Abstract—This paper is about a description and implementation of a relay switched by output limits defined by an user that excites the system for some defined conditions. After the end of the experiment, the strategy makes the process tend to a condition similar to the initial. The generated data can be used for parameter identification for ITD and IFOTD models. The strategy and experimental identification are demonstrated in laboratory scale thermal plant based on Peltier effect.

Keywords—Relay by Limits, System Identification, Integrating Systems, Didactic Plant

1 Introduction

Integrating time delayed (ITD) and integrating plus first order time delayed (IFOTD) processes are common model representation for low order systems transfer functions as water flow tanks (Roffel and Betlem, 2007), DC motors rotor position (Richard C. Dorf, 2010), drum-boiler (Åström and Bell, 2000) and others control engineering applications. The main characteristic of those processes are the instability, for open-loop tests. They do not have any equilibrium points and their outputs tend to infinity, since there is a pole at origin.

Hence, relay feedback methods are used to identify those systems because they avoid the divergence by making the process to oscillate in certain conditions. Many researches have developed studies for this strategy using an adaptation of (Åström and Hägglund, 1984) for parameters estimation and autotuning as (Liu and Gao, 2008), (Panda et al., 2011) for symmetric (Liu et al., 2013) or asymmetric relay parameters (Berner et al., 2014). These techniques use the hysteresis and noise levels to control relay switching, however, measuring or estimating them are not precise, because the stable operating points absence. Therefore, the obtained parametric models could have poor fitting.

Laboratory scale process have been used as an essential tool in academic studies, helping students to understand industrial applications and close the gap between theory and practice. Hence, didactic plants have become popular in most of the universities to demonstrate control loops operations with common industrial equipment, norms and protocols aiming to achieve the necessary learning objectives (Feisel and Rosa, 2005).

In this paper, it is proposed a new approach for estimating parameters of ITD and IFOTD systems from data resulted in an experiment using a relay feedback switched according to defined output limits. Thus, hysteresis and noise levels are not considered for input selection, only operational limits set by the user. The parameters estimation is done by data analysis in frequency domain. As integrating process are unstable, after the end of the experiment, they do not naturally return to initial conditions or converge to an operational point. Hence, it is proposed a input calculation that generates a pulse compensating the initial transitory effects, bringing the output to initial conditions. This strategy is tested in a laboratory scale thermal plant based on Peltier effect and two experimental models are obtained.

This paper is organized as follows: The experiment design is described in Section 2, the parameters estimation techniques used for ITD and IFOTD systems are explained in Section 3. Experimental results are presented and evaluated in Section 4 and the conclusions are discussed in Section 5.

2 Experiment Design

As many methods based on relay, this approach aims a simple and fast experiment with a little prior knowledge from the desired system, in order to obtain a low order parametric model. The only information required for the operator are the number of relay periods, input and output limits (lower and upper). Those respective variables are listed in Table 1.

The experiment design can be divided in two parts: System Excitation and Returning to Similar Initial Conditions. The first is based in a relay switching algorithm and the second uses the collected information from the previous part for the process return to the operation condition similar to the initial.
Table 1: Experiment Parameters

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relay Periods</td>
<td>Periods</td>
</tr>
<tr>
<td>Input Upper</td>
<td>InputUpper</td>
</tr>
<tr>
<td>Input Lower</td>
<td>InputLower</td>
</tr>
<tr>
<td>Output Upper Limit</td>
<td>OutputUpper</td>
</tr>
<tr>
<td>Output Lower Limit</td>
<td>OutputLower</td>
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</table>

2.1 System Excitation

After the input parameters were defined, a relay feedback is implemented in the process, as shown in the Figure 1, where $y(t)$ and $u(t)$ are the measured output and input respectively.

![Figure 1: Relay feedback schematic](image)

A switching algorithm was developed by adapting the relay switching equations in (Berner, 2015), where the hysteresis value was replaced by the output limits. The input will be controlled by the measured output ($y(t)$), the previous applied input ($u_{prev}$) and the parameters listed in Table 1 according to switching algorithm. This is shown in flowchart for a process with direct gain in Figure 2, where $J$ is a relay periods counter and it is verified to end the experiment.

The conditions 1 and 4 are the principle of the relay switched by limits, the current output is compared with the defined limits and if one of this is violated, the input receives the value that makes the process goes to an opposite behavior. Conditions 2 and 3 are necessary to avoid that noise influence in the system results in a wrong switching, improving the algorithm robustness. This is done by an additional comparison with $u_{prev}$. If all of those conditions are not valid, this means that the measured output has value between the defined limits, then a pulse needs to be generated for an initial excitation. Hence, the algorithm sets the input as $InputLower$ for direct gain or $InputUpper$ for reverse.

An illustrative example of relay algorithm behavior for two periods applied in process with direct gain is shown in Figure 3.

2.2 Returning to Similar Initial Conditions

As the integral processes are unstable, after the end of the System Excitation, the process naturally would increase or decrease to infinity, this
could cause immeasurable problems in industrial plants as accidents. Hence, it is proposed a strategy to generate a pulse with amplitude $Amp$ and time duration $D$ that forces the process to tend for a condition similar to the initial, even if the excited system is nonlinear.

The first step is to calculate the pulses area ratio $\alpha$ in the penultimate period:

$$\alpha = \left| \frac{A_{\text{lower}}}{A_{\text{upper}}} \right|$$  \hspace{1cm} (1)

Where $A_{\text{upper}}$ and $A_{\text{lower}}$ are the pulses area for the respective amplitudes and time intervals. If $\alpha$ is close to 1, the system can be considered linear, otherwise, it has nonlinear behavior. Other periods can be used to calculate $\alpha$, however, it is recommended to avoid using the first period because of the transients effects.

Analyzing the pulses areas sum ratio, it is possible to notice that:

$$\frac{\Sigma A_{\text{lower}}}{\Sigma A_{\text{upper}}} < \alpha$$  \hspace{1cm} (2)

Where $\Sigma A_{\text{upper}}$ and $\Sigma A_{\text{lower}}$ are the areas sum for the respective pulses.

Hence, the generated pulse area can be interpreted as an area increment in $\Sigma A_{\text{upper}}$ that maintains the areas sum ratio equal to the pulses area, $\alpha$, for one determined pulse:

$$\alpha = \frac{\Sigma A_{\text{lower}}}{\Sigma A_{\text{upper}} + \Delta A}$$  \hspace{1cm} (3)

As the generated pulse area is:

$$\Delta A = Amp \times D$$  \hspace{1cm} (4)

The equation 3 can be expanded to:

$$\alpha = \frac{\Sigma A_{\text{lower}}}{\Sigma A_{\text{upper}} + Amp \times D}$$  \hspace{1cm} (5)

Then, the time period $D$ can be calculated by:

$$D = \frac{(\Sigma A_{\text{lower}})/\alpha - \Sigma A_{\text{upper}}}{Amp}$$  \hspace{1cm} (6)

As equation 6 can have negative results and time period are positive:

$$D = \left| \frac{(\Sigma A_{\text{lower}})/\alpha - \Sigma A_{\text{upper}}}{Amp} \right|$$  \hspace{1cm} (7)

It is defined that the $Amp$ parameter has value equal to the last input amplitude generated by the System Excitation algorithm.

This strategy does not lead to the exactly initial condition because of numerical approximations related to the sampling time. Usually, the generated pulse duration $D$ is not a multiple of sampling time. Therefore, it is not possible to perform exactly the calculated value. However, this condition difference generates a DC value in frequency domain, that is essential to parameter estimation.

### 3 Parameters Estimation

The resulted data from the Experiment Design can be used for an offline linear parameter system estimation for ITD and IFOTD models.

#### 3.1 ITD

Based on the data generated from one relay period in the experiment, it is possible to estimate the gain $K$ and delay $\tau$ for ITD process:

$$P(s) = \frac{K}{s} e^{-s\tau}$$  \hspace{1cm} (8)

Applying the natural logarithm in equation 8:

$$\ln P(s) = \ln \left( \frac{K}{s} e^{-s\tau} \right)$$  \hspace{1cm} (9)

Using multiplication, quotient and power logarithm proprieties, it is possible to simplify equation 9 for:

$$\ln P(s) = \ln K - \ln s - s\tau$$  \hspace{1cm} (10)

Changing the Laplace domain to frequency, using $s \rightarrow j\omega$:

$$\ln P(j\omega) = \ln K - \ln(|\omega| + j\omega\tau) - j\omega\tau$$  \hspace{1cm} (11)

Expanding $\ln(j\omega)$:

$$\ln P(j\omega) = \ln K - (\ln(|\omega| + j\omega\tau)) - j\omega\tau$$  \hspace{1cm} (12)

$$\ln P(j\omega) = \ln K - \left( \ln(|\omega| + j\frac{\pi}{2}) - j\omega\tau \right)$$  \hspace{1cm} (13)

$$\ln P(j\omega) = (\ln K - \ln(|\omega|) + j \left( -\frac{\pi}{2} - \omega\tau \right))$$  \hspace{1cm} (14)

Then, the equation 14 real and imaginary part are:
\[ \Re \{ \ln P(j\omega) \} = \ln K - \ln |\omega| \quad (15) \]
\[ \Im \{ \ln P(j\omega) \} = -\frac{\pi}{2} - \omega \tau \quad (16) \]

The equations 15 and 16 are essential results that are used for the delay and gain estimation, discussed in next sections.

### 3.1.1 Delay

It is possible to notice that equation 16 is an affine function that can be decomposed in two vectors:

\[ \Im \{ \ln P(j\omega) \} = \left[ -\frac{\pi}{2} - \omega \right] \begin{bmatrix} 1 \\ \tau \end{bmatrix} = X \times b \quad (17) \]

Where \( X \) and \( b \) are the regressors and parameters vectors respectively.

Therefore, \( b \) can be estimated from Least Square Estimation (LSE) from \( \Im \{ \ln P(j\omega) \} \) and \( \omega \) measurements. In this approach, it is used only two data points to perform the LSE: \((\pi/2, 0)\) and \(\Im \{ \ln P(\omega_{relay}) \} / \omega_{relay}\). Those were chosen to simplify calculations and avoid undesired frequency components that could leave the system to wrongs results. The first data point is a theoretical result from the equation 16. To obtain the second data point, it is necessary to calculate the input signal frequency:

\[ \omega_{relay} = \frac{2\pi}{L \times h} \quad (18) \]

Where \( L \) is the data length collected in one period and \( h \) is the sampling time. Moreover, it is necessary to estimate the frequency response \( \hat{P}(\omega) \) by the Fourier transform from the input and output time domain data:

\[ \hat{P}(\omega) = \frac{\Im \{ u(t) \}}{\Re \{ u(t) \}} \quad (19) \]

Usually, frequency point calculation from time domain vectors is done by a Fast Fourier Transform (FFT) analysis, that obtain an good spectrum estimation. However, there is not any guaranty that a desired frequency point will be on FFT generated vector. As the only searched frequency is \( \omega_{relay} \), Goertzel algorithm is used to calculate the process response \( \hat{P}(\omega_{relay}) \) and the imaginary part of it is used.

Therefore, equation 17 can be modified to:

\[ \begin{bmatrix} \Im \{ \ln \hat{P}(0) \} \\ \Im \{ \ln \hat{P}(\omega_{relay}) \} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & -\omega_{relay} \end{bmatrix} \begin{bmatrix} -\frac{\pi}{2} \\ \tau \end{bmatrix} \]

\[ a = X \times b \quad (20) \]

Hence, the parameter vector \( b \) is obtained from:

\[ b = (X^T X)^{-1} X^T a \quad (22) \]

The delay \( \tau \) is the last element of \( b \).

### 3.1.2 Gain

The gain \( K \) is estimated by arranging the equation 15:

\[ K = e^{\Re \{ \ln P(j\omega_{relay}) \} + \Im \{ \omega_{relay} \}} \quad (23) \]

Where the term \( \Re \{ \ln P(j\omega_{relay}) \} \) is obtained using as the imaginary part.

### 3.2 IFOTD

Based on the data generated from all experiment, it is possible to estimate the gain \( K \), delay \( \tau \) and the pole \( L \) for IFOTD process:

\[ G(s) = \frac{K}{s(Ls + 1)} e^{-s \tau} \quad (24) \]

As the transfer function \( G(s) \) is the ratio between output and input data in Laplace domain, it is possible to organize the equation 24 to:

\[ G(s) = \frac{Y(s)}{U(s)} = \frac{K}{s(Ls + 1)} e^{-s \tau} = G_d(s) \quad (25) \]

\[ \frac{sY(s)}{U(s)/s} = \frac{K}{(Ls + 1)} e^{-s \tau} = G_d(s) \quad (26) \]

Then by deriving the output time response or integrating the generated input, it is possible to estimate the parameters using First Order Time Delay (FOTD) system techniques described in (Åström and Hägglund, 2006), using the equations:

\[ K = G_d(0) \quad (28) \]

\[ k = \frac{|G_d(\omega_{180})|}{G_d(0)} \quad (29) \]

\[ \tau = \frac{1}{\omega_{180}}(\pi - \arctan(k^{-2}) - 1) \quad (30) \]

\[ L = \frac{1}{\omega_{180}}(\sqrt{k^{-2} - 1}) \quad (31) \]

Where \( \omega_{180} \) is the frequency with phase equal to 180°. For systems with a high noise level, the gain \( K \) can be set as the same of ITD approach.

### 4 Experimental Tests

Aiming to test the developed techniques in real process, the presented strategy was applied to identify ITD and IFOTD systems models in a laboratory scale thermal plant developed for control and automation studies.
4.1 Plant Description

This didactic module is composed by two Peltier elements (1 and 2) with independent activation by H bridge circuits in a thermally coupled arrange. Each Peltier has a face coupled to a heat sink of a cooler, in order to improve the heat exchange with the environment. The other face is thermally coupled to a cylindrical piece of aluminum, composed of 3 disks with different diameters and two temperature sensors LM35. The complete module schematic is shown in Figure 4.

An arduino is used as microcontroller to read the temperature measurements and generate and PWM signal to the Peltier elements. The communication between the PC and arduino is based in OPC (OLE for process control) architecture client. Further plant details can be found at (Lima et al., 2014) and (Lima et al., 2015).

Based on the input and outputs available in the didactic module, it was chosen as input the PWM duty cycle (%) applied in Peltier 1 and the measured at Peltier 2 as output ($^\circ$C). Even, the theoretical modeling proves that this is a first order system, it behaves as a integrating model for small output deviations. The experiment design was implemented in a MATLAB® routine that communicates with the process by an OPC client.

4.2 Results

Initially the parameters required for the System Excitation algorithm were defined and listed in Table 2 according to plant’s operational limits and operator previous knowledge to avoid a nonlinear output behavior. Hence, a constant input of 50% was applied in the system until a certain temperature was reached between the defined output limits.

Then, the proposed strategy was applied in the system with time sampling as 1 second. The input and output results are shown in Figure 5.

Using the ITD parameter estimation technique using data from the penultimate period for calculation, resulted in the experimental model:

$$\hat{P}(s) = \frac{0.001177}{s}e^{-19.8s}$$

As the measured output has a high noise level, the IFOTD parameter estimation used the integration technique and the gain calculated in ITD process. This resulted in the model:

$$\hat{G}(s) = \frac{0.001177}{s(2.791s + 1)}e^{-13.4s}$$

Both estimated models were compared with the collected data using the normalized mean square error criteria (NRMSE), described by (34), where $y$ is real output value, $\hat{y}$ model output and $\text{mean}(y)$ the output mean. The experimental system models resulted in 82.72% and 80.59% fit for ITD and IFOTD estimated transfer functions respectively. The comparison between the real and the simulated data is illustrated in Figure 6.

$$NRMSE = 100 \left(1 - \frac{||y - \hat{y}||}{||y - \text{mean}(y)||}\right)$$

5 Conclusions

A relay switched by output limits algorithm is proposed to excite and make the process return for similar initial conditions for integrating system identification. This strategy is more robust than others because it considers the past data to select the next input value. After the end of the system excitation, a last pulse is generated to tend the system to a similar condition, returning the process in a safety output value between the defined

![Figure 4: Peltier didactic module schematic](image)

![Figure 5: Process input and output for the experiment](image)
output limits, avoiding any operational problems. Then, it is possible to obtain ITD and IFOTD experimental models by an frequency domain data. This was applied in a laboratory scale Peltier module.

Analyzing the results from the experiment, it is possible to excite an integrating system to generate satisfactory low order integrating models from this strategy to unstable real process with little prior knowledge from the operator. Besides that, the process returned to a condition similar to the initial from a generated pulse by the Returning to Similar Initial Conditions strategy.

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References


