ABSTRACT

This paper estimates the natural rate of interest (NRI) for Brazil between the third quarter of 1999 and the second quarter of 2019. The seminal model proposed by Laubach and Williams (2003), re-specified by Holston, Laubach and Williams (2017), is used. Results indicate an NRI with a downward trend, with the lowest values during the 2015-2016 recession. In the last observation of the sample, the NRI was 1.8% for the ex-ante rate. Analyzing the difference between the real interest rate (RIR) and the NRI, we identify three distinct periods in the conduct of Brazilian monetary policy. The first one, from 1999 to 2007, is called adaptation phase and is characterized by a contractionary policy after the adoption of the inflation-targeting regime. The average RIR was 12.2% for the period compared to an average NRI of 6.9%. The second period, from 2007 to 2014, the policy stance went from a neutral to expansionary, with an average RIR of 5.0% compared to an NRI of 7.1%. The third period began in 2015 and was characterized by a contractionary impulse in a recessionary economic environment. In that period, the average RIR was 5.6% against an NRI of 3.5%. For the sake of comparison and robustness, the NRI was estimated using an alternative method, based on Basdevant, Björksten and Karagedikli (2004). The estimation produced similar results to the ones mentioned above. Both models show a continuous reduction in the NRI over time, in line with the empirical literature for developing countries. It is important to notice that our estimates indicate a reduction in the NRI gap between Brazil and other countries. This result may imply that the period of abnormally high real interest rates in Brazil is over. They also show that Brazilian Central Bank extended the last contractionary cycle even after the inflation surge of 2015 had been purged.

Keywords: Natural rate of interest. Interest rate gap. Monetary policy. Kalman filter.
JEL Classification: E52, E43

1 INTRODUCTION

The natural rate of interest (NRI) is an equilibrium concept. Broadly speaking, the NRI is the real interest rate (RIR) consistent with potential output growth and price stability.
However, there is no standard definition in the literature, as that necessarily depends on the model used.

By construction, the NRI is an unobservable variable and its relevance is related to its importance as monetary policy benchmark. As sensibly argued by Bomfim (2001), the interest rate of an economy is only one part in assessing the stance of monetary policy. It is not just the level of the interest rate that matters, but how far is that level from its “equilibrium”. Therefore, it is crucial to have an equilibrium reference in order to assess monetary policy stances. In general, this role is played by the NRI. The monetary policy is considered to be expansionary when the RIR is below the NRI and contractionary otherwise. Hence, if the RIR is below the NRI, economic activity is encouraged and, consequently, inflationary pressures are heightened. Likewise, when the RIR is above the NRI, aggregate demand slows down and inflation falls.\(^2\)

The concept of NRI was introduced by Wicksell (1989) in the 19th century with the aim of better understanding inflation determinants. In his view, price stability would depend on maintaining the interest rate on credit activity in line with its natural or neutral level, which would vary according to economic shocks on capital productivity. As pointed out by Amato (2005), the concept has evolved remarkably since its introduction, but only in the late 20th century did it take on a central role in the economics literature.\(^3\) Two factors contributed towards that. The first one was the dissemination of sticky-price general equilibrium models based on the maximizing behavior of the private sector, also known as new Keynesian models. The role of NRI in such models is so decisive that Woodford (1997) referred to them as new Wicksellian models. In this context, the NRI would be more accurately characterized as the short-term interest rate in an economy where prices are perfectly flexible (AMATO, 2005).

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\(^1\) The difference between RIR and NRI is called interest rate gap. When the gap is positive, monetary stimulus is regarded as contractionary. However, when the gap is negative, economic activity is stimulated and the monetary policy is expansionary. If the gap is zero, the monetary policy is considered to be neutral.

\(^2\) Note that, just as in any unobservable variable, NRI estimation is surrounded by great uncertainty. Thus, the monetary authority should not be blindly guided by the mechanical notion present in the definition of the NRI; the NRI should be seen as a useful tool in the decision-making process. Orphanides and Williams (2002) showed how costly it could be for the monetary authority to use an NRI in its monetary policy rule that does not include the uncertainties about its estimation.

\(^3\) Some authors argue that Keynes might have used some of Wicksell’s ideas (LEIJONHUFVUD, 1989). Moreover, it is worth noting that the concept of NRI was a determinant factor for the construction of Friedman’s (1968) and Phelps’s (1968) theories about the existence of a non-accelerating inflation rate of unemployment (NAIRU). Nonetheless, this paper assumes that the NRI only began to enjoy prestige in the 1990s, owing to the larger number of studies that were conducted after the dissemination of the inflation-targeting regime as compared to what had been published previously.
The second factor related to the extended role of the NRI in the literature concerns its practical application to the conduct of monetary policy. After Taylor (1993) showed the existence of an implicit equilibrium rate in the reaction function of the U.S. central bank and the adoption of an inflation-targeting regime by several central banks worldwide, the NRI began to be seen as a source of relevant information for inflation stabilization. Consequently, there are studies that estimate the NRI for different economies around the globe.4

That was the case in Brazil as well. The inflation-targeting regime was adopted in 1999 and the NRI appeal has become increasingly evident ever since. Recent communications by the Monetary Policy Committee of the Central Bank of Brazil have constantly mentioned the NRI as reference for their decisions.5

In addition, there are a growing, yet recent, number of studies devoted to NRI estimation in Brazil. Miranda and Muinhos (2003), for instance, did one of the first studies applied to the Brazilian case. Among the different methods used by those authors to estimate an equilibrium interest rate, the estimation of the IS curve consistent with a zero output gap for the period after exchange rate anchoring (1999-2002) is noteworthy. The equilibrium interest rate ranged from 4.5% to 5% p.a.

Borges and Silva (2006) estimated the NRI for the period starting from 2000 until 2003 using a structural vector autoregressive (VAR) model à la Brzoza-brzezina (2003) and compared it with the RIR. They concluded that monetary policy was conservative in most of the analyzed period. Also, the NRI obtained proved to be more volatile than the RIR. The average NRI calculated by the authors was quite high: 10.0% p.a.

Barcellos Neto and Portugal (2009) estimated the NRI between 1999 and 2005. The authors used the Hodrick-Prescott filter, a band-pass filter, a dynamic Taylor rule, and a macroeconomic state-space model, following Laubach and Williams (2003). To the best of our knowledge, that was the first study to use a semistructural model for Brazilian equilibrium interest rates. The average NRI was 7.3% when the band-pass filter was used and 9.4% for the HP filter. The average NRI for the 1999-2005 period using the dynamic

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5 To be more precise, the Central Bank of Brazil uses the term structural interest rate, which corresponds to the NRI. See, for instance, Banco Central do Brasil (2018).
Taylor rule was similar to that obtained with the band-pass filter, but a lot less volatile. The application of the model proposed by Laubach and Williams (2003) yielded an average NRI of 9.4%, which was very similar to that obtained by the other methods. The authors wound up finding a strong correlation between macroeconomic peculiarities and NRI fluctuations.

Quite recently, Palma (2016) has applied the AR-trend-bound model proposed by Chan, Koop and Potter (2013) to estimate the NRI for the 2001-2015 period. In that model, the NRI is interpreted as an RIR trend. According to the author, the method combines statistical filters and unobservable components models. Palma (2016) obtained an average NRI of 5.4% for the 2001-2015 period.

Candido and Ronchi Neto (2017) applied the multivariate Kalman filter to estimate the neutral rate from 2002 to 2016 in a procedure developed by them which allows estimating four latent variables at once: the NRI, the potential output, the NAIRU, and the NAICU. The authors argue in favor of the smaller standard deviation of the NRI obtained by their model as compared to the conventional HP filter. Regarding the monetary policy, they observed a predominantly expansionary period from 2006 to 2014, whereas the remaining periods were mostly contractionary. According to them, the 2014-2016 crisis had a strong impact on the estimated NRI, decreasing it to 1.3% p.a. in 2016.

The common findings of these studies were the downward trend in the estimated NRI and the predominantly contractionary policy used by the Central Bank of Brazil. It must be stressed that the evolution of the NRI in Brazil is in line with estimates for developing countries, that also show a negative trend. The novelty of our estimates is that the difference between the NRI in Brazil and abroad is decreasing, that is the negative trend has been stronger in Brazil. This help to explain why it was possible to sharply reduce de interest rate gap between Brazil and the rest of the world without produce a raise in inflation.

Even though the current interest rate level is relatively low, Brazil is, from a historical perspective, one of the countries with the highest RIR. As demonstrated by Segura-Ubiergo

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6 Another interesting aspect of this method is the limitation on the NRI by arbitrarily truncate the error term distribution.
7 NAIRU stands for non-accelerating inflation rate of unemployment, and refers to the rate of unemployment that does not accelerate or decelerate inflation.
8 NAICU stands for non-accelerating inflation rate of capacity utilization, and refers to the installed capacity utilization that does not accelerate or decelerate inflation.
9 The interest rate differential between the Brazilian Selic rate and the Fed Funds rate, for instance, has been the lowest since the beginning of the inflation-targeting regime in 1999.
(2012), Brazilian interest rates was really high compared to those of other emerging countries, and understanding the reasons behind that is not trivial. One of the factors that help explain this phenomenon is certainly the history of high and volatile inflation in Brazil. In the 1970s, inflation was moderately high (on average 30% p.a.), but extremely high in the 1980-1988 period (above 200% p.a., on average), turning into hyperinflation between 1989 and 1994 (on average 1,400% p.a.). After five unsuccessful stabilization plans, the Real Plan, launched in 1994, managed to stabilize prices.\textsuperscript{10} The later adoption of the inflation-targeting regime in 1999 complemented the Plan, definitively curbing the high inflation rates. However, high interest rates and the consequent predominantly contractionary monetary policy seem to have been the price paid for stabilization.

The aim of this study is to contribute to the literature applied to Brazil, estimating the NRI for the period between the adoption of the inflation-targeting regime (third quarter of 1999) to the second quarter of 2019. This allows an overview of monetary policies over nearly 20 years, comparing the findings with those of previous studies. In addition, the study analyzes the behavior of the NRI during the massive recession faced by Brazil in the 2015-16 period. During these two years the gross domestic product (GDP) decreased by 6.8%, representing the worst recession of the time series started in 1901.

The chosen estimation method is that proposed by Holston, Laubach and Williams (2017), which estimates the NRI together with potential output. It is a version of the original Laubach and Williams (2003) model, widely acknowledged in the literature.\textsuperscript{11} Note that, because of the high inflationary inertia and of the low impact of monetary policy on the economic activity, its application was changed, taking good advantage of the multiple local maxima in the likelihood function. Thus, the estimates became more stable.

The results indicate an NRI with a downward trend, especially during the 2015-16 crisis, corresponding to 1.8% p.a. in the second quarter of 2019. The average NRI for the whole period was 6.2% p.a. By analyzing the difference between the real interest rate (RIR) and the NRI, three periods were identified in the conduct of monetary policy in Brazil. In the first one, from 1999 to mid-2007, contractionary stimuli prevailed, in some sort of adaptation after the adoption of the inflation-targeting regime. The second one, from 2007 to 2014,

\textsuperscript{10} For more information on stabilization plans and on the history of high inflation rates in Brazil, see Ayres et al. (2018), who summarize the history of the Brazilian currency over the past 50 years.

\textsuperscript{11} Several studies use the model proposed by Laubach and Williams (2003). For example, Messonier and Renne (2007), Danielsson, Helgason and Thorarinsson (2016), Barcelos Neto and Portugal (2009), Perrelli and Roache (2014), and Kuhn, Ruch and Steinbach (2017).
went from a relatively neutral to a strong expansionary policy after 2011. The third period, after 2014, was characterized by a contractionary policy even when the recession scenario unfolded.

For the sake of comparison and robustness, an alternative model was used. It is based on Basdevant, Björksten and Karagedikli (2004) to estimate the NRI using yield curve spreads and macroeconomic variables. However, unlike the original model, the present paper allows the estimated risk premium to vary over time. Brazil is an emerging country; so, the existence of a constant premium appears to be an extremely strong assumption.

This alternative model showed similar results than the obtained from the Holston, Laubach and Williams (2017) model, both in terms of rate trends and monetary policy stances.

This paper is organized into five sections. After this introduction, Section 2 describes in detail the methodology used in the study. Section 3 presents the estimation results. As an exercise, Section 4 estimates an alternative model and compares its results with the estimates obtained by the Holston, Laubach and Williams (2017) model. The last section presents the final remarks, highlighting the most relevant aspects.

The main contribution of this paper is twofold. On one hand we update the NRI estimates for Brazil using a traditional Holston, Laubach and Williams (2017) model and introduce an alternative estimation based on Basdevant, Björksten and Karagedikli (2004). On the other hand we show that the Brazilian NRI is converging to the more suitable values observed in other countries. Furthermore, our results indicate an inflation overkill by the Central Bank in the 2016/2017.

2 METHODOLOGY

The seminal model developed by Laubach and Williams (2003) adapted by Holston, Laubach and Williams (2017) is used for NRI estimation. As pointed out by Giammarioli and Valla (2004), the use of this method has two advantages. First, it does not depart from a previous assumption about structural economic issues. Second, it allows an NRI that varies over time, which could be used for the formulation of a monetary policy rule.
Laubach and Williams’ (2003) model is developed from the definition of NRI presented in Bomfim (1997): “the real short-term interest rate consistent with output converging to potential, where potential is the level of output consistent with stable inflation” (Bomfim (1997) apud Laubach and Williams, 2001). According to this definition, NRI represents an anchor for the medium-term RIR, corresponding to the intercept of reaction functions for the monetary policy, such as the Taylor rule (1993).

The authors focus on the concept of medium-term price stability, when all shocks and fluctuations have already dispersed. This notion differs, for instance, from the one used by Woodford (2003), who assumes that the NRI is the rate which maintains the prices in equilibrium period after period. In other words, while the NRI advocated by Woodford (2003) is an equilibrium rate subject to high-frequency fluctuations, Laubach and Williams’ (2003) rate seeks to capture changes in low-frequency components of the NRI. Therefore, unlike the NRI extracted from Laubach and Williams’ (2003) model, Woodford’s (2003) rate does not correspond to the intercept of an optimal monetary policy rule and would thus not serve the purpose of a monetary policy benchmark.

Laubach and Williams (2003) assume the theoretical relationship between NRI and potential output; therefore, the estimation of the NRI necessarily requires the estimation of potential output. There are two basic assumptions about the proposed method: (i) the output gap converges to zero whenever the interest rate gap is zero. In other words, when the RIR is the same as the NRI, the output will converge to its potential level; and (ii) inflation movements converge to zero if the output gap is null.

In the work by Laubach and Williams (2003), the NRI varies with the changes in individuals’ preferences, in productivity, and in technology. This relationship derives from conventional growth models, in which the optimization condition for the balanced-growth savings rate relates the RIR to technology and preference parameters. Based on this relationship, Laubach and Williams (2003) assume that the law of motion for the NRI is given by:

\[ r^* = c g_t + z_t \]  (1)
where $r^*$ is the NRI, $g_t$ is the growth trend of the natural rate of output, or the productivity growth rate, $c$ is a constant, and $z_t$ is a stochastic term that captures the other factors influencing the NRI, such as individuals’ preferences.

Laubach and Williams (2003) found a coefficient of $c$ that is very close to unity.\(^{12}\) Therefore, just as in Holston, Laubach and Williams (2017), $c = 1$ was used in the present paper.

For $r^*$ to be identifiable, a simplified macroeconomic model is chosen, which captures the short-term dynamics between inflation, output, and RIR. On the demand side, an IS curve relates the output gap\(^{13}\) to its lags, a moving average of the interest rate gap,\(^{14}\) and an error variable, which turns out to be serially uncorrelated:

$$h_t = ay_1h_{t-1} + ay_2h_{t-2} + \Sigma_{j=1}^{2} a_r \left( r_{t-j} - r^*_{t-j} \right) + \epsilon_{1t} \quad (2)$$

where $h_t = 100 \times (y_t - y^*_t)$ (output gap), $y_t$ and $y^*_t$ are the natural logarithms of the real GDP and of the potential GDP, respectively, $r_t$ is the RIR, and $r^*_t$ is the NRI. Both output gap lags and serially uncorrelated error control for transient shocks and for the short-term dynamics, whereas persistent movements in the relationship between output gap and the RIR are characterized by fluctuations in $r^*$.

The difference between the two types of shocks reflects the interpretation of the NRI as the intercept of a monetary policy rule. Transient shocks would keep the intercept unchanged, since the monetary authority’s response would be guided by output and inflation gaps. Only permanent shocks, such as those on potential output or on individuals’ preferences, would shift the intercept.

On the supply side, the authors propose a Phillips curve. Inflation is assumed to be determined by its lagged values, by the output gap, and by an error term:

$$\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + b_y h_{t-1} + \epsilon_{2t} \quad (3)$$

\(^{12}\) In Laubach and Williams (2003), the authors found a $\sigma$ close to 1. Since $c = \frac{1}{\sigma}$, this implies that $c$ would be close to unity. The assumption about this strategy is that the representative agent’s utility function would be logarithmic.

\(^{13}\) The output gap here is the percentage deviation of the real output from the potential output.

\(^{14}\) Difference between the real rate and the NRI.
where $\pi_t$ is the consumer price inflation and $\pi_{t-2,4}$ is the mean inflation observed in the second to the fourth lag. The specification of the supply curve is the main difference between Holston, Laubach and Williams (2017) and the original model proposed by Laubach and Williams (2003). The difference from the first study lies in not adding the terms of relative prices of imports and of crude oil.

The log of the potential output is assumed to follow a random walk with drift $g$, which, in turn, also follows a random walk, but with no drift:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{y^*t}$$  \hspace{1cm} (4)
$$g_t = g_{t-1} + \epsilon_{gt}$$  \hspace{1cm} (5)

Finally, the variable $z_t$ is also defined as a random walk,

$$z_t = z_{t-1} + \epsilon_{zt}$$  \hspace{1cm} (6)

It is also assumed that $\epsilon_{y^*t}$, $\epsilon_{gt}$ and $\epsilon_{zt}$ are normally distributed and serially uncorrelated with their standard deviations $\sigma_{y^*}$, $\sigma_g$, and $\sigma_z$, respectively. They are serially and contemporaneously uncorrelated. Equations (4) to (6) are the transition equations of the model in state space form, whereas equations (2) and (3) are measurement equations.

Given that NRI determinants are unobservable, the Kalman filter is used to estimate the model. All parameters are estimated by maximum likelihood. Note that merely applying the Kalman filter in this model would cause the estimates of the standard deviations of $z$ and $g$ to be biased towards zero. This would imply that $r^*$ would be much more stable than it would be desirable, reacting only to highly persistent shocks. This is known as pile-up problem, and it is discussed in Watson (1994). The solution proposed by Laubach and Williams (2003) is to use Stock and Watson’s (1998) estimator for $\lambda_g = \frac{\sigma_g}{\sigma_{y^*}}$ and $\lambda_z = \frac{a_r \sigma_z}{\sigma_h}$. When these ratios are imposed for estimation of the model (including $\sigma_h$ and $\sigma_{y^*}$), the problem is solved.
The estimation process follows three stages. In the first one, the Kalman filter is used to estimate the output gap, omitting the term related to the interest gap in the IS equation and assuming that $g$ is constant over time. Based on the estimates, Wald’s exponential test developed by Andrews and Ploegerber (1994) is performed for the structural break test at an unknown date for the preliminary estimation of the potential output, and $\lambda_g$ is then obtained. In the second step, the Kalman filter is applied by imposing the $\lambda_g$ obtained in the first stage and including the interest gap in the IS equation, assuming that $z$ is constant. The same structural break test applied in stage 1 is then performed on the potential output and $\lambda_z$ is then obtained. In the third stage, $\lambda_g$ and $\lambda_z$ obtained in the previous stages are imposed.

To facilitate numerical convergence, Holston, Laubach and Williams (2017) assume that the IS curve slope coefficient $a_r$ is negative and smaller than $-0.0025$ and that the Phillips $b_y$ curve slope coefficient should be greater than $0.025$. In this paper, $b_y \geq 0.25$ in the three estimation stages, so as to stabilize the results. This is discussed in further detail in the forthcoming section.

3 ESTIMATION AND RESULTS

3.1 DATA

There are two RIR categories: ex-ante and ex-post. To calculate the ex-post rate, the observed inflation is subtracted from the nominal interest rate. The ex-ante rate, more relevant for monetary policy, is obtained by deflating the nominal interest rates from the expected inflation. One could argue that the ex-ante rate is the most pertinent one, since it is used by economic agents in their decision making. However, the ex-post rate also plays a pivotal role in aggregate demand. Fluctuations in the ex-post rate produce an effect on the post-fixed debt of firms and households. Taking this discussion into account, this paper estimates the NRI in two ways, giving priority to the ex-ante rate in the comparison of the models.
To estimate the Holston, Laubach and Williams (2017) model, it is necessary to collect data on inflation, inflation expectations, interest rate and quarterly GDP. The seasonally adjusted quarterly GDP was the selected output variable, published by the Brazilian Institute of Geography and Statistics (IBGE) in its Quarterly Report on National Accounts. The inflation accumulated over four quarters measured by IBGE’s consumer price index (CPI), which is the official inflation index of the Brazilian inflation-targeting regime, was used for estimation of inflation. Holston, Laubach and Williams (2017) used the core consumer price index (seasonally adjusted). In this paper, however, the CPI was used without any exclusion, since the inflation target defined by the Central Bank of Brazil refers to CPI without exclusion. Moreover, instead of accumulating inflation quarterly and expressing it annually, inflation was accumulated over four quarters, without the need of a seasonal adjustment and rendering the data less volatility.

For inflation expectations, the series of smoothed inflation expectations for the subsequent four quarters, published by the central bank’s FOCUS survey, was used. As it is a daily series, it had to be changed to a quarterly one. Thus, the mean expectations of the three months that make up the quarter were used. In addition, the central bank’s series starts only in December 2001, whereas the aim of this paper is to estimate the NRI for the whole period after the adoption of the inflation-targeting regime. It was therefore necessary to extend the expectation series up to 1999. To do that, a simple model was developed, in which the endogenous variable is the expected inflation for the subsequent four quarters and the exogenous variable is the inflation observed in the past four quarters, using the data estimated by the model from the third quarter of 1999 to the third quarter of 2001.

The mean Selic rate target for the three months that make up the quarter, in annual terms, was used as the interest rate. The Selic rate is the benchmark Brazilian interest rate.

Note that in Laubach and Williams’ (2003) original paper, the authors also use data on the relative price of crude oil and imports. However, this paper estimates the NRI following the re-specification proposed by Holston, Laubach and Williams (2017), which disregards these factors.

The results for the estimated NRI vary only slightly when the annualized quarterly inflation rather than the rate accumulated in the past four quarters is used.

Focus is a Central Bank of Brazil’s daily report. It collects the expectations of market agents. There are more than 100 participants, including banks, research institutes, brokers, and financial agents. In this paper, we use the median of inflation expectations as proxy for the expectations.

The equation in the extension model can be written as: \( \pi_t^e = c + \phi \pi_{t-4} + \epsilon_t \). It is therefore assumed that agents build their expectations based on the observed inflation. The estimation results are shown in the Appendix.

The Selic rate is the benchmark Brazilian interest rate. The term “Selic” is an acronym in Portuguese for special system for settlement and custody, a computerized system that deals with the custody of federal public bonds. The rate value is given by the mean interest rate of daily financial operations backed by federal public bonds negotiated between financial institutions.
Once these series are obtained, the Holston, Laubach and Williams (2017) model is estimated for the period from the third quarter of 1999 to the second quarter of 2019, with a total of 80 quarterly observations for the NRI.

### 3.2 RESULTS

Estimating Holston, Laubach and Williams’ (2017) model for the Brazilian economy has proven to be challenging\(^1\). The main reason for that was the low coefficients estimated for the IS and Phillips curves (\(a_r\) and \(b_y\)). As argued in Bielecki, Brand and Penalver (2018), when coefficients approach to zero (relatively flat IS and Phillips curves) the uncertainty associated with \(r^*\) increases. Because of that, the use of the same assumptions about the range of variation of parameters has led to inconsistent results. In order to address this matter, we slightly changed the model run.

In their paper, Holston, Laubach and Williams (2017) impose \(b_y \geq 0.025\), i.e., that the slope of the Phillips curve should be positive and greater than such value. However, imposing the same constraint on the Brazilian data produced some inconsistencies. Parameter \(b_y\) tended towards zero, i.e., it did not show the effect of output gap on inflation.\(^2\) Consequently, the potential growth (\(g_t\)) became quite stable because the deviations of the output from its tendency did not have any impact on inflation. So, \(\lambda_g\) naturally tended towards zero,\(^3\) with \(\sigma_y^*\) biased towards zero and \(g_t\) basically constant. It does not seem reasonable to assume that potential growth rate has not changed during the massive crisis faced by Brazil in recent years. In addition, the estimation showed a bias of \(a_r\) towards zero and so high volatility of the estimated NRI associated with the \(z_t\) component.

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\(^1\) The model was also estimated with the annualized Selic rate accumulated over the quarter, but, given the nearly identical behavior of both series, the results changed only slightly.

\(^2\) We ran this model using the R code provided by the authors in https://www.newyorkfed.org/research/policy/rstar.

\(^3\) It could be argued that the movement is due to the use of extended CPI without exclusions as a way to measure inflation. As it is widely known, this may not be the measure of inflation most frequently correlated with the business cycle. However, even by running the model with measures of core inflation, the results were quite similar. Furthermore, the proposed model assumes that the central bank follows a Taylor rule in order to converge inflation to the inflation target, which is defined in terms of CPI without exclusions. Therefore, it would not be the case here to use a measure of inflation other than the official index of the Brazilian inflation-targeting regime.

\(^4\) With a very stable \(y_t^*\), Wald exponential statistic generated by Andrews and Ploegerer (1994) for the structural break test for an unknown date was too low, so it was not possible to obtain an unbiased estimator from Stock and Watson’s (1998) table 3 used by Holston, Laubach and Williams (2017) to solve the pile-up problem.
Hence, an adjustment was then made in the model run. The optimization algorithm was then truncated for the Phillips curve slope coefficient, which yielded more stable results. Constraint $b_y \geq 0.25$, which is tougher than that of Holston, Laubach and Williams (2017), was imposed. Thus, the optimization uncovered a new maximum for the likelihood function, yielding a significant higher coefficient. This led to a larger variability in the trend growth and a lower volatility of the $z_t$ component.

Table 1 summarizes the estimation results. Starting with the estimated parameters, the values of $\lambda_g$ and $\lambda_z$ show substantial variation in the potential output and NRI growth trends over nearly 20 years. Regarding slope coefficients, it is important to highlight the high value of $b_y$, as a result of $b_y \geq 0.25$ as solution to the hardship discussed in the previous paragraphs.

Table 1: Estimation results for Holston, Laubach and Williams’ (2017) model

<table>
<thead>
<tr>
<th>Sample</th>
<th>ex-ante RIR</th>
<th>ex-post RIR</th>
<th>ex-ante RIR</th>
</tr>
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<td>1999.3 - 2019.2</td>
<td>1999.3 - 2019.2</td>
<td>2004.4 - 2014.4</td>
<td></td>
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<tr>
<td>$\lambda_g$</td>
<td>0.142</td>
<td>0.142</td>
<td>0.116</td>
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<tr>
<td>$\lambda_z$</td>
<td>0.065</td>
<td>0.060</td>
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<td>0.546</td>
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<td>-0.007</td>
<td>-0.050</td>
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<tr>
<td>$b_y$</td>
<td>2.142</td>
<td>2.156</td>
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<td>$b_\pi$</td>
<td>1.009</td>
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<td>$\sigma_\pi$</td>
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<td>0.970</td>
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<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_r \leq -0.0025$</td>
</tr>
<tr>
<td>$b_y \geq 0.25$</td>
</tr>
</tbody>
</table>

Source: Data collected by the author. t statistics in brackets.
The sum of the coefficients for the IS curve lags demonstrated that the estimated output gap was much less persistent than expected and also smaller than that obtained by Holston, Laubach and Williams (2017) for other economies. At the same time, inflation became more sensitive to output gap fluctuations. Then, the filtered output gap had a lower range because its effect on inflation was higher with the change in the constraint.24

Coefficient $a_r$ was lower than that obtained by Holston, Laubach and Williams (2017) for Canada, the USA, and the euro zone, but higher than that reported for the United Kingdom.

As argued by Holston, Laubach and Williams (2017), the inaccuracy of $r^*$ estimates tends to be higher as $a_r$ draws near zero. This tendency was evident when the model was applied to a smaller sample. The results for the 2004 to 2014 sample, for example, showed a smaller mean standard deviation for $r^*$ estimates mainly because of the higher $a_r$.

Graphs 1 and 2 show the filtered ex-ante and ex-post estimates25 for the whole period (1999.3 – 2019.2). The periods in which the monetary policy was expansionary are indicated by the RIR below the NRI. Both rates are quite similar, with a lower standard deviation for the ex-ante NRI. There is also predominance of contractionary stimuli over expansionary ones in both estimates.

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24 Note that the imposition of a new constraint succeeded in stabilizing unobservable states. In the specific case of output gap, however, there was a loss of range. It will be shown further ahead that the measure kept demonstrating the trend of output gaps estimated by other methods.

25 Or one-sided. Given that the filter only includes information available up to time $t$, filtered estimates are the ones that best correspond to “real-time” estimates.
Another important aspect concerns the fact that the NRI has been very low since the fourth quarter of 2016. There was a sharp decrease in the neutral rate, likely indicating the effects of the economic crisis.

Note that this movement of Brazil’s NRI towards zero is in line with estimations made for other countries. Holston, Laubach and Williams (2017) showed that, after the subprime crisis, neutral rates drew closer to zero in some developed economies. In the case of the euro zone, for example, the NRI averaged -0.2% p.a. from the fourth quarter of 2013 to the second quarter of 2017. This movement was also observed in emerging economies, as pointed out by Us (2018) for Turkey.

Graph 3 shows the measure of interest gap obtained by subtracting the ex-ante RIR from the filtered NRI.

In this design, it is plainly evident how monetary stimuli were predominantly contractionary during the analyzed period. There have been clearly three different moments regarding the conduct of the Brazilian monetary policy since the adoption of the inflation-targeting regime. In the first one, from 1999 to mid-2007, there was predominance of contractionary impulses, in some sort of adaptation to the new regime. Owing to the long history of inflation in the Brazilian economy, it was necessary to use wider interest rate gaps. Moreover, some events required this period to be even longer. There was, for instance, the burst of the speculative bubble of information technology firms in the US stock market in the early 2000s, during the crisis known as “the dot-com crash.” Thereafter, there was the crisis of confidence in the election of Mr. Lula as president, causing a high depreciation of the Brazilian currency depreciation that contaminated inflation and its
expectations. Based on prices recorded for March 2018, the exchange rate was R$ 9.56 in October 2002, and the pass-through effect pushed inflation up to 17.2% p.a. in April 2003. In response, the central bank tightened the monetary screws. Later, in 2005, the political crisis resulting from the votes-for-cash scandal known as “mensalão” also prompted the monetary authority to increase interests again.

After the market turmoil during the adaptation period, the monetary policy entered a new stage, from 2007 to 2014. In that period, monetary policy went from initial neutrality to strong expansion after 2011. At the time, the expansionary policy was very controversial, with inflation above the target. Portugal and Silva (2015) found evidence that the Central Bank of Brazil was more concerned with obtaining a higher output gap.

In addition, unconventional devices were used to artificially curb inflation in this period, which was not well received by market agents. Provisional Measure 579 was amended in 2012. This measure renewed, in advance, the concessions for generating and transmission companies that would expire in 2015, providing they agreed to have the prices set by Aneel (Brazilian Electricity Regulatory Agency). Some costs were also reduced and contributions from the Treasury Department were established to make up for this reduction and for lower tariffs that would be applied in 2013 and 2014 and to indemnify the investments of concession companies as provided for in the contracts. Thus, the provisional measure artificially controlled inflation. At the same time, a tax (best known as CIDE - contribution to interventions in the economic domain) levied on fuels was reduced and, in 2013, price adjustments on city and intercity bus fares and toll charges were frozen.27

The third period, which began in 2015, was marked by a contractionary policy even in a strongly recessive environment. This occurred because administered pricing was allowed, propping up inflation beyond two digits in late 2015 (10.5% p.a.), forcing the monetary authority to increase interests and to reaffirm its commitment to the inflation target. Somehow, monetary tightening resulted from the use of artificial devices for inflation control in previous years.

As the crisis unfolded and inflation slowed down, the central bank began to cut interest rates. At the same time, NRI also decreased considerably during the period, reaching its lowest historical level, driven by a plummeting growth. Hence, even after the

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26 Mensalão is the votes-for-cash scandal surrounding the National Congress of Brazil between 2005 and 2006.
27 For further information on the strategies used for price control during the period, see Barrionuevo (2015).
central bank successively reduced nominal interest rates from October 2016 to March 2018 (its lowest historical level ever), very likely, the speed of fall was not enough for the RIR to be considered expansionary. The central bank may have been too slow in adopting monetary easing measures, leading to a sort of monetary overkill. In the last observations, monetary policy seems close to a neutral stance.

Graph 4 correlates the interest rate gap for the 2002.2 to 2016.2 period with inflation deviations from its target six quarters ahead (from 2004.1 to 2018.1). This lag was chosen based on Neiss and Nelson (2003), who found a high correlation between variables for this lag. Note that the inflation rate remained above the target for almost the entire period and, probably because of that, the monetary authority may have responded using a contractionary policy most of the time. Moreover, there is a clear negative correlation between the interest rate gap and future inflation, with a similar effect to that found by Neiss and Nelson (2003). As a choice, the period shown in Graph 4 does not contemplate the expectations shock related to former president Lula’s inauguration.

Graph 5 shows the potential output growth estimated by the model, comparing it with observed GDP growth. Note that, after the end of 2010, the model captures a reduction in potential growth, dropping from 3.2% p.a. in the first quarter of 2011 to -0.8% p.a. in the first quarter of 2016, its lowest rate. This downward trend of the potential output growth was a determining factor for the recent reduction in the NRI detected by the model.

Observe that the fast adjustment in the potential growth was only able because of the high signal-to-noise ratio coefficient ($\lambda_g$) obtained. It is also of relevance to point out that the strategy used to estimate the model significantly reduced the output gap range. This is not an unusual result from the literature perspective. As pointed out by Bielecki, Brand and Penalver (2018), the Laubach and Williams (2003) model may produce output gap estimates which are not aligned with other estimates.

**Graph 5:** Potential output growth and GDP
4 AN ALTERNATIVE MODEL AND SOME COMPARISONS

NRI can be estimated in several ways. Although the model of Laubach and Williams (2003) is quite robust and widely established in the literature, an alternative model is estimated here with the aim of comparing the results and observing the similarities and discrepancies between them. However, the relevance should not be the same as given to the Holston, Laubach and Williams (2017) model, especially because this alternative model is not exactly suitable for real-time monetary policy.

4.1 YIELD CURVE SPREAD MODEL

The model estimated was based on Basdevant, Björksten and Karagedikli (2004). In their paper, the authors propose two models to estimate the NRI: a predominantly financial one, which uses the yield curve spreads, and a slightly more structural one. The former is hinged upon the idea that it is possible to interpret simultaneous fluctuations in the short-term and long-term interest rates – after all cyclic fluctuations have dispersed – as changes in the NRI. It is therefore assumed that there exists a common stochastic trend between both rates. In the latter model, Basdevant, Björksten and Karagedikli (2004) propose a more structural approach to model the NRI. They use a Taylor rule and a no-arbitrage equation for the interest rate between short-term and long-term assets.

Here their two models are put together, resulting in a state-space model made up of the following measurement equations:

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29 We ran this model using the MARSS package in R.
\[ r_t = r^*_t + \pi^*_t + \beta(\pi_t - \pi^*) + \phi h_t + \epsilon_{1t} \]  
\[ R_t = r^*_t + \alpha_t + \pi^*_t + \epsilon_{2t} \]  

where \( r_t \) and \( R_t \) are, respectively, the RIR for 90 days and 10 years. The variable \( r^*_t \) is the NRI, \( \pi^*_t \) is the inflation expectation for the subsequent period, and \( \alpha_t \) is the risk premium or interest rate curve spread. Inflation expectation is assumed to be stable between the two maturities. Equation (7) is a Taylor rule where \( \pi_t \) is the observed inflation, \( \pi^* \) is the inflation target for the period and \( h_t \) is the output gap. Equation (8) denotes a no-arbitrage relationship between short-term and long-term financial assets, which basically establishes that the long-term NRI is identical with the short-term NRI added to a risk premium. Note that potential output is an observable variable in this model.

The state equations are given by:

\[ r^*_t = r^*_{t-1} + g_{t-1} \]  
\[ \alpha_t = \delta_0 + \delta_1 \alpha_{t-1} + \epsilon_{2t} \]  
\[ g_t = g_{t-1} + \epsilon_{3t} \]

where \( g_t \) is the annual growth rate of potential output, an observable variable as well, and \( r^*_t \) is the NRI, given by a random walk. The system made up of equations (7)-(11) represents a state-space model. As in Basdevant, Björksten and Karagedikli (2004), the Kalman filter was also applied.

To measure the output gap, the potential GDP estimated quarterly by the Brazilian Institute of Economic Research (IPEA) was used.\(^{30}\) The variable \( g_t \) is the IPEA’s annualized growth rate of potential output. The observed inflation is the price consumer index variation over a 12-month period, calculated by the Brazilian Institute of Geography and Statistics (IBGE), whereas \( \pi^*_t \) is the smoothed inflation expectations for the next 12 months collected from the Central Bank’s Focus report.\(^{31}\) To measure the short-term rate (\( r_t \)), the quarterly mean of interfinancial deposit (ID) swap contracts negotiated at the commodities and

\(^{30}\) IPEA’s potential output is estimated based on the production function approach described in Souza-Júnior and Caetano (2013) and Souza-Júnior (2015).

\(^{31}\) The rational expectations model (perfect foresight) was also estimated, but the results were very similar. Thus, they are not reported.
futures exchange (BM&F Bovespa) for the subsequent 90 days was used, whereas the mean rate of the same contracts with a maturity of 5 years (1800 days) was used for the long-term rate \( (R_t) \). Because of lacking data, the time horizon was shorter than that of the previous section. The sample includes the period from the second quarter of 2002 to the second quarter of 2019, totaling 69 observations.

4.2 RESULTS AND COMPARISONS

Table 2 shows the estimation results. Note that all parameters were significant at 5%. Deviations of inflation from its target were positively correlated with the interest rate gap. Conversely, output gap was negatively correlated with the interest rate gap. In other words, the larger the difference in the RIR from the neutral rate, the smaller the output gap.

<table>
<thead>
<tr>
<th>Table 2: Yield curve spread model</th>
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<tbody>
<tr>
<td>Coefficient</td>
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<tr>
<td>( \beta )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \delta_0 )</td>
</tr>
<tr>
<td>( \delta_1 )</td>
</tr>
<tr>
<td>Log-like.</td>
</tr>
<tr>
<td>Mean ( r^2 )</td>
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</table>

2002-2008 \( 9.426 \) 2015-2019 \( 3.526 \)
2009-2014 \( 4.865 \) Obs. Final \( 1.508 \)

Graph 6 shows the ex-ante RIR and the estimated NRI. By looking at the NRI, there is a clear trend towards a decrease, especially after the economic crisis loomed in 2014. The curve went from 9.2% p.a. in the second quarter of 2002 to 5.0% p.a. in the fourth quarter of 2014. In the second quarter of 2019, the NRI reached 1.5% p.a., one of the lowest rates so far.

By calculating the difference between the RIR and the NRI (interest rate gap), it is possible to assess monetary policy stances. Graph 7 reveals that monetary stimuli may have been primarily contractionary during the period, especially regarding the interval spanning from the beginning of the series to 2007. From then on, a reasonably neutral policy was established, which lasted to the beginning of 2011, when an expansionary process took place, changed later in 2014 into a contractionary policy. In the latest estimated values, monetary policy could be classified as neutral. Hence, the model
corroborates, to a large extent, the estimates obtained in Section 4, both in terms of trend of the NRI and of monetary policy stances.

Graph 6: NRI\textsuperscript{32} and RIR

Graph 7: Interest rate gap

Table 3 summarizes the estimates obtained with the alternative model, compared with those calculated by the model proposed by Holston, Laubach and Williams (2017). In general, the results of the alternative model concurred with those of the previous model. In the last observations, NRI was quite low, strengthening the results obtained in this study.

Although both estimates lead to similar conclusions about the conduct of monetary policy over the past 20 years, there is a significant difference between the estimated NRIs in some periods of the sample. Take, for example, the mean estimated values in the period from the first quarter of 2011 to the fourth quarter 2013. There is almost a three percentage points difference among the models, what highlights the intrinsic uncertainty involving non-observable variables.

Table 3: Comparison of estimates – mean NRI for the selected periods

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</thead>
<tbody>
<tr>
<td>HLW</td>
<td>6.2</td>
<td>8.3</td>
<td>7.5</td>
<td>7.2</td>
<td>5.4</td>
<td>1.7</td>
<td>1.8</td>
</tr>
<tr>
<td>Yield Curve Spread Model</td>
<td>6.3</td>
<td>10.0</td>
<td>6.8</td>
<td>4.3</td>
<td>5.0</td>
<td>2.4</td>
<td>1.5</td>
</tr>
<tr>
<td>RIR</td>
<td>8.6</td>
<td>13.3</td>
<td>6.9</td>
<td>4.0</td>
<td>7.0</td>
<td>3.9</td>
<td>2.9</td>
</tr>
</tbody>
</table>

\textsuperscript{*}From 2002.2 to 2018.1 for the yield curve spread model.
Source: Data collected by the author.

However, in the majority of the sample, the models seem to agree quite well. An interesting way of seeing it is taking the mean of the two estimates. Graph 8 shows the results of this exercise. As expected, the combined NRI captured the downward trend of

\textsuperscript{32} The smoothed rather than the filtered NRI was used. The reason for that lies in the fact that models like this are not for “real time” use by the monetary authority. Therefore, a filtered estimate would make no sense in this case.
both estimates. It is worth noticing too that the combined estimate didn´t show great fluctuations with the cycles. In this sense, the fast drop down in the mean NRI after 2016, caused by the simultaneous movement of the two estimates, is precisely the most striking result of this study. The mean estimate reinforces that the neutral interest rate should be in its lowest historical levels, probably lower than 2% p.a.

![Graph 8: Mean of NRI estimates](image)

Source: BCB and data collected by the author.

Finally, to compare the reliability of the models, one can analyze the correlation between the interest rate gap and deviations of inflation from the target established by the central bank. In this respect, a good model should have a negative correlation with inflation deviations from the target in the future.\(^{33}\)

Table 4 shows the correlation between interest rate gap and future inflation. Note that interest rate gap estimates are closely correlated with deviations of inflation in the periods ahead. The Holston, Laubach and Williams (2017) model showed a slightly stronger correlation than did the alternative model, but, in general, the results are similar.

5 FINAL REMARKS

This paper estimated the NRI for the period following the adoption of the inflation-targeting regime in Brazil (1999-2019). The seminal model proposed by Laubach and Williams (2003), with the re-specification made by Holston, Laubach and Williams (2017), was used. It is important to point out that we slightly changed the model run in order to obtain more stable results. The reason for that were the low coefficients estimated for the IS and Phillips curves \((a_r\) and \(b_y\)). As argued in Bielecki, Brand and Penalver (2018), when coefficients approach to zero (relatively flat IS and Phillips curves) the uncertainty associated with \(r^*\) increases. Because of that, the use of the same assumptions about the

\(^{33}\) Deviation of inflation from its target was calculated by \(\pi_t - \pi_{t*}\), where \(\pi_t\) is the inflation for four quarters.
range of variation of parameters has led to inconsistent results. So, the parameter \( b_y \) was truncated in order to address this matter.

The estimates indicate an NRI with a downward trend throughout the study period, pushed down in recent years by the acute 2014-2016 crisis and consequent decrease in potential growth rate. In the last observation (second quarter of 2019), the model indicated an NRI of 1.8% p.a..\(^{34}\)

The analysis of interest rate gap showed three different moments in the conduct of monetary policy. The first one went from 1999 to 2007 and is interpreted as a period of adaptation after the adoption of the inflation-targeting regime. The length of this adaptation was longer than it should have been, influenced by the memory of chronic inflation in Brazil and by some events which increased uncertainty during the period. Of large relevance, there were the “dot-com crash,” president Lula’s inauguration, and the votes-for-cash scandal known as “mensalão”, which increased the risk of the Brazilian economy and required interest rates above their natural levels. From 2007, an initial period of neutrality took place, followed by a strongly expansionary monetary policy between 2011 and 2014. The third period went from 2014 to the end of the sampling period and was characterized by contractionary policies. When government stopped using artificial tools to keep inflation low, characteristic of Dilma Rousseff’s term, inflation surged back into double-digits territory, prompting the Central Bank to raise the interest rates even in a recession scenario. When inflation finally lowered and the monetary authority began to cut interest rates, NRI had already been falling vertiginously because of the recession, and so the interest rate gap was not negative during that period. More recently, the monetary policy seemed to be in a neutral stance.

Given the uncertainties over the estimation of unobservable variables, an alternative model was estimated. It is a version of the Basdevant, Björksten and Karagedikli (2004) model, who estimated the NRI using financial and macroeconomic components. The innovation is allowing the risk premium to vary over time. The estimates were close to those obtained by the Holston, Laubach and Williams (2017) model, corroborating the sharp decrease in NRI during the 2015-2016 crisis and identifying similar monetary policy stances. The models agreed that the NRI was in its lowest levels in the middle of 2019.

\(^{34}\) The movement of neutral rates towards zero has also been described for other economies. See, for instance, Holston, Laubach and Williams (2017) for developed economies, and Us (2018) for emerging markets.
Some of the findings of this study have already been described in Brazilian publications, mainly the downward trend for the NRI and a predominantly contractionary bias in the monetary policy. Ribeiro and Teles (2013) estimated a version of the Laubach and Williams (2013) model and observed that the monetary policy was contractionary during most of the period investigated by them. The authors also highlighted the downward trend of the Brazilian NRI. Likewise, Perrelli and Roache (2014), Ferreira and Mori (2014), and Palma (2016) are some of the studies that provided similar evidence, suggesting that the results of the present study are plausible. In regard to the contractionary bias of the Brazilian monetary policy, it is believed that such bias is strongly associated with the history of high inflation persistently above the target. Segura-Ubiergo (2012) provided evidence of that.

As far as more recent estimates are concerned, previous studies, such as those by Barbosa, Camelo and João (2016), Ribeiro and Teles (2013), and Perrelli and Roache (2014) showed that the downward trend of the NRI would be reversed at the beginning of the current decade. However, as demonstrated in the present paper, this reversion has not taken place and the NRI dropped further, especially during recession. Candido and Ronchi Neto (2017) found similar evidence. The difference, however, is that those authors indicated a slight reversion in from 2016 onwards, whereas the present paper shows that the NRI remains low even after the end of recession.

By updating the NRI estimates we show a convergence of the Brazilian NRI to values estimated for other countries. This result may imply that the period of abnormally high real interest rates in Brazil is over.

In conclusion, analyzing Brazilian monetary policy since the adoption of the inflation-targeting regime brought interesting contributions. The extension of the estimation beyond the recent economic crisis revealed that the NRI has reached historically low levels.

REFERENCES


APPENDIX A: EXTENSION OF THE HISTORICAL SERIES OF SMOOTHED EXPECTATIONS ACCORDING TO THE FOCUS SURVEY

The model used to forecast the inflation expectations during the period in which no observations are available from the Focus survey is given by:

\[ \pi_t^e = c + \pi_t + \epsilon_t \]

Where \( \pi_t^e \) is the smoothed inflation expectations for the subsequent 12 months and \( \pi_t \) is the IPCA variation accumulated over 12 months. The model was used to estimate FOCUS expectations for the third quarter of 1999 to the third quarter of 2001. The estimation results are shown in Table A.

Table A: Results for the model used for extension of FOCUS expectations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard deviation</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>2.982</td>
<td>0.211</td>
<td>14.101</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0.363</td>
<td>0.029</td>
<td>12.366</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.708</td>
<td>SQR</td>
<td>28.175</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.704</td>
<td>Log likelihood</td>
<td>-65.063</td>
</tr>
</tbody>
</table>

Source: Data collected by the author.

Note that it is not within the scope of this paper to assess how agents build their expectations, but to extend the series backwards. Therefore, the paper does not make any assumption about the expectations and does not indicate the most suitable method for modeling them.