Distributed Model Predictive Controller of Nonlinear Processes based on Automatic Partitioning of the Incidence Information Matrix

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Summary

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• Problem Formulation – a new system decomposition
• Proposed Distributed MPC strategy
  – Controller Design
  – Closed-loop Stability
• Illustrative example
• Conclusion
Introduction

Why to control a system?

- Increasing process efficiency;
- Ensure product and process quality;
- Ensure the safety of the process;
- Reduce hand labor.
Introduction

Types of control

- Centralized control
- Decentralized control
- Distributed control
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Introduction

Partition models

Selection of states and inputs belonging to each sub-model based on:

- Intuition and perception of the physical structure and/or experience
Problem Formulation

DMPC 1
DMPC 2
DMPC 3
DMPC 4
Problem Formulation

**Step 1**: apply the proposed partitioning methodology

**Step 2**: find the control $\mathbf{u}_k$ using a DMPC-1 (non-cooperative) or DMPC-2 (cooperative) algorithm

**Step 3**: implement $\mathbf{u}_k$ obtained to the nonlinear plant

**Step 4**: measure or estimate the states $\mathbf{x}_k$

**Step 5**: with new $\mathbf{u}_k$ and $\mathbf{x}_k$ returns to Step 2
Problem Formulation

A new system decomposition

Complete Model

\[
\frac{dx_1}{dt} = f_1(x,u); \quad x_1(0) = x_{10} \\
\frac{dx_2}{dt} = f_2(x,u); \quad x_2(0) = x_{20} \\
\frac{dx_3}{dt} = f_3(x,u); \quad x_3(0) = x_{30} \\
\frac{dx_4}{dt} = f_4(x,u); \quad x_4(0) = x_{40} \\
\frac{dx_5}{dt} = f_5(x,u); \quad x_5(0) = x_{50}
\]

Submodels

\[
x = [x_1, x_2, x_3, x_4, x_5] \\
u = [u_1, u_2, u_3]
\]

\[
\begin{aligned}
\frac{dx_1}{dt} &= f_1(x,u) \\
\frac{dx_2}{dt} &= f_2(x,u) \\
\frac{dx_3}{dt} &= f_3(x,u) \\
\frac{dx_4}{dt} &= f_4(x,u) \\
\frac{dx_5}{dt} &= f_5(x,u)
\end{aligned}
\]
A new system decomposition

\[
\begin{align*}
\frac{dx_1}{dt} &= f_1(x,u); \quad x_1(0) = x_{10} \\
\frac{dx_2}{dt} &= f_2(x,u); \quad x_2(0) = x_{20} \\
\frac{dx_3}{dt} &= f_3(x,u); \quad x_3(0) = x_{30} \\
\frac{dx_4}{dt} &= f_4(x,u); \quad x_4(0) = x_{40} \\
\frac{dx_5}{dt} &= f_5(x,u); \quad x_5(0) = x_{50}
\end{align*}
\]

Complete Model

Submodels

\[
\begin{align*}
x &= [x_1, x_2, x_3, x_4, x_5] \\
u &= [u_1, u_2, u_3]
\end{align*}
\]

\[
\begin{align*}
\frac{dx_1}{dt} &= f_1(x,u) \\
\frac{dx_2}{dt} &= f_2(x,u) \\
\frac{dx_3}{dt} &= f_3(x,u) \\
\frac{dx_4}{dt} &= f_4(x,u) \\
\frac{dx_5}{dt} &= f_5(x,u)
\end{align*}
\]

How to select the \(x\) and \(u\) pairs for each submodel?
Complete Model

Consider the following generic model:
\[
\begin{align*}
\frac{dx_1}{dt} &= f_1(x_1, x_2, x_3, u_1, u_2, u_3) \\
\frac{dx_2}{dt} &= f_2(x_1, x_2, x_3, x_4, u_2) \\
\frac{dx_3}{dt} &= f_3(x_1, x_2, x_3, x_4, x_5, u_1, u_2, u_3) \\
\frac{dx_4}{dt} &= f_4(x_3, x_4, x_5, u_1, u_2) \\
\frac{dx_5}{dt} &= f_5(x_4, x_5, u_1, u_2, u_3)
\end{align*}
\]

\[y_1 = x_1 \quad y_2 = x_2 \quad y_3 = x_3\]
A new system decomposition

Step 1: Identify the states that each input affects

\[
E = \begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & 0 \\
1 & 1 & 1 \\
1 & 1 & 0 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

\[
\{u_1\} \rightarrow \{x_1 \ x_3 \ x_4 \ x_5\}
\]

\[
\Gamma_1 = \{x_1, x_3, x_4, x_5\}
\]

\[
\{u_2\} \rightarrow \{x_1 \ x_2 \ x_3 \ x_4 \ x_5\}
\]

\[
\Gamma_2 = \{x_1, x_2, x_3, x_4, x_5\}
\]

\[
\{u_3\} \rightarrow \{x_1 \ x_3 \ x_5\}
\]

\[
\Gamma_3 = \{x_1, x_3, x_5\}
\]
Problem Formulation

A new system decomposition

**Step 2:** Identify the effects between states

\[
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 \\
\end{bmatrix}
\]

\[s=\]

\[\{x_1\} \rightarrow \{x_1, x_2, x_3\}\]

\[\{x_2\} \rightarrow \{x_1, x_2, x_3\}\]

\[\{x_3\} \rightarrow \{x_1, x_2, x_3, x_4\}\]

\[\{x_4\} \rightarrow \{x_2, x_3, x_4, x_5\}\]

\[\{x_5\} \rightarrow \{x_3, x_4, x_5\}\]

\[\{x_4\} \rightarrow \{x_2, x_3, x_4, x_5\}\]

\[\{x_4\} \rightarrow \{x_2, x_3, x_4, x_5\}\]

\[\{x_5\} \rightarrow \{x_3, x_4, x_5\}\]

\[\{x_5\} \rightarrow \{x_3, x_4, x_5\}\]
A new system decomposition

**Step 3:** Identify which states affect the controlled outputs

- If the controlled outputs are: \( \{y_1, y_2, y_3\} \)

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

- \( \{u_1\} \rightarrow \{x_1, x_3, x_4, x_5\} \)
  - \( \omega_1 = \{x_1, x_3\} \)

- \( \{u_2\} \rightarrow \{x_1, x_2, x_3, x_4, x_5\} \)
  - \( \omega_2 = \{x_1, x_2, x_3\} \)

- \( \{u_3\} \rightarrow \{x_3, x_5\} \)
  - \( \omega_3 = \{x_1, x_3\} \)

\( \{u_1, u_3\} \rightarrow \{x_1, x_3\} \)
Problem Formulation

A new system decomposition

**Step 3:** Identify which states affect the controlled outputs

➢ If the controlled outputs are: \( \{y_1, y_2, y_3\} \)

**Subsystem #1:**

\[
\begin{align*}
\{u_1, u_3\} & \rightarrow \{x_1, x_3\} \\
\frac{dx_1}{dt} &= f_1(x_1, x_2, x_3, u_1, u_2, u_3) \\
\frac{dx_3}{dt} &= f_3(x_1, x_2, x_3, x_4, x_5, u_1, u_2, u_3)
\end{align*}
\]

**Subsystem #2:**

\[
\begin{align*}
\{u_2\} & \rightarrow \{x_1, x_2, x_3\} \\
\frac{dx_1}{dt} &= f_1(x_1, x_2, x_3, u_1, u_2, u_3) \\
\frac{dx_2}{dt} &= f_2(x_1, x_2, x_3, x_4, u_2) \\
\frac{dx_3}{dt} &= f_3(x_1, x_2, x_3, x_4, x_5, u_1, u_2, u_3)
\end{align*}
\]
Proposed Distributed MPC strategy

DMPC-1 – Non-cooperative Structure

Objective Function:

\[
J_i(k) = \sum_{j=H_{pi}}^{H_{wi}} \|\hat{y}_i(k+j|k) - r_{yi}(k+j)\|_{Q_i(j)}^2 + \\
+ \sum_{j=0}^{H_{wi}-1} \|\hat{u}_i(k+j|k) - r_{ui}(k+j)\|_{R_i(j)}^2 + \\
+ \sum_{j=0}^{H_{ui}-1} \|\Delta\hat{u}_i(k+j|k)\|_{W_i(j)}^2
\]

Restrictions per element:

\[
u_i^{min} < \hat{u}_i < u_i^{max}
\]

\[|\Delta\hat{u}_i| < \Delta u_i^{max}\]
Proposed Distributed MPC strategy

DMPC-2 – Cooperative Structure

Objective Function: \( J(k) = \sum_{i} J_{i}(k) \)

\[ J_{i}(k) = \sum_{j=H_{wi}}^{H_{pi}} \| \hat{y}_{i}(k + j|k) - r_{yi}(k + j) \|_{Q_{i}(j)}^{2} + \]
\[ + \sum_{j=0}^{H_{ui}} \| \hat{u}_{i}(k + j|k) - r_{ui}(k + j) \|_{R_{i}(j)}^{2} + \]
\[ + \sum_{j=0}^{H_{ui}-1} \| \Delta \hat{u}_{i}(k + j|k) \|_{W_{i}(j)}^{2} \]

Restrictions per element:
\[ u_{i}^{\text{min}} < \hat{u}_{i} < u_{i}^{\text{max}} \]
\[ | \Delta \hat{u}_{i} | < \Delta u_{i}^{\text{max}} \]
Proposed Distributed MPC strategy

Closed-loop Stability:

Among the several strategies to make a MPC algorithm robust, it was decided to use an additional constraint to the optimization problem in order to force this robustness.

\[ |J_{opt}(k - 1) - J_{opt}(k)| \leq 0 \]

where \( J_{opt} \) represents the optimal value found for the control problem.
Illustrative example

Plant present in Chillin et al. (2012)

\[ y = [T_1, T_2, T_3, T_4, T_5, C_{A4}, C_{B4}, C_{C4}, C_{D4}] \]

\[ u = [Q_1, Q_2, Q_3, Q_4, Q_5, F_1, F_4, F_6, F_{10}] \]
Illustrative example

Controller Parameters and Simulation Conditions:

\[ Q = I_{n_y} \times n_y; \]

\[ R = I_{m \times m}; \]

\[ W = I_{m \times m}; \]

\[ u_{\min} = [-1.0 \cdot 10^7; -1.0 \cdot 10^7; -1.0 \cdot 10^7; 0; 0; 0; 0; 0; 0]; \]

\[ u_{\max} = [0; 0; 0; 1.0 \cdot 10^7; 1.0 \cdot 10^7; 1.0 \cdot 10^{-2}; 1.0 \cdot 10^{-2}; 1.0 \cdot 10^{-2}; 1.0 \cdot 10^{-2}]; \]

\( n_y \rightarrow \text{number of controlled outputs} \)

\( m \rightarrow \text{number of manipulated inputs} \)

No tuning effort was made to improve controller behavior in the studied scenarios.
Illustrative example

Controller Parameters and Simulation Conditions:

\[ H_p = H_u = 5; \]
\[ r_y(t + k) = \alpha \cdot r_y(t + k - 1) + (1 - \alpha) \cdot sp_y(t + k) \rightarrow \alpha = 0,5 \]
\[ r_u(t + k) = \alpha \cdot r_u(t + k - 1) + (1 - \alpha) \cdot sp_u(t + k) \rightarrow \alpha = 0,5 \]
\[ T_S = 10 \text{ s.} \]
Illustrative example

Comparison of controlled variables $T_1$, $T_2$ and $T_3$:

Legend:
- Centralized MPC
- DMPC-1
- DMPC-2
Illustrative example

Comparison of controlled variables $T_4$ and $T_5$:

Legend:
- Centralized MPC
- DMPC-1
- DMPC-2
Illustrative example

Comparison of controlled variables $C_{A4}$ and $C_{B4}$:

Legend:
- Centralized MPC
- DMPC-1
- DMPC-2
Illustrative example

Comparison of controlled variables $C_{A4}$ and $C_{B4}$:

Legend:
- Centralized MPC
- DMPC-1
- DMPC-2
Conclusion

• A simple and promising procedure that can be applied to partitioning a nonlinear model for distributed model predictive control was introduced.

• The methodology can be applied to nonlinear systems. If guaranteed stability conditions are imposed to subsystems, the plant-wide control can be implemented.
Conclusion

• For the considered case study, the controllers developed herein presented equivalent responses to those obtained by the centralized controller, and the DMPC-1 presented the shortest processing time among all approaches evaluated.
Acknowledgements
References


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