

Univariate statistical tests	Linear model (without regime switching)			
	Inflation	Output gap	Monetary policy	Exchange rate
Serial autocorrelation (*p* values) standard 20 lags	(0.854)	(0.694)	(0.000)	(0.999)
Skewness	0.390	-1.719	0.392	0.745
Kurtosis	4.187	8.286	3.209	3.629
Jarque-Bera test (*p* values)	(0.047)	(0.001)	(0.298)	(0.030)
Serial autocorrelation in squared residuals (*p* values) standard 20 lags	(0.848)	(0.755)	(0.925)	(0.431)

For the sake of comparison, Table A5 also shows the same tests performed on the model without regime switching, confirming the difficulty in eliminating serial autocorrelation in the residuals. It should be remarked that the regime-switching mechanism in the inflation curve reduced kurtosis of the distribution of standardized errors.

The literature recognizes the difficulty in testing for linearity in Markov-switching models, since usual regularity conditions for likelihood-based inference are violated (Hansen, 1992; Carrasco, Hu & Ploberger, 2014; Di Sanzo, 2009). Under the null hypothesis of linearity some parameters are not identified, such as transition probabilities. Therefore, the asymptotic distribution of the test statistics of interest, such as LR, no longer has its conventional chi-square form. We submitted the MS model to Di Sanzo's (2009) test, which is based on a bootstrap distribution of the likelihood ratio under the null hypothesis. This test compares the likelihood ratio (LR) obtained from the linear (H_0) and the alternative (H_1) models, with a bootstrap distribution of a likelihood ratio calculated under the LR^* null in order to find the corresponding p-value.

The author gathers evidence that the bootstrap-based test works well in small samples and may be superior to those of Hansen (1992) and Carrasco, Hu & Ploberger (2014) in terms of power and size. In our experiment, the likelihood ratio value between the two models is $LR = 21.2070$. We calculated LR^* 5,000 times, but the result does not allow rejecting the null hypothesis of linearity, as the obtained *p*-value is $p_B = 0.2136$.

References

Baele, L.; Bekaert, G.; Cho, S.; Inghelbrecht, K.; Moreno, A. (2015) Macroeconomic regimes. *Journal of Monetary Economics*, v. 70, Mar. 2015, pp. 51-71.

Box, G. E. P.; Jenkins, G. M. ; Reinsel, G. C. (1994) *Time Series Analysis: forecasting and control*. 3rd ed. Englewood Cliffs, NJ: Prentice Hall, 1994.

Carrasco, M.; Hu, L.; Ploberger, W. (2014) Optimal test for Markov switching parameters. *Econometrica*, v. 82, n. 2, Mar. 2014, pp. 765-784.

Di Sanzo, S. (2009) Testing for linearity in Markov switching models: a bootstrap approach. *Statistical Methods and Applications*, v. 18, issue 2, Jul. 2009, pp. 153-168.

Hamilton, J. D. (1994) *Time Series Analysis*. Princeton University Press, 1994.

Hansen, B. E. (1992) The likelihood ratio test under nonstandard conditions: testing the Markov switching model of GNP. *Journal of Applied Econometrics*, v. 7, Dec. 1992, pp. S61-S82.