MULTIOBJECTIVE MANIPULABILITY IN TRAJECTORY TRACKING FOR CONstrained REDUNDANT ROBOT MANIPULATORS

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Abstract— This paper presents an approach to maximize the manipulability of constrained redundant serial manipulators for the tracking problem. It is considered that the constraints are within the manipulator chain. Two indices of manipulability are taken into account, the first refers to the Jacobian before the constraints and the second refers to the constrained Jacobian. A multiobjective solution that determines the joint velocities for trajectory tracking is presented in order to maximize the manipulability indices while the constraints in the manipulator chain are satisfied including the physical limits of joint angles and velocities. Simulations and experiments are performed in a Baxter® robot.

Keywords— manipulability, constrained robotic manipulators, trajectory tracking

1 Introduction

Redundant robots need to satisfy constraints while performing tasks, for example in minimally invasive surgery where the insertion point should not move transversely in order to not cause serious lesions in the epithelium (From, 2013). A manipulability analysis could help to improve a control strategy when redundant robots are subject to constraints because it is an indication of how close the manipulator is from a singular configuration.

A general discussion about manipulability for robot manipulators can be found in (Siciliano et al., 2009). Manipulability of constrained systems is discussed in (Wen and Willinger, 1999) and for constrained serial manipulators in (From et al., 2014). A geometrical approach can be found in (Park and Kim, 1998; Wen and O’Brien, 2003). A control scheme based on the constrained Jacobian in task space for constrained manipulators is discussed in (Pham et al., 2014; Coutinho, 2015). In industrial applications a confined environment can be seen as a kinematic constraint in the manipulator chain. (Simas et al., 2013; Everist and Shen, 2009). In (Yoshikawa, 1985) is presented a method for maximizing the manipulability for a non-constrained redundant manipulator using the null space of the geometric Jacobian. In (Zhang et al., 2012) an optimization problem is proposed in order to maximize the manipulability of self-motion planning in a redundant manipulator.

This paper presents a general formulation to determine the Jacobian of a serial manipulator with constraints in a point of this kinematic chain, the called constrained Jacobian. As stated by (From et al., 2014) the analysis of manipulability of a serial redundant constrained manipulator must take into account not only the constrained Jacobian, but also the manipulator Jacobian until the joint before the constraint. So a multiobjective approach, is presented in order to maximize two manipulability indices (corresponding to constrained Jacobian and the Jacobian until the joint before the constraint) while the end-effector follows a trajectory and the imposed constraints are satisfied. The analysis is addressed to an arm of the Baxter® robot with seven revolute joints and a plane constraint. Simulations and experiments results are presented.

The following notation and definitions are used throughout the paper. $\mathbb{R} := (-\infty, \infty)$ and $\mathbb{R}^+ := [0, \infty)$. The subscript $i$ means in general a reference for the $i$-th frame in the kinematic chain of the manipulator, a subscript $c$ denotes a reference for the constraint and a subscript $e$ a reference for the end-effector. A frame is represented by $F$. A subscript $(i, j)$ in a matrix or vector denotes the matrix or vector from $F_i$ to $F_j$. The joint angle vector is represented by $\theta$, a joint angle in the frame $i$ is denoted by $\theta_i$, a joint angle vector between $F_i$ and $F_j$ is represented by $\theta_{i,j} = [\theta_{i,1} \ldots \theta_{i,3} \theta_j \ldots \theta_j]$. The linear and angular velocities are denoted by $\overrightarrow{v} \in \mathbb{R}^3$ and $\overrightarrow{\omega} \in \mathbb{R}^3$, respectively. The velocity at a frame $i$ is defined by:

$$V_i = \begin{bmatrix} \overrightarrow{v}_i \\ \overrightarrow{\omega}_i \end{bmatrix}.$$  

The adjoint matrix $\Phi$ maps velocities between two frames, for instance, it maps $V_i$ to $V_{i+1}$:

$$V_{i+1} = \Phi_{i+1,i} V_i,$$  

$$\Phi_{i+1,i} = \begin{bmatrix} R_{i,i+1}^T & -R_{i,i+1}^T([\overrightarrow{p}_{i,i+1}]_i)_x \\ 0 & R_{i,i+1}^T \end{bmatrix}$$

where $R_{i,i+1} \in SO(3)$ is the orientation of $F_{i+1}$ with respect to $F_i$. $([\overrightarrow{p}_{i,i+1}]_i)_x \in \mathbb{R}^{3 \times 3}$ is the skew symmetric matrix of the distance vector, $([\overrightarrow{p}_{i,i+1}]_i)_i$ is the distance vector between frames $F_i$ and $F_{i+1}$ represented in $F_i$. The superscript $B$ means the variable is defined in the body frame, for example $V_i^B$ is the velocity in $F_i$ in the own frame $i$. 

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Figure 1: General manipulator with revolute joints.

The geometric Jacobian for a frame $F_i$ in body coordinates is defined by:

$$V_i^B = J_i^B(\theta_{1,i})\dot{\theta}_{1,i},$$

$$J_i^B(\theta_{1,i}) = \begin{bmatrix}
(\theta_1)_i \times (\mathbf{e})_i & \cdots & (\theta_{i-1})_i \times (\mathbf{e})_i & (\mathbf{e})_i
\end{bmatrix}$$

where $(\mathbf{e})_i$ is the axis of rotation of the joint $j$ in $F_i$ (without loss of generality, here, only revolute joints are considered). For any Jacobian matrix $J$, $J^T(JJ^T)^{-1}$ is the pseudo-inverse denoted by $J^\dagger$.

The pose of the manipulator is represented by $p(\theta) \in \mathbb{R}^{dim}$ while the desired pose by $p_d(t) \in \mathbb{R}^{dim}$, $dim$ is the number of dimensions used in the parameterization of pose. When considering the changing of variables over time, $t$ means the actual time and $T$ is the fixed time step of an algorithm.

2 Manipulability of constrained serial manipulators

A general formulation to obtain the constrained Jacobian and the manipulability indices of a constrained serial manipulator is presented below. A general system with $n$ revolute joints can be seen in Figure 1. The base frame $F_0$ and $F_1$ are initially located in the same place, frame $F_i$ ($i = 1, \ldots, n$) is associated to the $i$-th link. $F_k$ is the frame before the constraint, $F_{k+1}$ after the constraint and $F_c$ in the end-effector.

The velocity at $F_k$ and the joint velocity are related by:

$$V_k^B = J_k^B(\theta_{1,k})\dot{\theta}_{1,k}. \quad (1)$$

The velocity at $F_c$ and $F_k$ are related by:

$$V_c^B = \Phi_{c,k}V_k^B. \quad (2)$$

Suppose that a point $\mathbf{x} \in \mathbb{R}^{dim}$ in the kinematic chain of the manipulator is subject to a geometrical constraint where the point belongs to a surface $\mathcal{S}$. So a linear bilateral velocity constraint in $F_c$ can be defined using a matrix $H \in \mathbb{R}^{m \times 6}$ where $m$ is the dimension of the constraint, i.e.,

$$HV_c^B = 0. \quad (3)$$

Substituting (1) and (2) in (3), one has

$$H\Phi_{c,k}J_k^B(\theta_{1,k})\dot{\theta}_{1,k} = 0. \quad (4)$$

Defining

$$\wedge = H\Phi_{c,k},$$

the joint velocity vector satisfying (4) is given by:

$$\dot{\theta}_{1,k} = J_k^B(\theta_{1,k}) \wedge^\dagger u \quad (5)$$

where $\wedge^\dagger$ spans the null space of $\wedge$ and $u$ is a control degree of freedom.

The end-effector velocity is given by:

$$V_c^B = J_c^B(\theta)\dot{\theta}. \quad (6)$$

Separating $J_c^B(\theta)$, the end-effector velocity can be written as (the manipulator has a total of $n$ joints):

$$V_c^B = \begin{bmatrix} J_{c1}^B(\theta) & J_{c2}^B(\theta_{k+1,n}) \end{bmatrix} \begin{bmatrix} \dot{\theta}_{1,k} \\ \dot{\theta}_{k+1,n} \end{bmatrix}. \quad (6)$$

Replacing (5) in (6) creates:

$$v_c^B = \begin{bmatrix} J_{c1}^B(\theta) & J_{c2}^B(\theta_{k+1,n}) \end{bmatrix} \begin{bmatrix} \wedge^\dagger \dot{\theta}_{1,k} \\ \wedge^\dagger \dot{\theta}_{k+1,n} \end{bmatrix}. \quad (7)$$

In (7) $J_{c1}^B(\theta), J_{c2}^B(\theta_{k+1,n})$ only depends of $\theta_{k+1,n}$ in the condition that $J_k^B(\theta_{1,k})$ is not singular (see Coutinho, 2015). Thus $J_c^B(\theta_{k+1,n})$, called constrained Jacobian matrix, is defined as

$$J_c^B(\theta_{k+1,n}) = \begin{bmatrix} J_{c1}^B(\theta) & J_{c2}^B(\theta_{k+1,n}) \end{bmatrix} \wedge^\dagger.$$ 

A cartesian control can be considered defining:

$$\begin{bmatrix} u \\ \dot{\theta}_{k+1,n} \end{bmatrix} = J_{c1}^B(\theta_{k+1,n})u_c \quad (8)$$

where $u_c \in \mathbb{R}^{nd}$.

The manipulability is an index that represents the manipulator distance to singular configurations. For a given Jacobian matrix $J(\theta)$ a manipulability measure can be defined as:

$$w = \sqrt{\det(J(\theta)(J(\theta))^T)}.$$ 

In order to analyze the manipulability of a constrained serial manipulator two Jacobian matrices have to be taken into account, the geometric Jacobian until the joint before the constraint $(J_k^B(\theta_{1,k}))$ and the constrained Jacobian $(J_c^B(\theta_{k+1,n}))$.

The manipulability of $J_k^B(\theta_{1,k})$ is a measure of how efficiently the constrained manipulator can generate motions in $F_k$ in order to follow the desired trajectory of the end-effector:

$$w_M = \sqrt{\det(J_k^B(\theta_{1,k}))(J_k^B(\theta_{1,k}))^T}. \quad (9)$$

For the constrained Jacobian matrix $J_c^B(\theta_{k+1,n})$, which can only depend on the
restriction type and kinematics of the joints after the constraint, the manipulability indicates the possibility of generating the desired trajectory in the end-effector associated with the use of the constrained velocity vector in (8):

\[ w_C = \sqrt{\det((J^B_{rt}(\theta_{k+1,n}))^T(J^B_{rt}(\theta_{k+1,n})))}. \]  

(10)

3 Multiobjective problem formulation

There are two manipulability indices, \( w_M \) and \( w_C \), the control strategy in Section 2, control scheme in (1) to (8), that is applied in (Pham et al., 2014), does not address specifically those two indices, it strives to follow a trajectory with the constraints satisfied. In this section a multiobjective problem is proposed, the manipulator must follow the trajectory while maintaining the manipulability indices as high as possible and satisfying the constraints.

Whenever a manipulator follows a trajectory, the pose error in an instant \( t \) is the difference between the desired pose \( p_d(t) \) and the actual pose \( p(\theta(t)) \):

\[ e(t) = p_d(t) - p(\theta(t)). \]  

(11)

Considering a method that finds a solution \( \dot{\theta}^*(t) \), a joint velocity command, at a fixed step time \( T \) that drives the manipulator to bring the pose error in (11) to zero in a step time. The predicted error is:

\[ \hat{e}(t + T) = p_d(t + T) - p(\hat{\theta}(t + T)) \]  

(12)

where the predicted joint angle vector in (12) is

\[ \hat{\theta}(t + T) = \theta(t) + \dot{\theta}^*(t)T. \]  

(13)

Two objective functions are defined, \( f_1 \) and \( f_2 \). In an optimization problem we can maximize a function searching the minimum of the negative of this function, then the functions \( f_1 \) and \( f_2 \) are respectively the negative of \( w_M \) and \( w_C \) evaluated at the predicted joint angle vector in (13):

\[ f_1 = -\sqrt{\det((J^B_{rt}(\theta_{1,k}(t+T)))^T(J^B_{rt}(\theta_{1,k}(t+T)))^T)}, \]  

(14)

\[ f_2 = -\sqrt{\det((J^B_{rt}(\theta_{k+1,n}(t+T)))^T(J^B_{rt}(\theta_{k+1,n}(t+T)))^T)}. \]  

(15)

As a multiobjective problem a linear scalarization is used for functions \( f_1 \) and \( f_2 \) together with a parameter \( \alpha \in \mathbb{R} \) where \( 0 \leq \alpha \leq 1 \). For a constrained serial redundant manipulator with a constraint in a point of this chain and subject to follow a trajectory, using (14) and (15) a multiobjective problem (the solution is the joint velocity vector \( \dot{\theta}^*(t) \)) is defined as:

\[ \min \quad \alpha f_1 + (1-\alpha)f_2, \]  

(16)

\[ \text{s.t.} \quad -\delta \leq \hat{e}(t + T) \leq \delta. \]  

(17)

where \( \delta \in \mathbb{R}^+ \) is a constant, \( \theta^+ \) and \( \theta^- \) denote respectively the upper and lower joint angle limits while \( \theta^+ \) and \( \theta^- \) denote respectively the upper and lower joint velocity limits.

The decision variables of the multiobjective problem in (16) to (20) are the joint velocities \( \theta_i \). Although the decision variables are not explicit in (16), (17) and (19) the relation is defined in (13). The search method for the multiobjective problem in (16) to (20) could be any algorithm that solves nonlinear convex optimization problems. In the simulations a sequential quadratic programming (sqp) is used. The sqp solves a sequence of optimization subproblems, each subproblem optimizes a quadratic model (function of a decision variable \( \hat{\theta}_i \)) subject to a linearization of the constraints. Details about the sqp can be found in (Nocedal and Wright, 1999).

The objective function in (16) is minimized at each step of the sqp method reflecting a momentary value for \( w_c \) and \( w_M \). The inequality in (17) means that the predicted error is between a lower and an upper bound. The constraint (18), the same expression of (4), but now evaluated at \( \hat{\theta}(t+T) \) is the linear velocity constraint in the manipulator chain. The inequality constraints (19) and (20) are the manipulator physical constraints in terms of joint angle limits and joint velocity limits respectively.

In order to gather the values of \( w_M \) and \( w_C \) at each step of the sqp method in one index, the integral of the manipulability indices are taken into account: \( W_M = \int_0^T w_M dt \) and \( W_C = \int_0^T w_C dt \). So one solution \( s^* \) is defined as a pair \( W_M \) and \( W_C \) for a fixed \( \alpha \).

4 Simulations

In this section simulations are presented considering a kinematic model of the right arm in the Baxter® robot using the control scheme in (1) to (8) and the sqp method in the multiobjective problem of (16) to (20). All the simulations are performed with the software MATLAB®. Figure 2 shows a Baxter® robot, while Figure 3 shows a kinematic model of the Baxter’s right arm.

The Table 1 specifies the joint limits (angles and velocities) for the Baxter’s right arm. In order to restrict the search space in the sqp method the limits of joint velocities are set for \(-1.0 \text{ rad/s} \) to \(1.0 \text{ rad/s} \) for all \( \theta_i \), this is necessary to avoid that \( \dot{\theta}^*(t) \) go to the lower and upper physical velocity limits in consecutive steps of the sqp method.

In the end-effector trajectory only the position is considered (orientation is despised), so a
selection matrix $S$ is defined to represent only the movement in the $x$, $y$ and $z$ axes:

$$S = \begin{bmatrix} \vec{x} & \vec{y} & \vec{z} & 0_{3 \times 1} \end{bmatrix} ,$$

(21)

where $\vec{x}$, $\vec{y}$ and $\vec{z}$ are defined as canonical unitary vectors in direction of axes $x$, $y$ and $z$ respectively, i.e. $\vec{x} = [ 1 \ 0 \ 0 ]^T$, $\vec{y} = [ 0 \ 1 \ 0 ]^T$ and $\vec{z} = [ 0 \ 0 \ 1 ]^T$, $0_{3 \times 3}$ is a all zero matrix of 3 lines and 3 columns. The selection matrix in (21) premultiples $J^c_p(\theta)$, $J^c_p(\theta_{1,5})$ and $\Lambda^c \theta$.

The velocity constraint in the chain can be seen in Figure 3, between $F_4$ and $F_5$ in frame $F_c$, defined by the displacement $L = 50 \text{ mm}$. There is a plane restriction in $F_c$ meaning there is no movement on one axis, in this particular case the $x$ axis on the frame $F_c$. So the dimension of the constraint is the set $\mathbb{R}$, and the equation of the linear bilateral velocity constraint in (3) has the following vector:

$$H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} .$$

The graphics of the manipulability indices $w_M$ and $w_C$ using a plane restriction in axis $x$ can be seen respectively in Figures 4 and 5. The Figure 4 shows that a singular configuration is near in the lower limits of $\theta_2$ and $\theta_3$. Figure 5 shows that the singular configuration is reached when $\theta_6 = 0$.

The actual pose of the constrained manipulator (with orientation despised) is represented by a three element vector in this order, position in axes $x$, $y$ and $z$ (that can be found by the forward kinematics):

$$p(\theta(t)) = \begin{bmatrix} p_x(\theta(t)) \\ p_y(\theta(t)) \\ p_z(\theta(t)) \end{bmatrix} .$$

The cartesian control signal, $u_c$, used in the control scheme is a proportional plus feed-forward controller in the task space:

$$u_c(t) = K_p c(t) + \tilde{p}_d(t).$$

The manipulator must track the desired trajectory:

$$p_d(\theta(t)) = \begin{bmatrix} p_x(0) + 15 \sin(\omega t) \\ p_y(0) + 66 \cos(2\omega t) - 66 \\ p_z(0) + 30 \sin(2\omega t) \end{bmatrix} \text{ mm},$$

(22)

where $p_x(0)$, $p_y(0)$ and $p_z(0)$ are respectively the initial positions in axes $x$, $y$ and $z$ and $\omega = 2\pi/40 \text{ rad s}^{-1}$ is the frequency. A sketch of the desired trajectory is in Figure 6, the joint angle initial state is $\dot{\theta}(0) = [-0 \ -\pi/6 \ \pi/2 \ \pi/4 \ -\pi/3 \ \pi/4 \ 0]^T \text{ rad},$
the initial position is $p_x(0) = 762 \, \text{mm}$, $p_y(0) = 548 \, \text{mm}$ and $p_z(0) = 462 \, \text{mm}$, the task duration is $40 \, \text{s}$. In despite of follow the trajectory the manipulator has still to respect the constraint defined in (4).

Using the sqp method and the multiobjective problem in (16) to (20) a set of solutions $s^*$ can be generated for $\alpha = [0 \, 0.01 \, \cdots \, 0.99 \, 1]$. The set of solutions, pair of the integrals of manipulability indices, are classified in dominated and non dominated. A dominated solution means that there is at least one solution better in all objectives (maximize the values of $W_C$ and $W_M$ for the article) than this solution. A non dominated solution means that there is no solution better in all objectives than this solution. The solution plot is in Figure 7, for the 101 points 27 are non dominated.

The Figures 8, 9 and 10 show, respectively the variation of the manipulability indices, velocity in the constraint and trajectory error over time. The choices for $\alpha$ are 0, 0.77 and 0.9. There are also graphics for the control scheme in (1) to (8).

In Figure 8 can be noted that for a low $\alpha$ (close to 0) $W_C$ is maximized ($W_M$ is minimized) and for a high $\alpha$ (close to 1) $W_M$ is maximized ($W_C$ is minimized). In the control scheme the solution pair is $W_C = 196.781$ (the best one) and $W_M = 0.156$, if this solution is included in Figure 7 it will lead to only 6 non dominated solutions.

Figure 9 shows that for $\alpha = 0.77$ the velocity constraint was violated and the other solutions achieve a velocity near to zero. The Figure 10 shows that the four simulations represented achieve a good error performance, $e(t) \leq [2.0] \, \text{mm}$.

5 Experiments

In this section a experiment with the Baxter robot is presented. The experiment is implemented with the ROS (Robot Operating System) framework and the programming language Python.

The Figure 11 reflects the experiment using the control scheme in (1) to (8). In the manipulator the trajectory error. The legends are omitted for better visualization: $x$ axis blue; $y$ axis green; $z$ axis red.
Figure 11: Results in experiment with Baxter. In the first graphic the manipulability indices ($w_M$ multiplied by $10^3$), the second graphic is the velocity in the constraint and the third graphic the trajectory error.

Manipulability graphic, the result values $W_C = 191.595$ and $W_M = 0.165$ are very close to simulation using the control scheme and if this solution pair was in Figure 7 it will be non dominated. The velocity graphic shows the velocity in the constraint is nearly zero for all the experiment. The trajectory error is almost all time $e(t) \leq [5.0] \text{ mm}$ with the exception the $z$ axis for a brief period of time as can be seen in the third graphic. It was not possible to implement effectiveness the the sqp method with the multiobjective problem in (16) to (20) in the Baxter® robot because the convergence time of the sqp method varies between 500 ms and 800 ms, that is, several times the value of the frequency $\omega$ in (22). For comparison purposes in simulations the convergence time frequently varies between 20 ms and 40 ms.

6 Conclusions

Formulate a multiobjective problem using an optimization method is an alternative solution to the kinematic control. The advantage is the freedom to choose a specific index and explicit maximize or minimize it without using the null space of the Jacobian. The disadvantage is the convergence time related to the constraints in the formulation of the optimization problem, thus a smaller tolerance means a longer convergence time.

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References


